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Efficient electromagnetic boundary conditions to accelerate optimization of RF devices

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ABSTRACT

To achieve efficient field formulations and fast numerical computations, the reciprocal relations and equivalence between tangential and normal boundary conditions for electromagnetic fields are discussed in terms of the Maxwell's differential equations. Using the equivalence of each boundary condition, we propose the six essential boundary conditions, which may be applicable to matching electromagnetic discontinuities to efficiently design RF devices. In order to verify our approach, the reflection characteristics of a rectangular waveguide step are compared with respect to six essential boundary conditions.

Keywords: *Wireless Communication System, RF Device, Electromagnetic Wave, Boundary Condition.*

1. INTRODUCTION

In a wireless communication system [1]-[3], the performance of RF devices is of utmost importance to achieve high SNR (Signal to Noise Ratio) and reliable communication link. For better performance, a RF device should have low reflection coefficient, low insertion loss, simple structure, easy fabrication, etc. Since many researchers have investigated the RF scattering mechanism, the RF design procedures are well established.

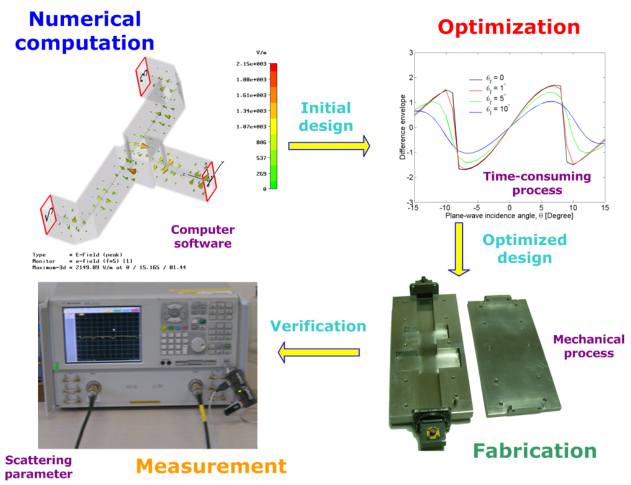

Fig. 1. Standard design procedure of RF devices

Fig. 1 illustrates the standard design procedure for RF devices which is composed of numerical computation, optimization [4], fabrication, and measurement. Among these steps, the optimization [4] is very time-consuming and deeply related to numerical computation scheme. However, when we have fast numerical computation algorithm, we can efficiently accelerate the optimization step. Numerical computation algorithms for RF devices include mode-matching technique, MoM (Method of Moments), FDM (Finite Difference Method), FEM (Finite Element Method), etc. For canonical structures such as waveguide structures, microwave resonators, and standard scatterers, the mode-matching technique is well known for fast computation time and low memory consumption. To apply the mode-matching technique, the electromagnetic boundary conditions (BCs) should be invoked beforehand. The enforcement of electromagnetic BCs is also one of fundamental problems in the application of a classical electromagnetics [5]-[12]. In [5]-[8], the concept of BCs has been extensively investigated and extended by means of four vector potentials, generalized impedance BCs related to higher order derivatives, differential forms, and the uniqueness theorem, respectively. The dependent relations of tangential and normal BCs are discussed and proved with the Stokes' theorem [9]. In [11], [12], the normal BCs for a planar surface are proposed based on the Maxwell's equations.

In this work, we present a simple yet rigorous proof of the reciprocal relations and equivalence between tangential and normal BCs in terms of the Maxwell's differential equations, thus facilitating more rapid and efficient numerical computations. Utilizing this equivalency, we will present six essential BCs. Numerical computations will be also performed to check the validity of applying the essential BCs. In Sect. 4, we will prove that the BCs composed of normal fields produce

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simpler matrix equations than those of tangential fields.

2. RELATIONSHIP BETWEEN TANGENTIAL AND NORMAL BOUNDARY CONDITIONS

Let's consider the sourceless Maxwell's equations as

$$\overline{\nabla} \times \overline{E} = i\omega \overline{B} \quad (1)$$

$$\overline{\nabla} \times \overline{H} = -i\omega \overline{D} \quad (2)$$

$$\overline{\nabla} \cdot \overline{D} = 0 \quad (3)$$

$$\overline{\nabla} \cdot \overline{B} = 0 \quad (4)$$

where we use the $e^{-i\omega t}$ time convention. In terms of tangential fields $\overline{E}_t, \overline{H}_t$ and normal fields $\overline{D}_n, \overline{B}_n$ with respect to an electromagnetic boundary layer in Fig. 2, the Maxwell's equations from (1) to (4) can be simplified as

$$\overline{\nabla}_t \times \overline{E}_t = i\omega \overline{B}_n \quad (5)$$

$$\overline{\nabla}_t \times \overline{H}_t = -i\omega \overline{D}_n \quad (6)$$

$$\overline{\nabla}_t \cdot \overline{E}_t = -\partial E_n / \partial n + \rho_e / \varepsilon \quad (7)$$

$$\overline{\nabla}_t \cdot \overline{H}_t = -\partial H_n / \partial n + \rho_m / \mu \quad (8)$$

where ρ_e, ρ_m are equivalent electric and magnetic charge densities, respectively, which are frequently introduced in waveguide discontinuity problems. The concept of equivalent charge densities will be explained in Sect. 3.

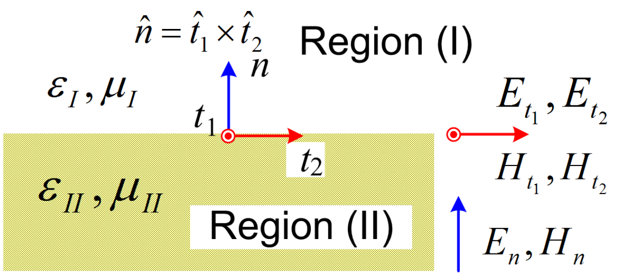

Fig. 2. Geometry of an electromagnetic boundary layer

For the tangential fields $\overline{E}_t, \overline{H}_t$ in a sourceless medium, using (1) and (2), the ordinary matching BCs are given by [13]

$$\overline{E}_t^I = \overline{E}_t^{II} \text{ and } \overline{H}_t^I = \overline{H}_t^{II} . \quad (9)$$

Subtracting (5) and (6) for regions (I) and (II) gives the relations between tangential and normal fields as

$$\frac{\partial}{\partial t_1}(E_{t_2}^I - E_{t_2}^{II}) - \frac{\partial}{\partial t_2}(E_{t_1}^I - E_{t_1}^{II}) = i\omega(B_n^I - B_n^{II}) \quad (10)$$

$$\frac{\partial}{\partial t_1}(H_{t_2}^I - H_{t_2}^{II}) - \frac{\partial}{\partial t_2}(H_{t_1}^I - H_{t_1}^{II}) = -i\omega(D_n^I - D_n^{II}), \quad (11)$$

where $\hat{t}_1 \perp \hat{t}_2 \perp \hat{n}$, $\hat{t}_1 \times \hat{t}_2 = \hat{n}$, and $\hat{t}_{1,2}$ and $\hat{n}$ are the tangential and normal unit vectors to a boundary layer in Fig. 2, respectively. Inserting (9) into (10) and (11), we obtain the auxiliary BCs as [9]

$$B_n^I = B_n^{II} \text{ and } D_n^I = D_n^{II} . \quad (12)$$

Even though the normal BCs (12) can be alternatively derived with a conceptual infinitesimal cylinder [13], (9) and (12) show that tangential BCs result in normal BCs, thus confirming that tangential and normal BCs are not independent each other.

Similarly, manipulating (7), (8), and (9), we also get the another auxiliary BCs as

$$\frac{\partial}{\partial n} E_n^I - \frac{\rho_e^I}{\varepsilon_I} = \frac{\partial}{\partial n} E_n^{II} - \frac{\rho_e^{II}}{\varepsilon_{II}} \quad (13)$$

$$\frac{\partial}{\partial n} H_n^I - \frac{\rho_m^I}{\mu_I} = \frac{\partial}{\partial n} H_n^{II} - \frac{\rho_m^{II}}{\mu_{II}} . \quad (14)$$

In view of the Maxwell's divergence equations, (3) and (4), and the uniqueness theorem in electromagnetics [13], the normal BCs (12) should be equivalent to (13) and (14) when tangential BCs (9) are given.

For the normal fields in a sourceless medium, we can select the matching BCs as

$$D_n^I = D_n^{II} \text{ and } B_n^I = B_n^{II} \quad (15)$$

$$E_{t_1}^I = E_{t_1}^{II} \text{ and } H_{t_1}^I = H_{t_1}^{II} . \quad (16)$$

It should be noted that we can replace (16) for the t_1-axis with that for the t_2-axis as

$$E_{t_2}^I = E_{t_2}^{II} \text{ and } H_{t_2}^I = H_{t_2}^{II} . \quad (17)$$

Applying (15) and (16) to (10) and (11), we may obtain the additional BCs as

$$\frac{\partial}{\partial t_1}(E_{t_2}^I - E_{t_2}^{II}) = 0 \quad (18)$$

$$\frac{\partial}{\partial t_1}(H_{t_2}^I - H_{t_2}^{II}) = 0 . \quad (19)$$

Based on (18) and (19), we define the tangential field relations for region (I) and (II) as

$$E_{t_2}^{I} = E_{t_2}^{II} + C_E(t_2, n) \quad (20)$$

$$H_{t_2}^{I} = H_{t_2}^{II} + C_H(t_2, n), \quad (21)$$

where $C_E(t_2, n)$ and $C_H(t_2, n)$ are arbitrary functions of only one-variable n. Inserting (20) and (21) into (1) and (2) yields

$$\overline{\nabla} \times \left[C_E(t_2, n)\hat{t}_2 \right] = i\omega\mu_{II} C_H(t_2, n)\hat{t}_2 \quad (22)$$

$$\overline{\nabla} \times \left[C_H(t_2, n)\hat{t}_2 \right] = -i\omega\varepsilon_{II} C_E(t_2, n)\hat{t}_2 \quad (23)$$

In order to simultaneously satisfy (22) and (23), we should have $C_E(t_2, n) = C_H(t_2, n) = 0$. This means that (15) and (16) yield (17) and in the same manner (15) and (17) result in (16).

3. ESSENTIAL BOUNDARY CONDITIONS AND THEIR EQUIVALENCY

Utilizing the reciprocal relationship between tangential and normal BCs in Sect. 2, we can define the essential BCs which are equivalent each other. Since the tangential BCs (9) produce the normal BCs (12), (13), and (14) and vice versa, we obtain the six equivalent BCs, any one of which makes us have unique solutions.

Table 1. Six essential boundary conditions

Condition	Subcondition (a)	Subcondition (b)
(I)	E_{t_1} and H_{t_1}	E_{t_2} and H_{t_2}
(II)	εE_n and μH_n	E_{t_1} and H_{t_1} or E_{t_2} and H_{t_2}
(III)	$\frac{\partial E_n}{\partial n} - \frac{\rho_e}{\varepsilon}$ and $\frac{\partial H_n}{\partial n} - \frac{\rho_m}{\mu}$	E_{t_1} and H_{t_1} or E_{t_2} and H_{t_2}
(IV)	εE_n and $\frac{\partial H_n}{\partial n} - \frac{\rho_m}{\mu}$	E_{t_1} and E_{t_2}
(V)	$\frac{\partial E_n}{\partial n} - \frac{\rho_e}{\varepsilon}$ and μH_n	H_{t_1} and H_{t_2}
(VI)	$\frac{\partial E_n}{\partial n} - \frac{\rho_e}{\varepsilon}$ and μH_n	εE_n and $\frac{\partial H_n}{\partial n} - \frac{\rho_m}{\mu}$

Table 1 shows the six essential BCs for electromagnetic fields, where each essential BC is composed of Subconditions (a) and (b) in Table 1. To understand Table 1, let's compare and get equivalency of the tangential and normal BCs. Continuity of tangential electric fields (9) is equivalent to that of normal magnetic flux densities (12) and normal derivative of normal electric fields (13). Similarly, continuity of tangential magnetic fields (9) does the same to that of normal electric flux densities (12) and normal derivative of normal magnetic fields (14). In addition, these continuities are vice versa. This indicates that the tangential and normal BCs in (9), (12), (13), and (14) have equivalent relations which are overtly organized in Table 1.

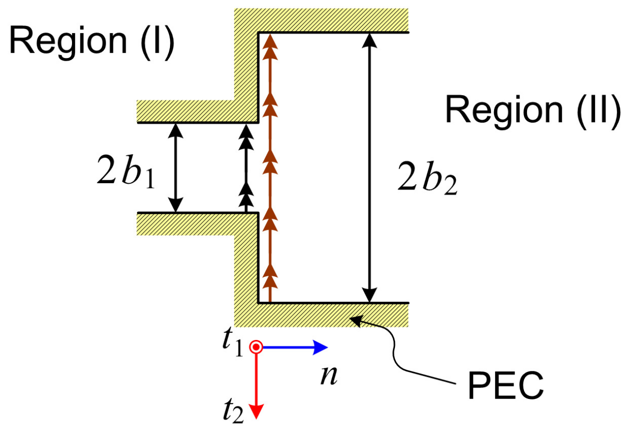

Fig. 3. Geometry of a PEC abrupt step

In the derivation of tangential and normal BCs, we assume there may be equivalent charge densities, ρ_e and ρ_m, in geometrical discontinuity. To figure out this assumption, let's regard a PEC (Perfect Electric Conductor) abrupt step in Fig. 3. For the Conditions (III) through (VI), we define the equivalent electric and magnetic surface charge densities ρ_{es} and ρ_{ms} at the waveguide step in Fig. 3. As such, we can get

$$\rho_{es}^{I} = -\varepsilon_1 \overline{E}_t^{I} \cdot \hat{n}_I \text{ and } \rho_{es}^{II} = 0 \quad (24)$$

$$\rho_{ms}^{I} = 0 \text{ and } \rho_{ms}^{II} = -\mu_2 \overline{H}_t^{II} \cdot \hat{n}_{II}, \quad (25)$$

where $\hat{n}_I$ and $\hat{n}_{II}$ are the outward normal vectors to the waveguides in regions (I) and (II), respectively. The definition in (24) and (25) can be understood in terms of electromagnetic BCs. As shown in Fig. 3, the tangential electric field $\overline{E}_t$ is normal to the PEC surface. This means that there should be surface electric charge density ρ_{es} on the PEC surface [13]. When we set up the simultaneous equations for Fig. 3, the integration interval for the electric field is from $-b_2$ to b_2. Thus, ρ_{es} is nonzero for region (I) and zero for region (II). This is because the edge points of region (I) at $t_2 = \pm b_1$ are within the integration interval $(-b_2, b_2)$. The formula (25) can also be figured out in terms of magnetic BC in which the normal magnetic field toward a PEC surface should be zero. The integration interval for the tangential magnetic field $\overline{H}_t$ is defined from $-b_1$ to b_1. The magnetic BC is satisfied for region (I), whereas that does not for region (II) due to truncation of the integral interval $(-b_1, b_1)$. As a result, we have to introduce ρ_{ms} for region (II)

4. NUMERICAL ANALYSIS

In order to verify the equivalency of matching BCs in Table 1, we consider a rectangular waveguide step shown in Fig. 4.

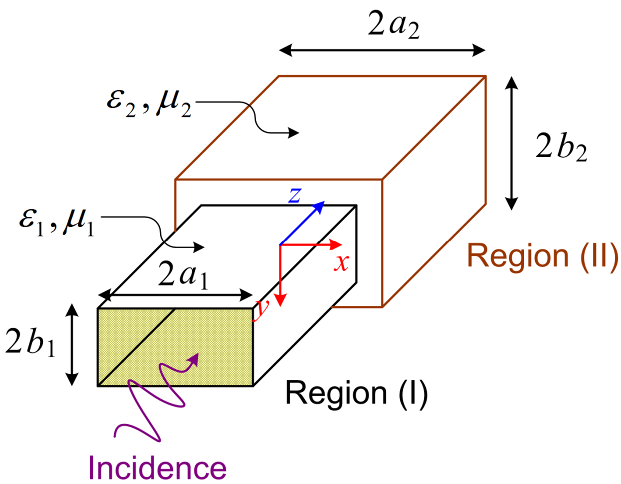

Fig. 4. Geometry of a rectangular waveguide step

Assume that an incident TE$_{m_s n_s}$ mode impinges upon a step. A standard mode-matching technique will be applied and the number of modes for regions (I) and (II) will be chosen to facilitate the mode selection criterion in [14]. The incident and reflected H_z fields in region (I) ($z \le 0$) are

$$H_z^i(\bar{r}) = \cos a_{m_s}^I (x+a_1) \cos b_{n_s}^I (y+b_1) e^{i\beta_{m_s n_s}^I z} \quad (26)$$

$$H_z^r(\bar{r}) = -\cos a_{m_s}^I (x+a_1) \cos b_{n_s}^I (y+b_1) e^{-i\beta_{m_s n_s}^I z}, \quad (27)$$

where $a_m^I = m\pi/2a_1$, $b_n^I = n\pi/2b_1$, $k_1^2 = (k_{mn}^I)^2 + (\beta_{mn}^I)^2$, and $(k_{mn}^I)^2 = (a_m^I)^2 + (b_n^I)^2$. In region (I), scattered electromagnetic fields are represented as

$$E_z^I(\bar{r}) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} p_{mn}^I \sin a_m^I (x+a_1) \sin b_n^I (y+b_1) e^{-i\beta_{mn}^I z} \quad (28)$$

$$H_z^I(\bar{r}) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} q_{mn}^I \cos a_m^I (x+a_1) \cos b_n^I (y+b_1) e^{-i\beta_{mn}^I z}, \quad (29)$$

where $m+n \ne 0$. In region (II) ($z>0$), transmitted fields are

$$E_z^{II}(\bar{r}) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} p_{mn}^{II} \sin a_m^{II} (x+a_2) \sin b_n^{II} (y+b_2) e^{i\beta_{mn}^{II} z} \quad (30)$$

$$H_z^{II}(\bar{r}) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} q_{mn}^{II} \cos a_m^{II} (x+a_2) \cos b_n^{II} (y+b_2) e^{i\beta_{mn}^{II} z}, \quad (31)$$

where $a_m^{II} = m\pi/2a_2$, $b_n^{II} = n\pi/2b_2$, $k_2^2 = (k_{mn}^{II})^2 + (\beta_{mn}^{II})^2$, and $(k_{mn}^{II})^2 = (a_m^{II})^2 + (b_n^{II})^2$.

Multiplying the E_x continuity at $z=0$ by $\cos a_l^{II}(x+a_2)\sin b_k^{II}(y+b_2)$ and integrating over $-a_2 < x < a_2$ and $-b_2 < y < b_2$ results in

$$\sum_{m,n}[(Y_{mn}^I a_m^I p_{mn}^I + b_n^I q_{mn}^I) A_{mn}^I G_{ml}(a) F_{nk}(b) + (Y_{mn}^{II} a_m^{II} p_{mn}^{II} - b_n^{II} q_{mn}^{II}) A_{mn}^{II} a_2 b_2 \alpha_m \delta_{ml}^{nk}] = 0, \quad (32)$$

where $\delta_{ml}^{nk} = \delta_{ml}\delta_{nk}$,

$$\alpha_m = \delta_{m0} + 1 = \begin{cases} 2 & (m=0) \\ 1 & (m=1,2,\cdots) \end{cases} \quad (33)$$

$$G_{ml}(a) = \frac{2a_l^{II}}{(a_l^{II})^2 - (a_m^I)^2} \cos(a_l^{II} a_2 - a_m^I a_1) \sin(a_l^{II} - a_m^I) a_1 \quad (34)$$

$$F_{nk}(b) = \frac{2b_n^I}{(b_k^{II})^2 - (b_n^I)^2} \cos(b_k^{II} b_2 - b_n^I b_1) \sin(b_k^{II} - b_n^I) b_1 . \quad (35)$$

By enforcing the E_y continuity at $z=0$ and multiplying it by $\sin a_l^{II}(x+a_2)\cos b_k^{II}(y+b_2)$, we obtain

$$\sum_{m,n}[(Y_{mn}^I b_n^I p_{mn}^I - a_m^I q_{mn}^I) A_{mn}^I F_{ml}(a) G_{nk}(b) + (Y_{mn}^{II} b_n^{II} p_{mn}^{II} + a_m^{II} q_{mn}^{II}) A_{mn}^{II} a_2 b_2 \alpha_n \delta_{ml}^{nk}] = 0. \quad (36)$$

Similarly, from the H_x and H_y continuities at $z=0$, we get, respectively,

$$\sum_{m,n}[(b_n^I p_{mn}^I - Z_{mn}^I a_m^I q_{mn}^I) B_{mn}^I a_1 b_1 \alpha_n \delta_{ml}^{nk} - (b_n^{II} p_{mn}^{II} + Z_{mn}^{II} a_m^{II} q_{mn}^{II}) B_{mn}^{II} F_{lm}(a) G_{kn}(b)] = -2Z_{m_s n_s}^I a_{m_s}^I B_{m_s n_s}^I a_1 b_1 \alpha_{n_s} \delta_{m_s l}^{n_s k} \quad (37)$$

$$\sum_{m,n}[(a_m^I p_{mn}^I + Z_{mn}^I b_n^I q_{mn}^I) B_{mn}^I a_1 b_1 \alpha_m \delta_{ml}^{nk}$$
$$-(a_m^{II} p_{mn}^{II} - Z_{mn}^{II} b_n^{II} q_{mn}^{II}) B_{mn}^{II} G_{lm}(a) F_{kn}(b)]$$
$$= 2Z^I_{m_s n_s} b^I_{n_s} B^I_{m_s n_s} a_1 b_1 \alpha_{m_s} \delta^{n_s k}_{m_s l} \,. \quad (38)$$

In terms of normal fields, we enforce the εE_z and μH_z continuities at $z = 0$ to obtain the scattering relations as, respectively,

$$\sum_{m,n}[p_{mn}^I a_1 b_1 \delta_{ml}^{nk} - \frac{\varepsilon_2}{\varepsilon_1} p_{mn}^{II} F_{lm}(a) F_{kn}(b)] = 0 \quad (39)$$

$$\sum_{m,n}[q_{mn}^I G_{ml}(a) G_{nk}(b) - \frac{\mu_2}{\mu_1} q_{mn}^{II} a_2 b_2 \alpha_m \alpha_n \delta_{ml}^{nk}] = 0 \,. \quad (40)$$

Similarly, applying the $\partial E_z/\partial z - \rho_e/\varepsilon$ and $\partial H_z/\partial z - \rho_m/\mu$ continuities at $z = 0$ yields, respectively,

$$\sum_{m,n}\{[Y_{mn}^I (k_{lk}^{II})^2 p_{mn}^I + C_{ml}^{nk} q_{mn}^I] A_{mn}^I F_{ml}(a) F_{nk}(b)$$
$$+ p_{mn}^{II} \beta_{mn}^{II} a_2 b_2 \delta_{ml}^{nk}\} = 0 \quad (41)$$

$$\sum_{m,n}\{q_{mn}^I \beta_{mn}^I a_1 b_1 \alpha_m \alpha_n \delta_{ml}^{nk}$$
$$+[D_{ml}^{nk} p_{mn}^{II} + Z_{mn}^{II} (k_{lk}^I)^2 q_{mn}^{II}] B_{mn}^{II} G_{lm}(a) G_{kn}(b)\}$$
$$= 2\beta^I_{m_s n_s} a_1 b_1 \alpha_{m_s} \alpha_{n_s} \delta^{n_s k}_{m_s l} \,, \quad (42)$$

where

$$C_{ml}^{nk} = \frac{(a_l^{II})^2 (b_n^I)^2 - (a_m^I)^2 (b_k^{II})^2}{a_m^I b_n^I} \quad (43)$$

$$D_{ml}^{nk} = \frac{(a_l^I)^2 (b_n^{II})^2 - (a_m^{II})^2 (b_k^I)^2}{a_m^{II} b_n^{II}} \,. \quad (44)$$

Manipulating (32), (36) through (42), we can compose of the six essential BCs shown in Table 1. When $\hat{t}_1 = \hat{x}$, $\hat{t}_2 = \hat{y}$, $\hat{n} = \hat{z}$, the essential BCs for the geometry in Fig. 4 are formulated in Table 2.

Table 2. Essential boundary conditions for a waveguide step

Condition	Subcondition (a)	Subcondition (b)
(I)	(32) and (36)	(37) and (38)
(II)	(39) and (40)	(32) or (36) (37) or (38)
(III)	(41) and (42)	(32) or (36) (37) or (38)
(IV)	(39) and (42)	(32) and (36)
(V)	(41) and (40)	(37) and (38)
(VI)	(41) and (40)	(39) and (42)

For instance, let's compare the simultaneous equations for Conditions (I) and (VI). The equation set for Condition (I) shown in Table 2 is obtained with the standard tangential field matching and that for Condition (VI) does with the normal field matching proposed in this work. Condition (I) results in much more complex equations than Condition (VI), thus verifying that Condition (VI) is numerically very efficient for most practical applications.

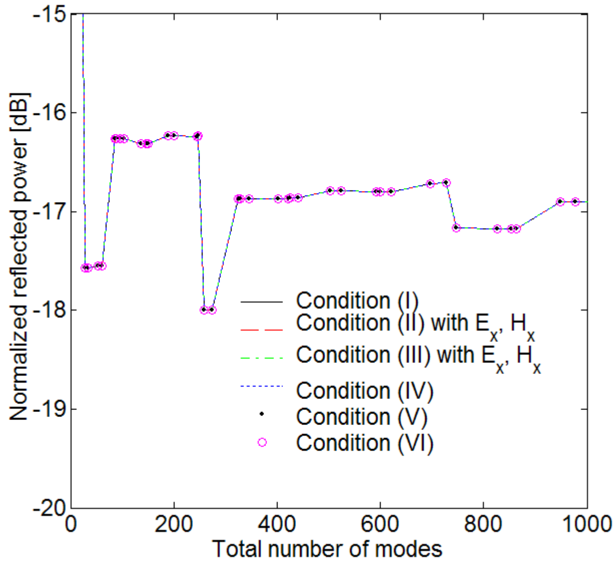

Fig. 5. Convergence characteristics of normalized reflected power with f = 5 [GHz], $\varepsilon_1 = \varepsilon_0$, $\mu_1 = \mu_0$, $\varepsilon_1 = 2\varepsilon_0$, $\mu_1 = 2\mu_0$, $a_1 = 2b_1$ = 20.19 [mm], $a_2 = 2b_2$ = 54.61 [mm]

Fig. 5 shows the convergence characteristics of normalized reflected power versus the total number of modes with respect to the TE_{10} mode incidence. Numerical computations illustrate that the convergence characteristics of the BCs (I) through (VI) agree well with each other.

5. CONCLUSION

By studying the equivalency of tangential and normal electromagnetic boundary conditions, we propose the six essential boundary conditions. We showed that the six essential boundary conditions composed of tangential and normal fields are identical and interchangeable each other. For normal boundary conditions, a novel concept of equivalent charge density is important to obtain proper electromagnetic solutions. Numerical computations for a waveguide step show that our essential boundary conditions are almost identical each other. The boundary conditions composed of normal fields are more efficient than those for tangential fields in terms of computational complexity. In further work, we will investigate the generalization of boundary conditions for general media including metamaterial, plasma, and anisotropic material.

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