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Elaboration of Real Options Model and the Adequacy of Volatility

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Abstract When evaluating the economic value of technology or business project, we need to consider the period and cost for commercialization. Since the discounted cash flow (DCF) method has limitations in that it can not consider consecutive investment or does not reflect the probabilistic property of commercialization cost, we often take it desirable to apply the concept of real options with key metrics of underlying asset value, commercialization cost, and volatility, while regarding the value of technology and investment as the opportunity value. We at this moment provide more elaborated real options model with the effective region of volatility, which reflects the uncertainty in the option pricing model (OPM).

Keywords Technology valuation, real options method, black-scholes model, volatility, propriety of technology investment

I. Introduction

In recent years, investment in technology venture start-ups has been increasing, and technology credit assessment has been further playing an important role. However, the technology credit-rating mechanism widely used by financial institutions failed to systematically reflect the profitability of the targeted technology. Hence the applications of technology valuation, which has been widely utilized as a reference for negotiating technology transactions, bank security or technology surety, is expanding rapidly. This is important to business angels, venture capital (VC) and private equity companies as they rely on information on the profitability of the target technology, or the portion of technology share out of the total asset, in the creation of a technology venture.

Besides general technologies in manufacturing or the service industry, there are often situations where it is necessary to consider the timing, and the required

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cost of commercialization for future investments as the value of the technology is assessed in fields such as biotechnology, pharmaceuticals and medical. The existing discounted cash flow (DCF) method has several limitations in that it can not consider consecutive investment over a certain time span or does not reflect the input cost for the commercialization of technology-applied products. Thus, it is desirable to apply the concept of real options because the value of technology or investment should be regarded as the opportunity value, and the decision-making information for resource allocation should be taken into consideration.

When the valuation of the enterprise value (EV) of a firm is carried out, there are cases where we utilize the variance of 'initial call value in the following day to the final call value in the other day' of the stock price by introducing the concept of 'volatility.' To apply the concept for the valuation of technology, both 'the continuity of stock price (i.e. relatively a minute change)' and 'positivity condition of the variance' need to be guaranteed. In many cases, we might be unable to deduct the volatility of cash flows of the technology business over a short period of two to three years of sales estimation, unlike that of stock price.

As discussed in most of the literature, there is the necessity to investigate the relationship between the value of the underlying asset, the commercialization cost and the volatility in the Black-Scholes model for estimating the value of technology based on real options. The study proposes a more elaborated real options-based technology valuation model for a wide variety of usages such as technology transfer, business feasibility analysis, among others. It does so by mathematically assessing whether the present value of the underlying asset and the commercialization cost reflecting the uncertainty in the option pricing model (OPM) are identified or not into the "no action taken" (NAT) area under critical condition. The study then develops and presents the calculation logic of the option value of a technology in accordance with the observation variable (or input value).

The research is organized as follows. In Section II, we explain the reason for introducing the concept of real options and the theoretical background for measuring the volatility in the Black-Scholes model. In Section III, we describe the research methodology to ascertain whether there exists an effective region of volatility when applying the Black-Schoes model into the valuation of technology. In Section IV, we present the findings that include the critical ratio of 'the commercialization cost to underlying asset' for the calculation of the option value, the effective region of volatility and the way to determine the option value of technology for a specific business model. Section V summarizes the findings and lay out implications.

II. Literature Review and Theoretical Backgrounds

1. Technology Valuation by DCF and Real Options Method

In general, the types of approach in technology valuation are divided into income approach, market approach, and cost approach. The cost approach is the mechanism of estimating the minimum level of technology value by taking into account the cost and margin put into the development of similar or identical technology, based on the principle of substitution. However, it is not recommended for other uses like decision-making based on future profitability.

The market approach is an unreliable source to refer to statistical data of limited information if the active market for technologies does not exist or if there is only a small number of transactions in the field of the technology considered. Lastly, in the income approach, the discounted cash flow (DCF) method can quantify the value of technology based on the short-term profitability of intangible assets. However, since the DCF method has a limitation in presenting supplementary information associated with the specificity of the business model or the future uncertainty in deriving the optimal decision-making, the real options method has been proposed as a feasible alternative.

Seol and Yoo (2002) identified the limitations of the DCF method in the valuation of technology or its relevant investment business and proposed the concept of technology-based real options. Myers (1987) and Kester (1984) had already pointed out that the most notable challenge of the DCF method is that it does not consider consecutive investment and does not reflect the uncertainty inherent in the project such as business fluctuations, changes of price indices, etc. Myers (1987) recognized that R&D activities and outcomes themselves are composed of option values, and Nichols (1994) found out that, since the utilization of options-based technology valuation enables the quantitative evaluation of consecutive, multistage projects, it would be used more flexibly in simulation techniques as in sensitivity analysis, Monte Carlo simulation, etc.

Sung (2002) conducted an empirical study to demonstrate the results of the adequacy of technology value of multi-phased technology investment using multiple options. Park et al. (2009) described a practical hybrid model, which consists of the former part of decision-making tree over an early-stage project accompanying certain risk factors, followed by the post-stage binomial lattice where market risk factors exist. In order to assess the value of a two-staged R&D investment business mutually correlated, Geske (1979), Geske and Johnson (1984), and Buraschi and Dumas (2001) put forward the concept of ‘dual option' and performed a linear regression analysis of the factors, which have individual influence on the dual option.

According to Benaroch and Kauffman (1999, 2000), the DCF method does not take into account the potential opportunity value due to the investment committed, and thus they proposed a real options method as an alternative. Luehrman (1998) compared real options value with the net present value of future cash flows to assess the value of an investment inherent with uncertainty. Panayi and Trigerogis (1998) attempt to assess the economic valuation of a long-distance telecommunication project using the concept of complex options.

Later in Korea, Kim, D.H. (2003) Applied the real options method to measure the enterprise value of a venture firm based on five variables including market capitalization, strike price, volatility, exercise period and risk-free interest rate. Kim and Yoon (2014) estimated the option value of a technology assuming that there exists uncertainty in technology transactions between technology suppliers and technology demanders. They completed the calculation by considering both the cost approach-based value on the side of technology suppliers and the real options-based value on the side of technology demanders.

Sung et al. (2013) undertook a case study about how the variables in Black-Scholes model influence the ultimate technology values of promising technologies that small and medium sized enterprises (SMEs) carried forward. Sung et al. (2017) implemented the calculation logic of real options-based technology valuation directly into a web-based valuation system, called as ‘STAR-Value system’.

Until recently, the body of research about the DCF method and real options method has been progressing steadily, but today research is needed to examine the potential practical applicability of the real options method instead of the DCF method, which has limitations as regards specific business models, the strategy of business entity, and future uncertainty with cash flow fluctuations.

2. Black-Scholes Model and Volatility

It is well accepted that the typical models of real options include the Black-Scholes model and the binomial model, as well as the dynamic DCF, options-reflected DCF, and options tree models. Black and Scholes (1973) first proposed the Black-Scholes model based on financial options that can be most widely used assuming continuity in decision-making. Later, Brennan and Schwartz (1985) expanded it to real options-based valuation. Since then, Santos (1991), Grenadier and Weiss (1997), McGrath (1997), Heo (2000), and Seol and Yoo (2002) proposed the applicability of the Black-Scholes model to overcome the limitation of net present value about technology or new investment business.

Lee et al. (2004), Lee and Jeong (2011), and Chang and Lee (2014) conducted some empirical studies on the Black-Scholes model to use it in uncertaintyreflected decision-making about investment project related to technology asset. They applied the research findings to practical cases such as neural network-based optimization with financial data, R&D evaluation of pharmaceutical technologies, and the quantitative valuation of the production process of polysilicon (the core material of solar cell).

Chakravarthy (1997) defined 'strategic flexibility value' as the competence in CEO's strategy to create new added value by responding with sensitivity to future changes in market and technology. Huchzermeier and Loch (1999) maintained that the concept of strategic flexibility value has a linearity with the volatility associated with the project, i.e., a number of 'volatility' concepts of market demands, achievement, and effective duration, while making the point that the linearity enables optional decision-making such as expansion, reduction, abandonment and conversion of the subject project. Dahlberg and Porter (2000) suggested that real options value based on Black-Schole model needs to be considered under the situation where the stability of future growth patterns is not guaranteed.

Park et al. (2009) proposed to estimate the volatility (σ2) for the business model and business entity in real options value by the volatility of stock prices. Kim et al. (2013) suggest evaluating the option values of 20 technologies by using the volatility of profitability in each industry classification. In the past, Razgatis (1999) once mentioned a volatility of 30% when a business entity enters a new market where the existence of a real market is guaranteed for a specific technology.

According to Mun (2002), in assessing the value of investment business, one of the most important factors in applying the Black-Scholes model is the measure of volatility, which can be estimated by various indices of natural logarithmic-based profitability, historical volatility, Monte Carlo simulation volatility, and substitution in the market.

III. Research Methodology

When the real options method is applied to assess the value of a new technology or investment project, there is a chance of little significance of real options-based value from directly reflecting the variance under the condition that the revenue period is relatively short, i.e., 2 to 3 years, or the degree of fluctuation in cash flows is too rapid.

To cope with the issue above, we look at how to define 'volatility' in the Black-Scholes model and how to determine the region of effective volatility that can be used for decision-making by mirroring future uncertainty in option value.

The Black-Scholes model has been systematized as the option-pricing model (OPM) by reflecting the volatility of stock prices and the principle of 'hedge' as regards the problem of determining the price of financial options, and is widely being used up to now. (Sung, 2005). The theory of the Black-Scholes model is derived based on the following assumptions:

- Stock prices are compliant with the geometric Brownian motion.
- Short-term interest rates are known and remain constant over the effective period of options.
- There is no intermediate dividend payment of stock as an underlying asset.
- It assumes the complete market without transactions cost, taxes and short-term selling restrictions.
- The volatility of 'rate-of-return' of underlying assets does not change over the options period.

The uncertainty of the underlying asset is calculated from the equations (i.e. Equation (1) to (2)) expressed in the form of Gauss Wiener process, and by the assumptions above regarding the OPM, the options value (i.e. call options) is ultimately obtained as in Equation (3).

$$dS_t = aS_t dt + \sigma S_t dz \quad (1)$$

$$d\text{In}S_t = \left[a - \frac{1}{2}\sigma^2\right] dt + \sigma \cdot dz \quad (2)$$

where S_t : present value of underlying assets (price of financial options)
a : growth rate of underlying assets
σ : volatility of underlying assets (i.e. squared root of variance)
dz : increment in Gauss Wiener Process

$$\text{V} = \text{S} \cdot (d_1) - Xe^{-rT} N(d_2) \quad (3)$$

$$d_1 = \frac{In\left(\frac{S}{X}\right) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \quad (4)$$

$$d_2 = \frac{In\left(\frac{S}{X}\right) + \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = d_1 - \sqrt[\sigma]{T} \quad (5)$$

where V : options value ((ultimate) call option value in financial options)
S : present value of underlying assets (PV in financial options)
X : required cost for commercialization (strike price)
T: required time for commercialization (residual period to maturity)

r : risk-free interest rate
σ^2: volatility of underlying assets (variance of fluctuations)
$\mathrm{N}(\cdot)$: cumulative normal distribution

In Equation (3), we mean $\mathrm{S} \cdot N(d_1)$ by the expectation of options value when the investment value at maturity is greater than the required cost for commercialization, and $N(d_2)$ by the risk neutral probability in which the investment value at maturity is expected to be greater than the required cost for commercialization. Here Xe^{-rT} also means the present value of the cost for commercialization, where the term e^{-rT} is the element that gives the commercialization cost its present value and thus reduces the amount of present value according to the combination of r and T. In addition, the options value V is an incremental function for $\frac{S}{X}$, rT, and $\sqrt[\sigma]{T}$, and in order for the real options to have an intrinsic value, i.e. in-the-money (ITM), the value of underlying assets S must be greater than or equal to the cost for commercialization X.

According to Mun et al. (2002), volatility can be measured in a variety of methods, but it can be obtained from natural logarithmic cash flows method and Monte Carlo simulation method as follows.

1. Volatility by Natural Logarithmic Cash Flows Method

This is a method of calculating volatility directly from the logarithm of cash flows of the following year versus cash flows of the previous year, similar to the method of calculating the stock return.

$$\sigma^2 = \frac{1}{n-1}\sum_{t=1}^{n}(In\left(\frac{CF_t}{CF_{t-1}}\right) - \overline{R_{CF_{n-1}}})^2 \qquad (6)$$

where F_t : cash flow at t-th year

$\overline{R_{CF_{n-1}}}$: mean value over $(n-1)$ distinct values of $In\left(\frac{CF_t}{CF_{t-1}}\right)$

The natural logarithm cash flows method does not need to additionally perform Monte Carlo simulations, and it is specially a recommendable way to measure volatility for current financial assets with plenty of time-series data. In case of fluctuations in a certain amplitude, that is, in stocks (or financial assets) for which continuous variation is anticipated, there is such abundant data in "initial prices in the following day relative to closing prices in the previous day" that we make it easy to measure volatility either monthly or annually.

However, in case of a technology or investment project with a duration of less than 30 years, which is applied to technology valuation, if the number of samples is too small and cash flows has a chance to have negative values, we cannot take the natural logarithm of the ratio (i.e. values in the following day to those in the previous day), so it is difficult to apply in practice.

As mentioned above, unless cash flows generated by a technology business in one sector compensate one in another sector, as in case of a start-up that just launched a new business, then we often encounter negative cash flows at early stages of the business, which would be inappropriate. Therefore, by making use of financial ratios of the relevant industry or corporations, volatility can be estimated from the proxy based on pre-tax operating profit (or earnings before interest and taxes (EBIT)), but we should examine thoroughly the formula for options value (i.e. Equation (3)) in Black-Scholes model whether it can be considered together with the present value of underlying assets.

2. Volatility by Monte Carlo Simulations Method

This is a method of calculating volatility by conducting Monte Carlo simulations regarding the base-year sum of net present values for cash flows (PV_0) and the first year sum of present values of cash flows (PV_1). It can be measured from the parameter $Y = In\left(\frac{PV_1}{PV_2}\right)$ as follows.

$$Y = In\left(\frac{PV_1}{PV_0}\right) = In\left[\frac{\sum_{t=1}^{n}\frac{CF_t}{(1+r)^{t-1}}}{\sum_{t=0}^{n}\frac{CF_t}{(1+r)^{t}}}\right] \quad (7)$$

where PV_0 : sum at base year over present values of all cash flows
PV_1 : sum at first year over present values of all cash flows
r : risk-adjusted discount rate

The advantage of this method lies in that it can be applied to most cases only if the integrated values of both PV_0 and PV_1 have positive values, although yearly cash flows have a negative value in a specific year. When all CF_i's (i.e. CF_0 to CF_n) are provided for a practical case, we are able to repeatedly generate distinct CF_i's under uniform distribution at each iteration of Monte Carlo simulations where the lower and upper bound are determined by $\min(CF_i)$, and $\max(CF_i)$, respectively, and estimate volatility from the variance of $Y = In\left(\frac{PV_1}{PV_2}\right)$ according to the number of iterations for Monte Carlo simulations.

3. Effective Region of Volatility for Estimating the Options Value

As described in the previous section, it is not always possible to estimate an options value regardless of the effective region of volatility. In order to resolve the challenge based on the decision-making principle of financial options, it is necessary to consider the thresholds of the area where the present value of underlying assets (considering growth rate) is less than that of the cost for commercialization from the risk neutral probability.

When present value for underlying assets (S), commercialization cost (X), risk-free interest rate (r) and time for commercialization (T) are all known at their specific values, the region for no action taken (NAT) which corresponds to 'zero' options value can be calculated beyond the ratio of commercialization cost to present value for underlying assets (i.e. $\mathrm{R} = \frac{S}{X}$).

In the 'real options method' logic implemented in web-based valuation system (named as 'STAR-Value 5.0 PLUS'), for the negative value of volatility (σ^2) the algorithm embedded recursively performs the step of cash flows calculation so that the options value returns no less than or equal to zero.

In next the section, the realistic timespan for commercialization is considered from zero to 15 years, which is widely known as the time necessary from pre-clinical trial to clinical approval in biotechnology and pharmaceuticals, with the risk-free interest rate of 5% and a maximum of 30 years of revenue period.

For specific value of R, options value is not always valid for all values of volatility (σ^2). Therefore, we need to explore the region below (R,σ^2)-curve where options value is zero such that present value of underlying assets (considering growth rate) is less than that of commercialization cost (based on the risk-neutral probability).

IV. Research Results

1. Critical Ratio of Commercialization Cost

In order to observe the region of options abandonment, previously defined as 'no action taken' (NAT), according to the ratio of cost for commercialization to present value of underlying assets (R) and volatility (σ^2), we will look into general cases which have high frequencies of occurrence in practice.

Considering that risk-free interest rate has been varying from 1.3% to 3% in 2016, and the required time for commercialization was on average 1.9 to 2.3 years from 40 actual projects conducted by Korea Institute of Science and Technology Information (KISTI) in 2015 and 2016, we obtain parametric numbers of (r, T) = (2.65%, 2.1 years). However, in the study we focus on the three cases below, including (r, T) = (5%, 30 years) which could be the most severe condition for real options (or technology investment projects).

(1) Case A : (r, T) = (5%, 30 years)
We take into account that risk-free interest rate to be about 3 to 3.5% from 2008 to 2012 and the time to maturity of real options is set at 30 years under harsh condition. In case of biotechnology and pharmaceutical technology, the period from pre-clinical through clinical phase 3 to preparation for approval is about 10 to 15 years.

(2) Case B : (r, T) = (2%, 3 years)
In case of general technologies in manufacturing and service industry, the period and cost of commercialization are not required in many cases, and the ready-to-commercialization time, or risk-free interest rate of the government bond for 3 years is applied. For a general technology, the parametric numbers afore-mentioned might be applied to comparative analysis between real options method and DCF method.

(3) Case C : (r, T) = (3%, 5 years)
For biotechnology and pharmaceutical technology, time for commercialization usually takes 4 to 6 years to prepare for approval.

For the first case A, we calculated a threshold value of R^{th}= 0.2231 for option abandonment or no taken action (NAT) as shown in Figure 1. It is obtained by numerical analysis and iterations, and implies that if variance is less than 0.0018 (i.e. σ^2=0.18%) in Table 1, options value becomes zero. However, the volatility of cash flows during the revenue period is greater than 0.2% in many cases, and then effective options value can be calculated.

In Figure 1, we realize that the threshold value (R^{th}) obtained corresponds to x-intercept (i.e. $\frac{N(d_2)}{N(d_1)} \cdot e^{-rT}$) , and there exists the region of NAT with options value of zero where the ratio (R) is below R^{th} corresponding to specific *S* and *X* such that the value of underlying assets is 0.2231 times commercialization cost.

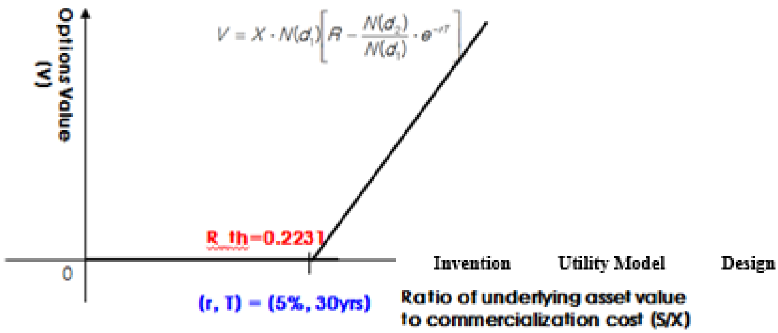

Figure 1 Real options value (V) and Threshold value (R_{th}) (R) under (r, T) = (5%, 30 years)

Table 1 Real options value (V): (R=0.2231) under (5%, 30 years)

variable	explanation	Options value according to variation of volatility												
sigma_2	1) Volatility	0.0009	0.001	0.0015	0.0018	0.0019	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
S	2) Present value of underlying assets	0.2231												
X	3) Comm. cost	1												
R=S/X	4) Ratio of Assets PV to Comm. Cost	0.2231	0.2231	0.2231	0.2231	0.2231	0.2231	0.2231	0.2231	0.2231	0.2231	0.2231	0.2231	0.2231
r=0.05	5) Risk-free int. rate	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
T=30	6) Time to Comm.	30	30	30	30	30	30	30	30	30	30	30	30	30
rT=1.5		1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
e^(-rT)		0.22313	0.22313	0.22313	0.22313	0.22313	0.22313	0.22313	0.22313	0.22313	0.22313	0.22313	0.22313	0.22313
d_2		5.081646	4.812212	3.893799	3.535171	3.4346	-1.21717	-1.48711	-1.71741	-1.92133	-2.10613	-2.27629	-2.43478	-2.5837
N(d_2)		1	0.999999	0.999951	0.999796	0.999703	0.88823	0.931507	0.957048	0.972655	0.982403	0.988586	0.99255	0.995113
d_1		5.245962	4.985417	4.105931	3.76755	3.673347	1.782833	1.976992	2.155573	2.321314	2.476446	2.622689	2.76137	2.893527
N(d_1)		1	1	0.99998	0.999918	0.99988	0.962693	0.975979	0.984442	0.989865	0.993365	0.995638	0.997122	0.998095
V/X	7) Options value to Comm. Cost	-3.01E-05	-3E-05	-2.4E-05	-3.1E-06	9.34E-06	0.016586	0.009894	0.006083	0.00381	0.002416	0.001544	0.00099	0.000635
V/S	8) Options value to Assets PV	-0.000135	-0.00013	-0.00011	-1.4E-05	4.19E-05	0.074344	0.044346	0.027264	0.017079	0.010829	0.006919	0.004438	0.002848

As shown in Table 2 and Table 3, if the value of $\boldsymbol{R}$ (i.e. $\frac{S}{X}$) becomes 2 and 0.25, respectively, then valid values for real options are calculated at all intervals without being restricted by volatility (σ^2). In both cases, options value will always be no less than technology value from DCF method. In the ratio of options value to underlying assets value (i.e. $\frac{V}{S}$ in item 8)) if underlying assets value (S) is twice of commercialization cost (X), then the options value will range between 88.8% and 90.8% of underlying assets value at all intervals of volatility (σ^2).

Table 2 Real options value (V): (R=2) under (5%, 30 years)

variable	explanation	Options value according to variation of volatility												
sigma_2	1) Volatility	0.01	0.03	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
S	2) Present value of underlying assets	2	2	2	2	2	2	2	2	2	2	2	2	2
X	3) Comm. cost	1	1	1	1	1	1	1	1	1	1	1	1	1
R=S/X	4) Ratio of Assets PV to Comm. Cost	2	2	2	2	2	2	2	2	2	2	2	2	2
r=0.05	5) Risk-free int. rate	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
T=30	6) Time to Comm.	30	30	30	30	30	30	30	30	30	30	30	30	30
rT=1.5		1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
e^(-rT)		0.22313	0.22313	0.22313	0.22313	0.22313	0.22313	0.22313	0.22313	0.22313	0.22313	0.22313	0.22313	0.22313
d_2		3.014355	1.424111	0.858162	0.1738	-0.48948	-0.89966	-1.21214	-1.47147	-1.69681	-1.89827	-2.08186	-2.25147	-2.40979
N(d_2)		0.998712	0.922793	0.804599	0.568989	0.687748	0.815849	0.88727	0.929418	0.955134	0.97117	0.981322	0.987822	0.992019
d_1		3.562077	2.372794	2.082907	1.905851	1.960012	2.100343	2.251963	2.401516	2.545827	2.684305	2.817123	2.944685	3.067434
N(d_1)		0.999816	0.991173	0.98137	0.971665	0.975003	0.982151	0.987838	0.991836	0.994549	0.996366	0.997577	0.998384	0.99892
V/X	7) Options value to Comm. Cost	1.776789	1.776443	1.78321	1.816372	1.796548	1.782261	1.777699	1.776292	1.775979	1.776035	1.776192	1.776354	1.776492
V/S	8) Options value to Assets PV	0.888395	0.888221	0.891605	0.908186	0.898274	0.89113	0.888849	0.888146	0.887989	0.888017	0.888096	0.888177	0.888246

Table 3 Real options value (V): (R =0.25) under (5%, 30 years)

variable	explanation	Options value according to variation of volatility												
sigma_2	1) Volatility	0.01	0.03	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
S	2) Present value of underlying assets	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
X	3) Comm. cost	1	1	1	1	1	1	1	1	1	1	1	1	1
R=S/X	4) Ratio of Assets PV to Comm. Cost	0.25	0.2231	0.2231	0.2231	0.2231	0.2231	0.2231	0.2231	0.2231	0.2231	0.2231	0.2231	0.2231
r=0.05	5) Risk-free int. rate	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
T=30	6) Time to Comm.	30	30	30	30	30	30	30	30	30	30	30	30	30
rT=1.5		1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
e^(-rT)		0.22313	0.22313	0.22313	0.22313	0.22313	0.22313	0.22313	0.22313	0.22313	0.22313	0.22313	0.22313	0.22313
d_2		1.365545	0.420055	0.080425	-0.37614	-0.87835	-1.21717	-1.48711	-1.71741	-1.92133	-2.10613	-2.27629	-2.43478	-2.5837
N(d_2)		0.913959	0.662778	0.53205	0.646595	0.810122	0.88823	0.931507	0.957048	0.972655	0.982403	0.988586	0.99255	0.995113
d_1		1.913268	1.368739	1.305169	1.355907	1.571143	1.782833	1.976992	2.155573	2.321314	2.476446	2.622689	2.76137	2.893527
N(d_1)		0.972143	0.91446	0.904082	0.912436	0.941925	0.962693	0.975979	0.984442	0.989865	0.993365	0.995638	0.997122	0.998095
V/X	7) Options value to Comm. Cost	0.039104	0.05613	0.082984	0.05929	0.029381	0.016586	0.009894	0.006083	0.00381	0.002416	0.001544	0.00099	0.000635
V/S	8) Options value to Assets PV	0.156416	0.251592	0.37196	0.265753	0.131694	0.074344	0.044346	0.027264	0.017079	0.010829	0.006919	0.004438	0.002848

In a similar way with Figure 1, for the second and third cases (i.e. (r, T) = (2%, 3 years), (3%, 5 years)) by numerical analysis technique we then obtain the effective regions which are compliant with positive options value corresponding to option abandonment region (or NAT region), as shown in Figure 2. In addition, Figure 1 and Figure 2 imply that there exist threshold or critical values of the ratio (R^{th}) under specific values of risk-free interest rate (r) and time for commercialization (T) such that Black-Scholes model is activated or properly operating.

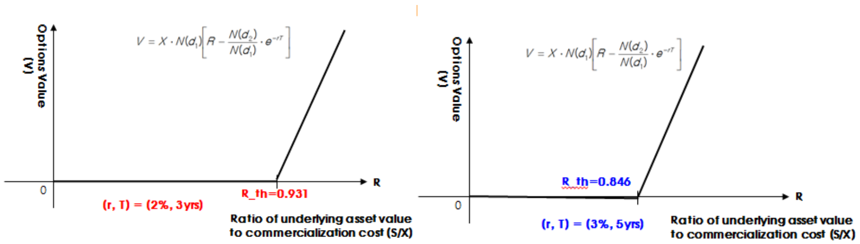

Figure 2 (V) and threshold value (R^{th}) under (2%, 3 yrs) (L) and (3%, 5 yrs) (R)

In the left of Figure 2, i.e. (r, T) = (2%, 3 years), we found that the threshold value of option abandonment (or NAT) region (R^{th}) is 0.931 from Table 4, and by the numerical analysis it then implies that if volatility is greater than 0.99 (i.e. 99%), options value becomes 0, and otherwise options value is positive.

Table 4 Real options value (*V*): (*R* =0.931) under = (2%, 3 years)

variable	explanation	Options value according to variation of volatility												
sigma_2	1) Volatility	0.01	0.03	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.98	0.99	1
S	2) Present value of underlying assets	0.931	0.931	0.931	0.931	0.931	0.931	0.931	0.931	0.931	0.931	0.931	0.931	0.931
X	3) Comm. cost	1	1	1	1	1	1	1	1	1	1	1	1	1
R=S/X	4) Ratio of Assets PV to Comm. Cost	0.931	0.931	0.931	0.931	0.931	0.931	0.931	0.931	0.931	0.931	0.931	0.931	0.931
r=0.02	5) Risk-free int. rate	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
T=3	6) Time to Comm.	3	3	3	3	3	3	3	3	3	3	3	3	3
rT=0.06		0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
e^(-rT)		0.9417645	0.941765	0.941765	0.941765	0.941765	0.941765	0.941765	0.941765	0.941765	0.941765	0.941764534	0.941764534	0.941765
d_2		0.0805385	-0.0535	-0.1189	-0.22101	-0.34992	-0.44383	-0.5213	-0.58874	-0.64924	-0.70459	-0.840437613	-0.844886088	-0.84931
N(d_2)		0.5320955	0.521334	0.547323	0.587456	0.636802	0.671416	0.698919	0.721981	0.741909	0.759468	0.799668466	0.800912779	0.802146
d_1		0.2537436	0.246499	0.268397	0.326716	0.424672	0.504857	0.57415	0.63601	0.692398	0.744546	0.874205207	0.878482706	0.88274
N(d_1)		0.6001532	0.597352	0.605803	0.628059	0.664462	0.69317	0.717067	0.737615	0.755656	0.771727	0.808996744	0.810159091	0.811311
V/X	7) Options value to Comm. Cost	0.0576339	0.065161	0.048553	0.031477	0.018896	0.013026	0.009372	0.006784	0.004812	0.003238	7.65689E-05	-1.31362E-05	-0.0001
V/S	8) Options value to Assets PV	0.0619054	0.06999	0.052151	0.03381	0.020297	0.013992	0.010066	0.007287	0.005169	0.003478	8.22437E-05	-1.41098E-05	-0.00011

Table 5 Real options value (*V*): (*R* =1) under = (2%, 3 years)

variable	explanation	Options value according to variation of volatility												
sigma_2	1) Volatility	0.01	0.03	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
S	2) Present value of underlying assets	1	1	1	1	1	1	1	1	1	1	1	1	1
X	3) Comm. cost	1	1	1	1	1	1	1	1	1	1	1	1	1
R=S/X	4) Ratio of Assets PV to Comm. Cost	1	1	1	1	1	1	1	1	1	1	1	1	1
r=0.02	5) Risk-free int. rate	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
T=3	6) Time to Comm.	3	3	3	3	3	3	3	3	3	3	3	3	3
rT=0.06		0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
e^(-rT)		0.9417645	0.941765	0.941765	0.941765	0.941765	0.941765	0.941765	0.941765	0.941765	0.941765	0.941764534	0.941764534	0.941765
d_2		0.2598076	0.05	-0.03873	-0.16432	-0.30984	-0.4111	-0.49295	-0.56338	-0.6261	-0.68316	-0.735866836	-0.785068999	-0.83138
N(d_2)		0.6024939	0.519939	0.515447	0.565259	0.621658	0.659499	0.688976	0.713413	0.734375	0.752749	0.769094129	0.783793443	0.797122
d_1		0.4330127	0.35	0.348569	0.383406	0.464758	0.537587	0.602495	0.661362	0.715542	0.765973	0.813326503	0.858098673	0.900666
N(d_1)		0.6674972	0.636831	0.636293	0.649291	0.678948	0.704569	0.726578	0.74581	0.762863	0.778154	0.791984557	0.80458101	0.816117
V/X	7) Options value to Comm. Cost	0.1000898	0.147171	0.150864	0.11695	0.093492	0.083476	0.077724	0.073943	0.071254	0.069242	0.067678983	0.066432144	0.065416
V/S	8) Options value to Assets PV	0.1000898	0.147171	0.150864	0.11695	0.093492	0.083476	0.077724	0.073943	0.071254	0.069242	0.067678983	0.066432144	0.065416

Table 6 Real options value (*V*): (*R* =0.89) under = (2%, 3 years)

variable	explanation	Options value according to variation of volatility												
sigma_2	1) Volatility	0.019	0.02	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
S	2) Present value of underlying assets	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89
X	3) Comm. cost	1	1	1	1	1	1	1	1	1	1	1	1	1
R=S/X	4) Ratio of Assets PV to Comm. Cost	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89
r=0.02	5) Risk-free int. rate	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
T=3	6) Time to Comm.	3	3	3	3	3	3	3	3	3	3	3	3	3
rT=0.06		0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
e^(-rT)		0.9417645	0.941765	0.941765	0.941765	0.941765	0.941765	0.941765	0.941765	0.941765	0.941765	0.941764534	0.941764534	0.941765
d_2		-0.080043	-0.08414	-0.1694	-0.25672	-0.37518	-0.46444	-0.53915	-0.60471	-0.66382	-0.71809	-0.768535446	-0.81586926	-0.8606
N(d_2)		0.5318985	0.533527	0.567261	0.601302	0.646235	0.678835	0.705109	0.727313	0.746598	0.763649	0.77891543	0.792712542	0.805272
d_1		0.1587038	0.160809	0.217894	0.291005	0.399421	0.48424	0.556294	0.620039	0.677819	0.731049	0.780657892	0.827298412	0.871447
N(d_1)		0.5630489	0.563878	0.586244	0.614476	0.655208	0.685892	0.710995	0.732384	0.751057	0.767625	0.782498134	0.795966028	0.808245
V/X	7) Options value to Comm. Cost	0.0001904	-0.00061	-0.01247	-0.0194	-0.02547	-0.02886	-0.03126	-0.03314	-0.03468	-0.03599	-0.037131587	-0.038138793	-0.03904
V/S	8) Options value to Assets PV	0.0002139	-0.00068	-0.01401	-0.0218	-0.02861	-0.03243	-0.03512	-0.03723	-0.03896	-0.04044	-0.041720885	-0.042852576	-0.04386

However, as shown in Table 5, when the value of *R* exceeds the threshold of R^{th}= 0.931, options value is obtained with adequacy guaranteed at all intervals, regardless of volatility (σ^2). Also, we observe that options value is taken about 6.5% to 15.1% of *S* at all intervals of volatility. On the other hand, in case that the value of *R* becomes 0.89, options value is positive only if volatility is less than 0.019 (i.e. 1.9%) and for most case options value is calculated as zero.

Lastly, in the right of Figure 2, i.e. (r, T) = (3%, 5 years), the threshold value of option abandonment (NAT) region was calculated as R^{th}= 0.846. In Table 2, we see that if volatility (σ^2) is greater than 0.99 (i.e. 99%), options value becomes zero and otherwise it returns positive value. In other words, option values takes zero only within upper 1% of volatility, and as an example with *R* =0.847 in Table 8, options value is valid at all intervals for *R*, which is greater than the threshold value of R^{th}= 0.846.

Table 7 Real options value (*V*): (*R* =0.846) under = (3%, 5 years)

variable	explanation	Options value according to variation of volatility												
sigma_2	1) Volatility	0.01	0.03	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.99	1
S	2) Present value of underlying assets	0.846	0.846	0.846	0.846	0.846	0.846	0.846	0.846	0.846	0.846	0.846	0.846	0.846
X	3) Comm. cost	1	1	1	1	1	1	1	1	1	1	1	1	1
R=S/X	4) Ratio of Assets PV to Comm. Cost	0.846	0.846	0.846	0.846	0.846	0.846	0.846	0.846	0.846	0.846	0.846	0.846	0.846
r=0.03	5) Risk-free int. rate	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
T=5	6) Time to Comm.	5	5	5	5	5	5	5	5	5	5	5	5	5
rT=0.15		0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
e^(-rT)		0.860708	0.860708	0.860708	0.860708	0.860708	0.860708	0.860708	0.860708	0.860708	0.860708	0.860708	0.860707976	0.860708
d_2		0.2342074	0.00612	-0.09526	-0.24414	-0.42263	-0.5492	-0.6524	-0.74164	-0.82136	-0.89406	-0.96131	-1.077654381	-1.08343
N(d_2)		0.592588	0.502442	0.537946	0.596437	0.663717	0.708566	0.742928	0.770846	0.794278	0.814355	0.831803	0.859405988	0.860692
d_1		0.4578142	0.393419	0.404741	0.462972	0.57737	0.675545	0.761816	0.839503	0.910695	0.976771	1.038685	1.147205165	1.152635
N(d_1)		0.676457	0.652995	0.657166	0.678308	0.718155	0.750335	0.776915	0.799406	0.818772	0.835659	0.850524	0.874351581	0.87547
V/X	7) Options value to Comm. Cost	0.0622374	0.119978	0.092948	0.06049	0.036293	0.024915	0.017826	0.012824	0.00904	0.006046	0.003604	3.84891E-06	-0.00016
V/S	8) Options value to Assets PV	0.0735667	0.141818	0.109868	0.071501	0.042899	0.029451	0.021071	0.015159	0.010685	0.007146	0.00426	4.54953E-06	-0.00019

Table 8 Real options value (*V*): (*R* =0.847) under = (3%, 5 years)

variable	explanation	Options value according to variation of volatility												
sigma_2	1) Volatility	0.01	0.03	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
S	2) Present value of underlying assets	0.847	0.847	0.847	0.847	0.847	0.847	0.847	0.847	0.847	0.847	0.847	0.847	0.847
X	3) Comm. cost	1	1	1	1	1	1	1	1	1	1	1	1	1
R=S/X	4) Ratio of Assets PV to Comm. Cost	0.847	0.847	0.847	0.847	0.847	0.847	0.847	0.847	0.847	0.847	0.847	0.847	0.847
r=0.03	5) Risk-free int. rate	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.03
T=5	6) Time to Comm.	5	5	5	5	5	5	5	5	5	5	5	5	5
rT=0.15		0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
e^(-rT)		0.860708	0.860708	0.860708	0.860708	0.860708	0.860708	0.860708	0.860708	0.860708	0.860708	0.860708	0.860707976	0.860708
d_2		0.2365018	0.007445	-0.09423	-0.24341	-0.42212	-0.54878	-0.65203	-0.74131	-0.82106	-0.89378	-0.96106	-1.02394558	-1.0832
N(d_2)		0.5934783	0.50297	0.537538	0.596156	0.66353	0.708422	0.742811	0.770748	0.794194	0.814281	0.831739	0.84706951	0.860641
d_1		0.4601086	0.394743	0.405767	0.463697	0.577883	0.675964	0.762179	0.839827	0.910991	0.977045	1.038942	1.097374764	1.152865
N(d_1)		0.6772809	0.653484	0.657543	0.678568	0.718329	0.750468	0.777023	0.799497	0.81885	0.835727	0.850584	0.8637612	0.875517
V/X	7) Options value to Comm. Cost	0.0628453	0.12059	0.094276	0.061631	0.037319	0.025902	0.018796	0.013786	0.009997	0.007002	0.004561	0.002526252	0.000802
V/S	8) Options value to Assets PV	0.0741976	0.142374	0.111305	0.072763	0.04406	0.030581	0.022191	0.016276	0.011803	0.008267	0.005385	0.002982588	0.000947

Therefore, by the ratio of *R* we are able to determine effective regions of specific (r, T) where the limitation of DCF method-based technology value is overcome with real options value.

2. Effective Region of Volatility

Now, we investigate the region of (R, σ^2) such that options value is zero or positive under a certain condition of (r, T) as shown in Figure 3 to Figure 5. The region of (R, σ^2) can be found given that income statement, cash flows table, and advance planning for commercialization preparation are provided, by calculating 'volatility' or 'variance of cash flows' with S and X determined. Ultimately, if (R, σ^2) is first determined and Monte Carlo simulations are then performed, we are able to check the validity of volatility obtained above and.

For example, in Figure 3 if $R > 0.2231$, options value is then valid for $\sigma^2 >> 0.99$and if $0.13 < R < 0.22$, options value becomes positive only below shaded area (i.e. σ^2 lies in between 0.01 (1%) and 0.88 (88%)). In fact, if $R < 0.13$, then options value always return zero regardless of volatility (σ^2).

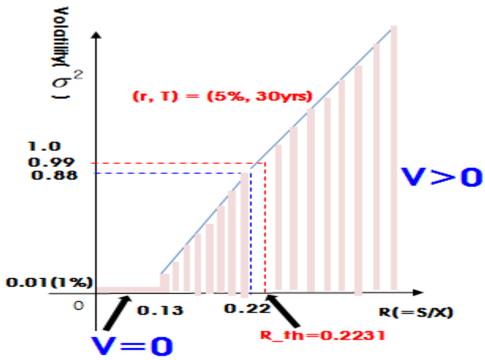

Figure 3 Feasible region of options value (*V*) (5%, 30 years)

Similarly with the above, for the case of (r, T) = (2%, 3 years) which corresponds to most practical technologies in manufacturing or service industry, we recognize that if $R > 0.931$, then we obtain valid options value under $\sigma^2 >>$ 0.99, and if $R < 0.89$ $R < 0.89$, then options value becomes zero regardless of volatility (σ^2), as shown in Figure 4.

For the case C of (r, T) = (3%, 5 years) in the previous section such that it often applies to biotechnology and pharmaceutical technology, if the required time for commercialization for a technology corresponding to phase 2 or phase 3 clinical trials is as a whole 4 to 6 years. In Figure 5, under the condition that (r, T) = (3%, 5 years) and $R > 0.846$ are known, we can guarantee that options value is always valid in the interval of $\sigma^2 >> 0.99$. In the other way, if the value of R is less than 0.75, then options value is zero regardless of volatility (σ^2).

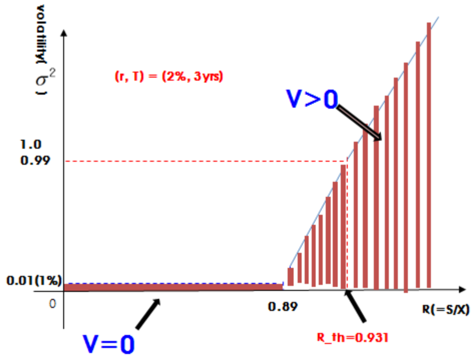

Figure 4 Feasible region of options value under (2%, 3 years)

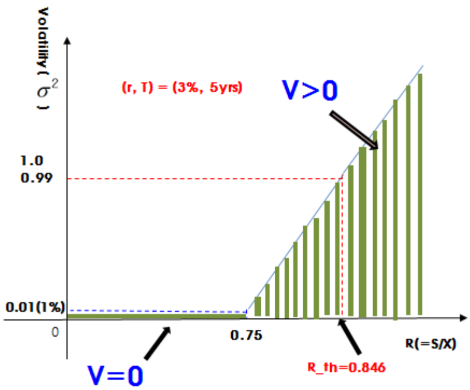

Figure 5 Feasible region of options value under (3%, 5 years)

3. Determination of Options Value under Effective Region

Consider the case of (r, T) = (2%, 3 years) in the previous section. We have realized that options value yields a positive value in the range of $\sigma^2 \leq 0.98$ for the threshold value of option abandonment (or NAT) region of $R^{th} = 0.931$. In Table 9 to Table 11, when the value of R is 0.77, 0.92, and 1, respectively, options value in yellow-colored interval becomes zero.

In Table 10, the value of R =0.92 is less than the threshold $R^{th} = 0.931$, and options value is zero at specific interval of volatility (i.e. $\sigma^2 \geq 0.35$) because we recognize $0.89 < R < 0.931$ in Figure 4 is calculated as 0 only for a certain interval. For general cases, the logic for adequacy of both volatility and options value has been embedded on web-based valuation system, referred to as 'STAR-Value 5.0 PLUS'.

Table 9 Real options value (*V*): *R*=0.77 under (2%, 3 years)

variable	explanation	Options value according to variation of volatility												
sigma_2	1) Volatility	0.01	0.037	0.038	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
S	2) Present value of underlying assets	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77
X	3) Comm. cost	1	1	1	1	1	1	1	1	1	1	1	1	1
R=S/X	4) Ratio of Assets PV to Comm. Cost	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77
r=0.02	5) Risk-free int. rate	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
T=3	6) Time to Comm.	3	3	3	3	3	3	3	3	3	3	3	3	3
rT=0.06		0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
e^(-rT)		0.9417645	0.941765	0.941765	0.941765	0.941765	0.941765	0.941765	0.941765	0.941765	0.941765	0.941764534	0.941764534	0.941765
d_2		-0.395538	-0.32719	-0.3273	-0.37156	-0.45638	-0.53075	-0.59657	-0.65606	-0.7107	-0.76149	-0.809136758	-0.854148544	-0.89692
N(d_2)		0.6537772	0.628238	0.62828	0.644888	0.675941	0.702202	0.724603	0.744108	0.761366	0.776819	0.790781756	0.803488655	0.815119
d_1		-0.222333	0.005975	0.010339	0.176167	0.318218	0.417938	0.498875	0.568682	0.630937	0.687644	0.740056581	0.789019129	0.835132
N(d_1)		0.5879728	0.502384	0.504124	0.569919	0.62484	0.662004	0.691066	0.715214	0.735959	0.754161	0.770367169	0.784949587	0.798178
V/X	7) Options value to Comm. Cost	-0.162965	-0.20482	-0.20352	-0.1685	-0.15545	-0.15157	-0.15028	-0.15006	-0.15034	-0.15088	-0.151547492	-0.152285937	-0.15305
V/S	8) Options value to Assets PV	-0.211643	-0.266	-0.26431	-0.21883	-0.20188	-0.19684	-0.19517	-0.19488	-0.19525	-0.19594	-0.196814925	-0.197773944	-0.19877

Table 10 Real options value (*V*): *R* =0.92 under (2%, 3 years)

variable	explanation	Options value according to variation of volatility												
sigma_2	1) Volatility	0.01	0.03	0.05	0.1	0.2	0.33	0.34	0.35	0.6	0.7	0.8	0.9	1
S	2) Present value of underlying assets	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92
X	3) Comm. cost	1	1	1	1	1	1	1	1	1	1	1	1	1
R=S/X	4) Ratio of Assets PV to Comm. Cost	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92	0.92
r=0.02	5) Risk-free int. rate	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
T=3	6) Time to Comm.	3	3	3	3	3	3	3	3	3	3	3	3	3
rT=0.06		0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
e^(-rT)		0.9417645	0.941765	0.941765	0.941765	0.941765	0.941765	0.941765	0.941765	0.941765	0.941765	0.941764534	0.941764534	0.941765
d_2		0.0507365	-0.07071	-0.13223	-0.23043	-0.35659	-0.47359	-0.48142	-0.48913	-0.65309	-0.70815	-0.759241693	-0.807107026	-0.85229
N(d_2)		0.5202323	0.528185	0.552599	0.591122	0.6393	0.682102	0.684892	0.687626	0.743151	0.760575	0.776146005	0.790197588	0.802974
d_1		0.2239416	0.229293	0.255069	0.317292	0.418008	0.521401	0.528529	0.535562	0.688551	0.740984	0.789951646	0.836060647	0.879759
N(d_1)		0.5885986	0.590679	0.600665	0.624489	0.662029	0.698956	0.701434	0.703869	0.754447	0.770648	0.785221996	0.798439607	0.810505
V/X	7) Options value to Comm. Cost	0.0515744	0.045999	0.032194	0.017832	0.006997	0.00066	0.000312	-2.2E-05	-0.00578	-0.00729	-0.008542544	-0.009615624	-0.01055
V/S	8) Options value to Assets PV	0.0560592	0.049999	0.034994	0.019383	0.007605	0.000717	0.00034	-2.4E-05	-0.00628	-0.00792	-0.009285374	-0.010451765	-0.01146

In Table 11, since the value of $R = 1$ is greater than 0.931, options value are always positive, which is compliant with the shaded region in Figure 4.

Table 11 Real options value (*V*): *R* =1.0 under (2%, 3 years)

variable	explanation	Options value according to variation of volatility												
sigma_2	1) Volatility	0.01	0.03	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
S	2) Present value of underlying assets	1	1	1	1	1	1	1	1	1	1	1	1	1
X	3) Comm. cost	1	1	1	1	1	1	1	1	1	1	1	1	1
R=S/X	4) Ratio of Assets PV to Comm. Cost	1	1	1	1	1	1	1	1	1	1	1	1	1
r=0.02	5) Risk-free int. rate	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
T=3	6) Time to Comm.	3	3	3	3	3	3	3	3	3	3	3	3	3
rT=0.06		0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06
e^(-rT)		0.9417645	0.941765	0.941765	0.941765	0.941765	0.941765	0.941765	0.941765	0.941765	0.941765	0.941764534	0.941764534	0.941765
d_2		0.2598076	0.05	-0.03873	-0.16432	-0.30984	-0.4111	-0.49295	-0.56338	-0.6261	-0.68316	-0.735866836	-0.785068999	-0.83138
N(d_2)		0.6024939	0.519939	0.515447	0.565259	0.621658	0.659499	0.688976	0.713413	0.734375	0.752749	0.769094129	0.783793443	0.797122
d_1		0.4330127	0.35	0.348569	0.383406	0.464758	0.537587	0.602495	0.661362	0.715542	0.765973	0.813326503	0.858098673	0.900666
N(d_1)		0.6674972	0.636831	0.636293	0.649291	0.678948	0.704569	0.726578	0.74581	0.762863	0.778154	0.791984557	0.80458101	0.816117
V/X	7) Options value to Comm. Cost	0.1000898	0.147171	0.150864	0.11695	0.093492	0.083476	0.077724	0.073943	0.071254	0.069242	0.067678983	0.066432144	0.065416
V/S	8) Options value to Assets PV	0.1000898	0.147171	0.150864	0.11695	0.093492	0.083476	0.077724	0.073943	0.071254	0.069242	0.067678983	0.066432144	0.065416

From the above results, we are able to understand what the variation of options value is like that of the relationship between the ratio of underlying assets value (S) to commercialization cost (X) (that is, $R = \frac{S}{X}$) given that all parameters such as r, T and *X* are concretely specified, and thus the effective region of volatility can be determined.

The significance of this study lies in overcoming the limitation of real options model which has remained only in the theoretical background, not applicable to practical uses up to present, and providing the concrete, effective regions of volatility (σ^2) for the utilization of real options method under certain conditions with the present value of underlying assets (*S*) and commercialization cost (*X*). We will also anticipate that the elaborated study herein leads to enhance the reliability of 'optimal decision-making' reflecting uncertainty for further promotion in commercialization.

V. Summary and Conclusion

The purpose of this study is to investigate the issue of volatility of the Black-Scholes model that complements the limitations of the discounted cash flow (DCF) model used as a representative model of the income approach in technology valuation. In particular, we examine the estimation methods for volatility based on the cash flow-based profitability method in natural logarithm and the Monte Carlo simulation method that calculates the volatility in the Black-Scholes model, and we look into the validity interval of the volatility on the condition that the specific ratio of 'additional expenditures for commercialization to the present value of the underlying asset obtained from the free cash flows (FCFs)' is given.

In addition, throughout the approach by numerical analysis, we mathematically examine whether the present values of the underlying asset and the commercialization cost reflecting the uncertainty in the option pricing model (OPM) are identified into the "no action taken" (NAT) area under a certain critical condition or not, and then present the table for options value according to the observation variable (or input value).

This study aims to underline the significance of the elaboration of technology valuation models based on real options in order to overcome the limitations of applying the discounted cash flow model with regards to business models that cannot be determined by deterministic variables. We expect that it will contribute to invigorating the technology-based market with numerous goals such as technology transfer, technology licensing, and technology financing, amongst others.

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