Stochastic Petri Nets Modeling Methods of Channel Allocation in Wireless Networks

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ABSTRACT

To obtain realistic performance measures for wireless networks, one should consider changes in performance due to failure related behavior. In performability analysis, simultaneous consideration is given to both pure performance and performance with failure measures. SRN is an extension of stochastic Petri nets and provides compact modeling facilities for system analysis. In this paper, a new methodology to model and analyze performability based on stochastic reward nets (SRN) is presented. Composite performance and availability SRN models for wireless handoff schemes are developed and then these models are decomposed hierarchically. The SRN models can yield measures of interest such as blocking and dropping probabilities. These measures are expressed in terms of the expected values of reward rate functions for SRNs. Numerical results show the accuracy of the hierarchical model.

The key contribution of this paper constitutes the Petri nets modeling techniques instead of complicate numerical analysis of Markov chains and easy way of performance analysis for channel allocation under SRN reward concepts.

Keywords: *SRN, channel allocation, handoff, hierarchical model.*

1. INTRODUCTION

With the increasing popularity of wireless communications systems, customers are expecting the same level of service, reliability and performance from the wireless communications systems as the traditional wire-line networks. Due to the dynamic environment, such as the roaming of mobile subscribers, maintaining a high radio frequency (RF) availability is one of the most challenging aspects in wireless networks. RF availability depends on natural environment, infrastructure, and subscriber handsets. There are many research and development results [1], [2] in each of the above areas.

In wireless networks, an RF channel is assigned to a call either during the call set-up process when a new call is initiated or during the handoff process when an ongoing call subscriber roams into the cell. Different channel allocation schemes have been studied [3]-[8].

A common assumption in these studies has been that the channel in use never fails. However, in a practical environment, wireless networks, like any other physical system, are subject

to failures. To obtain realistic performance measures for wireless networks, one should consider changes in performance due to failure related behavior. In performability analysis [9,10], simultaneous consideration is given to both performance and reliability/availability measures.

One approach is to combine the performance and availability models into a single monolithic model. The advantage of this approach is that it yields accurate results. However, this direct approach generally faces two problems, namely, largeness and stiffness.

Another widely applied approach in combined performance and availability analysis is the hierarchical modeling technique [11]. There are several advantages in using this approach. First, the largeness problem can be avoided through the divide and conquer' strategy, where a large system is decomposed into several sub models [12]. Second, the stiffness problem can be resolved by separating the fast and slow rates from each other [13].

Stochastic reward net (SRN) is an extension of stochastic Petri nets and provides compact modeling facilities for system analysis. To get the measures of interest, appropriate reward rates are assigned to its SRN. SRN models are presented as Markov reward models, which are obtained by associating

reward rates with Markov chain's state. SRN models can yield measures of interest such as blocking and dropping probabilities. These measures are expressed in terms of the expected values of reward rate functions for SRNs. If a cyclic dependency exists, a fixed point iteration scheme is used to obtain the model's solution.

In this paper, we will illustrate SRN modeling techniques for composite performance and availability analysis through evaluating an M/M/C/C queueing system in a wireless network. Basic modeling paradigm that we use is that of Markov reward models. In order to automatically generate and solve such models, we use the framework of stochastic reward nets. SRN models of channel allocation with channel failure/recovery are developed to compare system availability and performance.

The paper is organized as follows. In Section 2, we give a brief introduction to SRNs. In Section 3, three SRN models, a pure performance model for channel allocation, a composite performance and availability model for channel allocation with channel failure and repair, and a hierarchical model, are presented. In Section 4, Measures of interest are introduced and explained how to express the numerical measures in terms of reward assignments in SRN. In Section 5, the numerical results for the composite and hierarchical model are compared and showed the accuracy of the approximation of hierarchical modeling. Finally, we make our conclusions in Section 6.

2. STOCHASTIC PETRI NETS

2.1 Stochastic Petri Nets (SPNs)

A Petri net (PN) is a bipartite directed graph with two disjoint sets called places and transitions [14]-[16]. Directed arcs in the graph connect places to transitions (called input arcs) and transitions to places (called output arcs). Places may contain an integer number of entities called tokens. The state or condition of the system is associated with the presence or absence of tokens in various places in the net. The condition of the net may enable some transitions to fire. This firing of a transition is the removal of tokens from one or more places in the net and/or the arrival of tokens in one or more places in the net. The tokens are removed from places connected to the transition by an input arc; the tokens arrive in places connected to the transition by an output arc. A marked PN is obtained by associating tokens with places. The marking of a PN is the distribution of tokens in the places of the PN. A marking is represented by a vector $M = (\#(P_1), \#(P_2), \#(P_3), ..., \#(P_n))$ where $\#(P_i)$ is the number of tokens in place i and n is the number of places in the net.

In a graphical representation of a PN, places are represented by circles, transitions are represented by bars and the tokens are represented by dots or integers in the circles (places). The input places of a transition are the set of places which connect to that transition through input arcs. Similarly, output places of a transition are those places to which output arcs are drawn from that transition. A transition is considered enabled to fire in the current marking if each of its input places contains the number of tokens assigned to the input arc (called the arc multiplicity).

A marking depicts the state of a PN which is characterized by the assignment of tokens in all the places. With respect to a given initial marking, reachability set is defined as the set of all marking reachable through any possible firing sequences of transitions, starting from the initial marking. A transition is enabled if all of its input places have at least as many tokens as required by the multiplicities of corresponding input arcs. When enabled, the firing of a transition is an atomic action in which the designated number of tokens are removed from each input place of that transition and one or more tokens as specified by the output arc multiplicity) are added to each output place of that transition, possibly resulting in a new marking of the PN. For any given marking, if more than one transition is simultaneously enabled, the conflict can be resolved by applying a race model so that the transition with its firing time elapses first is the next one to fire [15]. Fig 1. shows a simple PN.
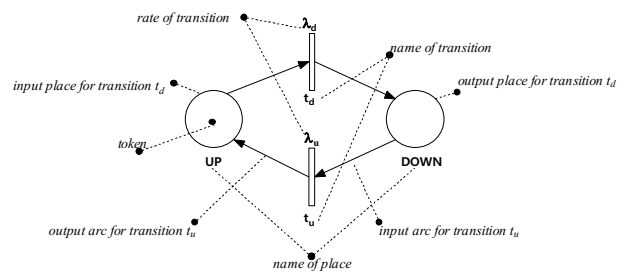

Fig. 1. Graphical representation of PN.

If exponentially distributed firing times correspond with the transitions, the result is a SPN [17], [18]. Allowing transitions to have either zero firing times (immediate transitions) or exponentially distributed firing times (timed transitions) gives rise to the GSPN [19]. The transitions with exponentially distributed firing time are drawn as unfilled rectangles; immediate transitions are drawn as lines.

2.2 Generalized Stochastic Petri Nets (GSPNs)

Generalized stochastic Petri nets (GSPNs) [19] extend the PNs by assigning a firing time to each transition. Transitions with exponentially distributed firing times are called timed transitions while the transitions with zero firing times are called immediate transitions. The transitions with exponentially distributed firing time are drawn as unfilled rectangles; immediate transitions are drawn as lines.

GSPN also introduces inhibitor arcs. An inhibitor arc from a place to a transition disables the transition if the place contains at least as many tokens as the cardinality of the inhibitor arc. Graphically, an inhibitor arc is represented by a line terminated with a small circle.

A marking in a GSPN is called vanishing if at least one immediate transition is enabled; otherwise it is called a tangible marking. For a given GSPN, an extended reachability graph (ERG) is generated with the markings of the reachability set as the nodes and some stochastic information attached to the arcs, thus connecting the markings to each other. Under the condition that only a finite number of transitions can fire in finite time with non-zero probability, it can be shown that a given ERG can be reduced to a homogeneous continuous time

Markov chain (CTMC) [19].

2.3 Stochastic reward nets (SRNs)

In order to make more compact models of complex systems, several extensions are made to GSPN, leading to the SRN. One of the most important features of SRN is its ability to allow extensive marking dependency. In an SRN, each tangible marking can be assigned with one or more reward rate(s). Parameters such as the firing rate of the timed transitions, the multiplicities of input/output arcs and the reward rate in a marking can be specified as functions of the number of tokens in any place in the SRN.

Another important characteristic of SRN is the ability to express complex enabling/disabling conditions through guard functions. This can greatly simplify the graphical representations of complex systems. For an SRN, all the output measures are expressed in terms of the expected values of the reward rate functions. To get the performance and reliability/availability measures of a system, appropriate reward rates are assigned to its SRN. As SRN is automatically transformed into a Markov Reward Model (MRM) [20,21], steady state and/or transient analysis of the MRM produces the required measures of the original SRN.

2.4 Markov reward models

SRNs provide the same modeling capability as Markov reward models (MRMs). A Markov reward model is a Markov chain with reward rates (real numbers) assigned to each state [How71]. A state of an SRN is actually a marking (labeled $(\#(P_1), \#(P_2), \#(P_3), ..., \#(P_n))$ if there are n places in the net). We label the set of all possible markings that can be reached in the net as ⟨⟩. These markings are subdivided into tangible markings ⟨⟩$_T$ and vanishing markings ⟨⟩$_V$. For each tangible marking i in ⟨⟩$_T$ a reward rate r_i is assigned. This reward is determined by examining the overall measures to be obtained. Several measures are obtained using Markov reward models. These include the expected reward rate both in steady state and at a given time, the expected accumulated reward until either absorption or a given time, and the distribution of accumulated reward either until absorption or a given time.

The expected reward rate in steady state can be computed using the steady state probability of being in each marking i for all $i \in$ ⟨⟩$_T$. For steady state distribution π_i, the expected reward rate is given by

$$E[R] = \sum_{i \in \langle \rangle_T} r_i \pi_i \qquad (1)$$

2.5 The steps in analyzing an SRN Model

The SRN model can be solved by using SPNP, a software package for the automated generation and solution of Markovian stochastic systems developed by researchers in Duke University [22]. SPNP can calculate the throughput of each transition, which is used to find the fixed point. The SPNP is a versatile modeling tool for solution of SPN models.

The measures are defined in terms of reward rates associated with the markings of the SRN. Parametric sensitivity analysis

allows the user to evaluate the effect of changes in an input parameter on the output measures. In Fig. 2, we summarize the steps in analyzing an SRN model.
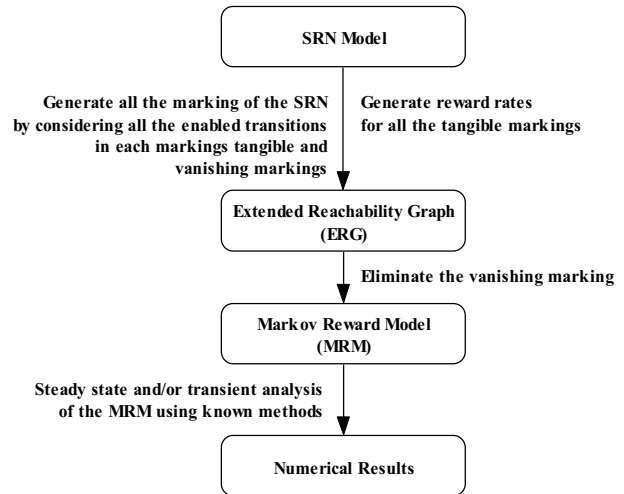

Fig. 2. The steps in analyzing an SRN model.

2.6 Example: SRN model of M/M/1/K queue

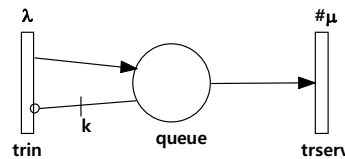A SRN model of M/M/1/K queueing system is shown in Fig. 3 as an example.

Fig. 3.   SRN model for the M/M/1/K queue.

Place **queue** is a queue with size of k. Transition **trin** and **trserv** represent the arrivals of customers with rate λ and the service with rate μ. The number of tokens in place **queue** represents the number of customers. The queue capacity is reflected by the multiple inhibitor arc from transition **trin** to place **queue** with k. SPNP source for M/M/1/K queue is given in Appendix A.1. The reachability graph (RG) and CTMC, which is isomorphic to RG, are automatically generated by SPNP. They are shown in Appendix A.2 when $k = 2$. The CTMC for the SRN model of Fig. 3 is shown in Fig. 4.
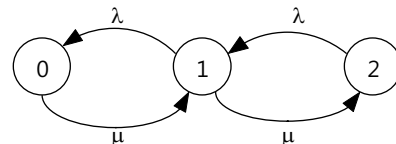

Fig. 4 . CTMC for the M/M/1/K queue.

3. SRN MODELING OF CHANNEL ALLOCATION

3.1 System Description

In a wireless network, mobile subscribers (MSs) are provided with service within a geographical area. The service

area is divided into multiple adjacent cells. MSs communicate via radio links to base stations (BSs), one for each cell. When an MS moves across a cell boundary, the channel in the old BS is released and an idle channel is required in the new BS. This phenomenon is called handoff. Handoff is an important function of mobility management.

In hard handoff, the old radio link is broken before the new radio link is established, and an MS always communicates with one BS at any given time. In the handoff procedure, the network needs to set up the new path for the handoff call. Some CDMA systems [23] and GSM with macro diversity [24] utilize soft handoff where an MS may communicate with the outside world using multiple radio links through different BSs at the same time. During handoff, the signaling and voice information from multiple BSs are typically combined (or bridged) at the mobile switching center (MSC). In some soft handoff systems, an MS may connect up to three or four radio links at the same time. Thus, within the overlay area of cells, an MS can connect to multiple BSs. Loss probabilities or formulas for new call blocking and handoff call dropping have been widely employed to evaluate call admission control (CAC) strategies for wireless cellular networks. When a new call (NC) is attempted in an cell covered by a BS, the NC is connected if an idle channel is available in the cell. Otherwise, the new call is blocked. Similarly, if an idle channel exists in the target cell, the handoff call (HC) continues nearly transparently to the MS. Otherwise, the HC is dropped.

The dropping of a HC is considered more severe than the blocking of a NC. One method [25] to reduce the dropping probability of HCs is to reserve a fixed or an adaptive (natural or fractional) number of channels exclusively for HCs. These exclusively reserved channels are referred to as guard channels. For example, if the total number of channels is *S* and the number of guard channels is *g*, then the number of RF channels available for both NCs and HCs is *S-g*. It should be noted that no specific channels are reserved as guard channels but only a specific number of channels are reserved.

3.2 A Pure Performance Model

We first present a performance model under the assumption that the channels in a wireless network never fail. We consider a single cell in mobile cellular networks. Let λ_{vn} be the rate of the Poisson arrival stream of new calls and λ_{vh} be the rate of Poisson stream of handoff arrivals. An ongoing call (new or handoff) completes service at the rate μ_{vt} and the mobile engaged in the call departs the cell at the rate μ_{vout}. There are a limited number of channels *S*, in the channel pool. When a HC arrives and an idle channel is available in the channel pool, the call is accepted and a channel is assigned to it. Otherwise, the HC is dropped. When a NC arrives, it is accepted provided that *g+1* or more idle channels are available in the channel pool; otherwise, the NC is blocked. Here, *g* is the number of guard channels. We assume that *g < S* in order not to exclude new calls altogether.

• **SRN model**

To make the model easier to understand, we present the model in form of a SRN. Fig. 5 shows an SRN of a performance model for the guard channel allocation scheme.
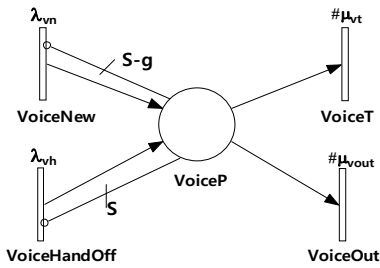

Fig. 5 SRN performance model of channel allocation

The number of tokens in place ***VoiceP*** represents the number of channels that are occupied by either a NC or a HC. The firing of transition ***VoiceNew*** and ***VoiceHandoff*** represent the arrival of NCs and HCs respectively. A HC will be dropped only when all channels are occupied (i.e. # ***VoiceP*** = *S*). This is realized by an inhibitor arc from place ***VoiceP*** to ***VoiceHandoff*** in with multiplicity. A new call, however, will be blocked if there are no more than *g* idle channels. This is simply reflected in the SRN by the inhibitor arc from place ***VoiceP*** to transition ***VoiceNew*** with multiplicity *S-g*. The firings of transition ***VoiceT*** and *VoiceOut* represent the completion of a call and the departure of an outgoing HC, respectively. The rates of transitions ***VoiceT*** and ***VoiceOut*** are marking-dependent, as indicated by the two '#' symbols next to the arcs from the place ***VoiceP***.

Guard channel for HCs can be modeled using enabling function. Enabling function of transition ***VoiceNew*** is

Table 1. Enabling function of transition ***VoiceNew***

if((#("VoiceP")) < S - g)   return(1);else return(0);

Enabling function of transition ***VoiceHandoff*** is

Table 2 Enabling function of transition ***VoiceHandoff***

if((#("VoiceP")) < S) return(1);else return(0);

Here #(*"VoiceP"*) represent the number of tokens in place ***VoiceP***.

• **Reachability Graph and Markov Chain**

The corresponding CTMC of Fig. 5 is shown in Fig. 6, where state(***i***) represents that there are ***i*** talking channels in ***VoiceP***. The state-dependent arrival and departure rates in the birth-death process of Fig. 6 are given by

$$\Lambda(j) = \begin{cases} \lambda_{vn} + \lambda_{vh} & , j < S - g \\ \lambda_{vh} & , j < S \end{cases} \qquad (2)$$

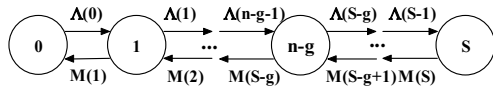and $M(j) = j(\mu_1 + \mu_2), j = 1,2,...,S$.
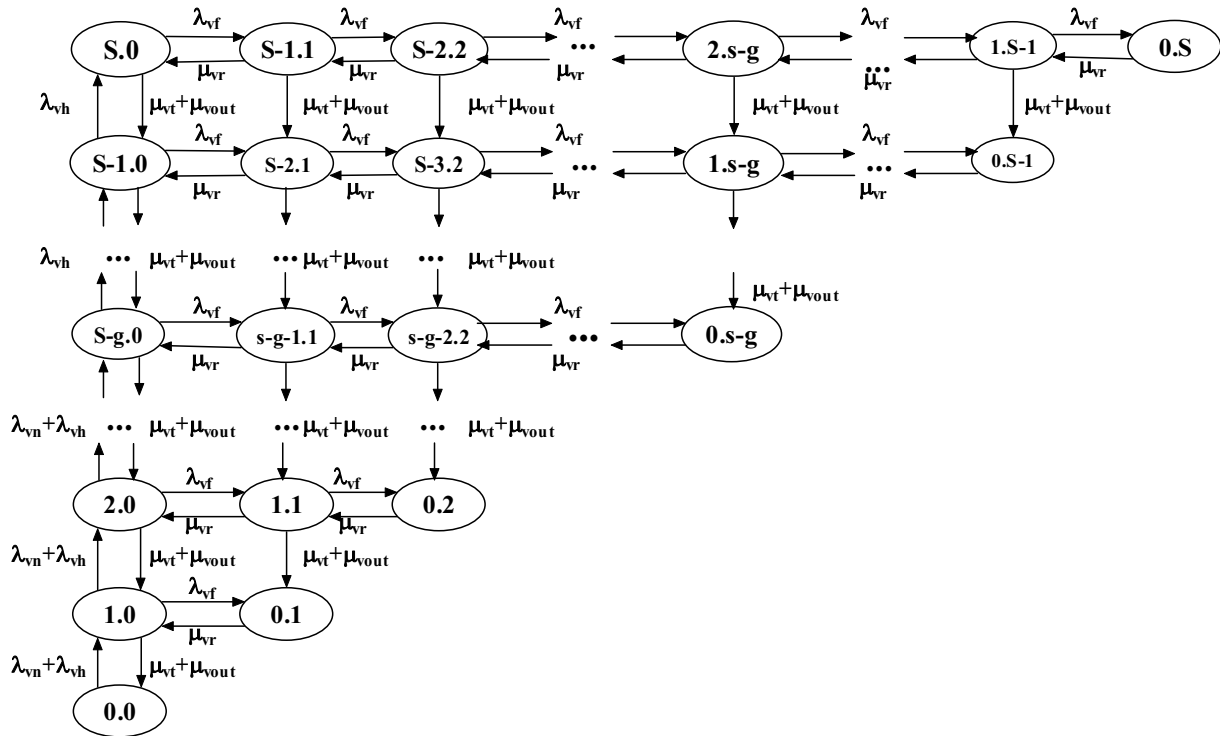
Fig. 6 CTMC for the SRN model in Fig. 5

Fig. 8. The exact CTMC model of the M/M/S/S queueing system with channel failure/repair

channel while transition **VoiceRepair** represents the repair of a channel. Considering failure, enabling function of transition **VoiceNew** and **VoiceHandoff** are shown in Table 3 and Table 4, respectively.

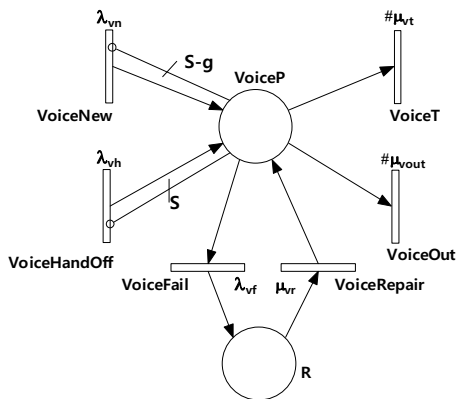## 3.3 Composite Performance and Availability Model

Fig. 7 Exact SRN composite model

Table 3. Enabling function of **VoiceNew**.

if((#("VoiceP")+#("R")) < S - g)   return(1);
else return(0);

Table 4. Enabling function of **VoiceHandoff**.

if((#("VoiceP")+#("R"))) < S) return(1);
else return(0);

The corresponding CTMC model of Fig. 7 is shown in Fig. 8, where state (u, v) represents that there are u talking channels and v channels are in failure status.

3.4 Hierarchical Model

Fig. 9. Upper level of hierarchical model.

We consider the exact composite model. All failure events are assumed to be mutually independent. Times to channel failure and repair are assumed to be exponentially distributed with mean $1/\lambda_{vf}$ and $1/\mu_{vr}$. The monolithic SRN model is shown in Fig. 7. Compared with Fig. 5, Fig. 7 has one more place (place **R**) and two more transitions (transitions **VoiceFail** and **VoiceRepair**). Place **R** represents the place where the channels are being repaired or waiting to be repaired. Transition **VoiceFail** represents the failure of a
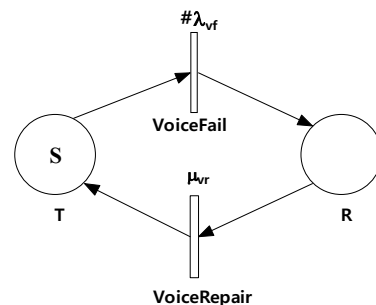
For systems with large number of channels, constructing the underlying Markov chain and seeking the solution is not trivial. We use the decomposition method [26] to build a two-level. We compute the dropping and blocking probabilities using the performability model [27]. We first present an availability model which accounts for the failure repair behavior of channel, and second we use a performance model to compute the measures of interest such as blocking and dropping probabilities. Finally we combine them together and get performability measures of interest. The hierarchical mode is a two-level Markov reward model. Such a two-level model is an approximation since we assume that in each state of the upper level model, the lower level model reaches steady state. The upper level model, as shown in Fig. 9, describes the failure and repair behavior of the system. The number of tokens in place **T** represents the number of channels that are currently non-failed in the cell. The number of tokens in place **R** represents the number of channels that have failed. Transition **VoiceRepair** with rate μ_{vr} represents the repair of a channel, while transition **VoiceFail** with λ_{vf} represents the failure of a channel. The actual firing rate of **VoiceFail** equals the number of tokens in place **T** multiplied by λ_{vf}; this is indicated by the '#' next the arc from place **T** to transition **VoiceFail**.

3.5 Fixed Point Iteration

With different NC arrival rates, the corresponding HC arrival rates vary accordingly. To capture this dynamic behavior, we apply a fixed-point iteration scheme [28] to determine the HC arrival rates. The arrival rate λ_{vh} of HCs should be equal to the actual throughput of transition **VoiceOut** denoted by Λ_{ho}. The value of Λ_{ho} can be calculated as follows,

$$\Lambda_{ho} = \sum_{j \in} (\#[VoiceP_j]) \mu_{out} \pi_j (\lambda_{vh}) = \sum_{j \in} (\#[VoiceP_j]) \mu_{out} \pi_j (\Lambda_{ho}) \quad (3)$$

By iteration, Equation (3) turns to be

$$\Lambda_{ho}^{s+1} = \sum_{j \in} (\#[VoiceP_j]) \mu_{out} \pi_j \Lambda_{ho}^{s} \quad (4)$$

where   is the set of tangible markings of the SRN model, $\#[VoiceP_j]$ denotes the number of tokens in **VoiceP** in marking (state) j, and π is the steady-state probability vector of the SRN model. Notice that π is a function of Λ_{ho}, because the firing rate of transition **VoiceHandoff** should be equal to Λ_{ho}. When all the other parameters are fixed, an increase/decrease in $\lambda_{vh}(\Lambda_{ho}^{s})$ implies an increase/decrease in Λ_{ho}^{s+1}. Therefore, Λ_{ho} is a monotone increasing/decreasing function of λ_{vh}. In [28], Equation (3) is defined as the fixed point equation and Λ_{h}^{o} is termed as the iteration variable. SPNP source for fixed point iteration is given in Appendix B.

4. MEASURES OF INTEREST

Two steady-state measures are of interest from the SRN model of Fig. 5, namely, the NC blocking probability, P_b, and the HC dropping probability, P_d. We obtain these two measures by computing the expected steady-state reward rate for the SRN model with the proper assignment of reward rates to the markings.

$$P_b = \sum_{i \in} r_{bi} \pi_i \quad (5)$$

$$P_d = \sum_{i \in} r_{di} \pi_i \quad (6)$$

The reward rate assigned to marking i for computing the new-call blocking probability is

$$r_b = \begin{cases} 1, \#(VoiceP) \geq S & g \\ 0, \#(VoiceP) < S & g \end{cases} \quad (7)$$

And that for the handoff dropping probability is

$$r_d = \begin{cases} 1, \#(VoiceP) = S \\ 0, \#(VoiceP) < S \end{cases} \quad (8)$$

To reduce the dropping probability of HCs, a fixed number of guard channels is reserved exclusively for the HCs [25]. By using the guard channel policy, dropping probability can significantly be reduced. However, reserving guard channels exclusively for HC could result in blocking probability increase. To find a balance between these two measures, we define a new measure, GoS (Grade of Service) to consider the composite effect of dropping and blocking probabilities. GoS is defined by the following equation [29],

$$GoS = Pb + \omega Pd \quad (9)$$

Where ω is a weighting factor that decides how much emphasis is placed on HCs.

5. NUMERICAL RESULTS

Table 5 summarizes the parameters used.

Table 5. Summerize parameters in numerical result.

Parameter	Meaning	Value
S	Number of total channel	10
λ_{vn}	New call arrival rate	4
λ_{vh}	Handoff call arrival rate	λ_{vn} * 0.6114
μ_{vt}	Service rate	0.5
μ_{vout}	Departure rate	0.1
λ_{vf}	Call failure rate	0.000016677
μ_{vr}	Call repair rate	0.0167
ω	Weighting factor of GoS	10

Fig. 10 compares the NC blocking and HC dropping probability of the exact composite model. In Fig. 10, we have plotted P_b and P_d as functions of the number of guard channel g. We assume that a set of S =10 channels are assigned to each cell. As g increases, P_b increases and P_d decreases.
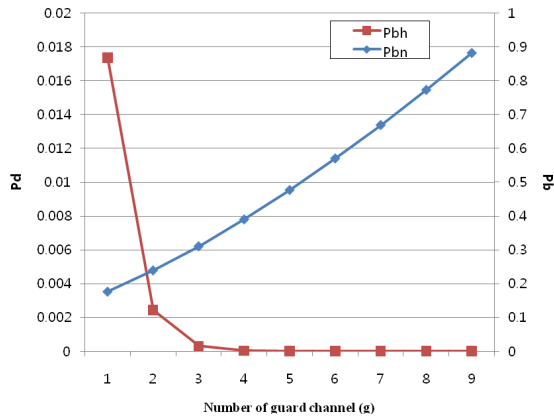

Fig. 10. Blocking and dropping probability of the exact composite model.
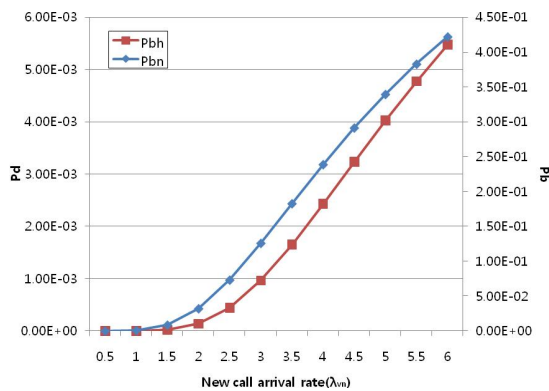

Fig. 11 Blocking and dropping probability of the exact composite model under NC arrival rates

In Fig. 11, we have measures P_b and P_d for fixed the number of total channels, S =10 and guard channels, g=2 and for various values of inverse of mean NC arrival rate from 0.5 to 6. We observe that P_b and P_d are increased as the NC arrival rate increases.
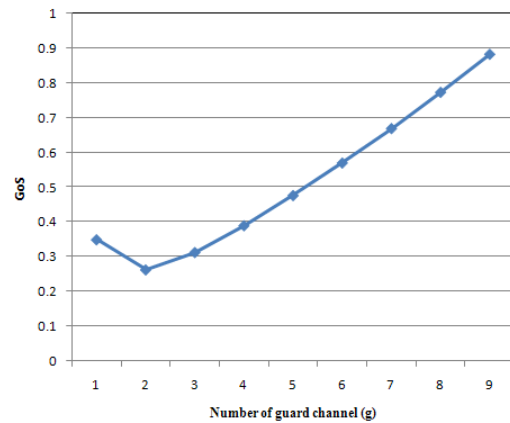

Fig. 12. GoS of the exact composite model.

Numerical experiments are conducted to obtain the optimal value for g. According to the results shown in Fig. 12, we choose to use g=2 to obtain the numerical results in this section.

We tabulate some results form composite model and hierarchical model to show the accuracy of hierarchical model. Table 6 compares blocking and dropping probabilities from both models. The presented results from the two modes show negligible difference. The proven high accuracy of hierarchical models with respect to accurate composite models allows us to carry out a variety of experiments with much higher computational efficiency.

Table 6. Comparison of blocking and dropping probabilities from composite and hierarchical model.

s	Pd			Pb		
	composite(A)	Hierarchical(B)	|A-B|	composite(C)	Hierarchical(D)	|C-D|
3	0.880994088	0.881246904	0.000253	0.003413959	0.003402496	0.000011
4	0.771909016	0.772152614	0.000244	0.0053636	0.005352896	0.000011
5	0.668047212	0.668295379	0.000248	0.00583823	0.005831471	0.000007
6	0.569206796	0.569462658	0.000256	0.005558851	0.005554965	0.000004
7	0.475943412	0.476207542	0.000264	0.004904766	0.004902965	0.000002
8	0.389107844	0.389379258	0.000271	0.004087448	0.004087193	0.000000
9	0.309705408	0.309981517	0.000276	0.003235956	0.003236834	0.000001
10	0.238792149	0.239068509	0.000276	0.002433525	0.002435178	0.000002
11	0.177344031	0.17761413	0.000270	0.001733078	0.001735177	0.000002
12	0.126082219	0.126337658	0.000255	0.001163554	0.001165789	0.000002
13	0.085270903	0.085502173	0.000231	0.000732937	0.00073504	0.000002

6. CONCLUSION

We have developed a performability model of wireless handoff scheme using a SRN and presented the exact composite Markov chain models for performance and availability study of wireless networks. We then have followed a reward-assigning approach to develop the two-level hierarchical peformability models. Measures of interest, such as NC blocking probability,

HC dropping probability, and GoS are given by using reward rates in SRN models. A fixed point iteration based scheme to determine handoff arrival rate into a cell is used. Compared with composite models, the more robust and less time-consuming hierarchical models are proven to be able to provide high accuracy.

APPENDIX

A. M/M/1/K queue
A.1 SPNP source

```
…
/*Marking dependent firing rate*/
double rate_serv() {
  if ( mark("queue") < m ) return( mark("queue")*mu);
  else return(m*mu);
}

/*function to define an SRN*/
void net() {
    place("queue");
    rateval("trin",lambda);
    ratefun("trserv",rate_serv);
    oarc("trin"," queue "); mharc("trin"," queue ",k);
    iarc("trserv"," queue ");
}
…
```

A.2 Reachability graph (k=2)

```
_nplace = 1;
_ntrans = 2;
_places =
0: buf;
_transitions =
0: trin;
1: trserv;
_ntanmark = 3;
_nabsmark = 0;
_nvanmark = 0;
_nvanloop = 0;
_nentries = 4;
_reachset =
# buf
0_t :
1_t 1
2_t 2
_reachgraph =
0 1:trin:3.000000e+00;
1 0:trserv:3.300000e+00 2:trin:3.000000e+00;
2 1:trserv:3.300000e+00;
```

B. SPNP source for Fixed Point Iteration

```
void net() {
    /* parameters */
    parm("lam_vh");

    ..

    /* transition */
    ..
    rateval("VoiceHandOff",1.0);
    useparm("VoiceHandOff","lam_vh");

    ..
    /* assign parameters */
    bind("lam_vh", lam_vh);
    …
}

void ac_final() {
    /* iteration   */
    int i;
    double tp,dtp,err;

    for (i=1; i<MAX_ITERATIONS; i++) {
        pr_value("lam_vh", lam_vh);
        bind("lam_vh", lam_vh);
        solve(INFINITY);
        tp = expected(hotput);
        pr_value("Throughput of VoiceOut", tp);
        err = fabs((lam_vh-tp)/tp);
        pr_value("Error", err);
        if( err < MAX_ERROR ) break;
        lam_vh = tp;
    }
…
}
```

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