[Field Research]

# Model and Heuristics for the Heterogeneous Fixed Fleet Vehicle Routing Problem with Pick-Up and Delivery*

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## Abstract

**Purpose** – This paper discusses the heterogeneous fixed fleet vehicle routing problem with pick-up and delivery (HFFVRPPD), for vehicles with different capacities, fixed costs, and travel costs.

**Research Design, data, methodology** – This paper made nine assumptions for establishing a mathematical model to describe HFFVRPPD. It established a practical mathematical model, and because of the non-deterministic polynomial-time hard (NP-hard), improved the traditional simulated annealing algorithm and tested a new algorithm using a certain scale model.

**Result** – We calculated the minimum cost of the heterogeneous fixed fleet vehicle routing problem (HFFVRP) with a single task and, on comparing the results with the actual HFFVRP for the single task alone, observed that the total cost of HFFVRPPD reduced significantly by 46.7%. The results showed that the new algorithm provides better solutions and stability.

**Conclusions** – This paper, by comparing the HFFVRP and HFFVRPPD results, highlights certain advantages of using HFFVRPPD in physical distribution enterprises, such as saving distribution vehicles, reducing logistics cost, and raising economic benefits.

**Keywords** : Vehicle Routing Problem, VRP with Pick-Up and Delivering, Simulated Annealing Algorithm, HFFVRP, HFFVRPPD.

**JEL Classifications** : C51, C93, D23, L92.

## 1. Introduction

Anily(1996) concluded that the Vehicle Routing Problem (VRP) is one of the most studied combinatorial optimization problems and is concerned with the optimal design of routes to be used by a fleet of vehicles to serve a set of customers. "VRPPD (VRP with Pick-Up and Delivering) as the expansion of VRP need to consider goods distribution and recovery and it is also an important part for an optimal logistics system" introduced by Angelelli and Mansini(2002). VRPPD can be traced back to the 1980s, when the concept of reverse logistics was put forward, as everyone's concern of reverse logistics, the recovery services of forward logistics, and reverse logistics distribution blended together, forming a VRPPD problem.

The VRP with Pickup and Delivery (VRPPD), a heterogeneous vehicle fleet based at multiple terminals must satisfy a set of transportation requests. Each request is defined by a pickup point, a corresponding delivery point, and a demand to be transported between these locations. The requested transport could involve goods or people. This latter environment is called dial-a-ride. The objective function generally minimizes system costs.

Researches on the VRPPD arecarried out by assuming that vehicles that used to provide the pick-up and delivering service are homogeneous. Thatmeans all the vehicles have the same capacity, the same fixed cost, the same maximum distance that allowed in each service plan and so on. Furthermore, the number of vehicles that can be use is also infinite. But inreal distribution, the vehicles of the company are always a fleet with heterogeneous , and the vehicles have a different capacity, the different unit cost for travel, and different fixed cost. Because of the constraints a capital, numbers of each kind of vehicles are also different. Therefore the problem we studied in this paper was proposed: Heterogeneous Fixed Fleet Vehicle Routing Problem with Pick-Up and Delivering, (HFFVRPPD).

This case is the problem we have been come across, and thus in this work, we consider a new version of the VRP problem. We decided to work on its exact real world formulation, without any concession to judicious simplification, which would have probably allowed us to borrow results from existing successful solution techniques.

The paper is organized as follows: Section 2 we present the problem description Section 3 we discuss Algorithm design Section 4 shows the experimental analysis Finally, Section 5 we draw some conclusions and discuss future work.

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## 2. Problem description and model foundation

The Vehicle Routing Problem was first introduced by Dantzig and Ramsey (1959) in what they called The Truck Dispatching Problem. It was formulated as a branch of the Traveling Salesman Problem (TSP) with multiple vehicles and routes. Subsequently, many other extensions that include time windows, different depots, pick-up and delivery options, heterogeneous fleet and periodic routing have been developed by Toth and Vigo(2001). As Liu and Xuan(2005) concluded that most of the researches are focus on improve the algorithm, transforming constraint conditions and the uncertainty of customer information, etc. We review here the articles in the literature that deal with those variants of the HFFVRPPD that are the closest to our problem. Baldacci & Battara (2007) gave an overview of approaches to solve the case of a fleet of vehicles characterized by different capacities, the HVRP (H for Heterogeneous). Some recent construction heuristics for the HFFVRPPD include the ones based on column generation methods by Tailard(1999).

### 2.1. Problem description

From the graph theory, HFFVRPPD can be defined as follow: Assuming that there is a digraph $G=(V,E)$, where $V=\{0,1,2...,I\}$ defines the node-set where '0' is stands for the logistics center and $E=(i,j),i,j\in V,i\neq j$ defines the edge-set. $V'=\{1,2...,I\}$ is defined the node-set of customersthat need service and their requirements pick-up $d_i$ and delivering $p_i$ are known. The distance from customer $i$ to the neighboring customer $j$ is $D_{ij}$. A fleet of vehicles start forms the depot and proved pick-up and delivering service to the customers. $\varphi=\{1,2...,K\}$ is defined the kinds of vehicles, vehicles of kind $k$ have the capacity $Q_k$ and the maximum travel distance $L_k$, and the fixed cost is $f_k$. The variable costs of vehicles of type $k$ between the couple-node $(i,j)$ is $v_k$. The number of vehicles in each type is fixed to $n_k$. The objective of the optimal in HFFVRPPD is that: Arrange the best vehicles routes to provide service to all the customers who need both of the pick-up and delivering service, so that the total cost is a minimum. The total cost contains all the vehicles' fixed cost, travel cost and routes must satisfy the following requirements: (1) The service vehicles must start from the logistic center, and back to the logistics center after the service has been to the customers one by one; (2) Each demand point can only be serviced once, as a service stop, can meet pick-up and delivery requirements; (3) The load capacity of a vehicle at any customer load points cannot exceed vehicle capacity (4)The length of each delivery path does not exceed the maximum travelled distance of the vehicle.

### 2.2. Model assumption

In order to establish the mathematical model to describe HFFVRPPD, the article made the following assumptions:

(1) There is only one logistic center;
(2) Coordinates and required volume of the logistic center and demand points are already known;
(3) The numbers of various types of vehicles are already known and the fixed cost, travel cost and the capacity of various types of vehicles are already known.
(4) There are no time constrains for customers;
(5) One vehicle service, a loop, starting from the logistic center and eventually back to the logistic center;
(6) The weight in a moving vehicle does not exceed vehicle capacity volume restrictions;
(7) The distribution distance of each vehicle is not more than the maximum travel distance;
(8) Each demand point can only be serviced once, as a service stop can meet pick-upand delivery requirements;
(9) Other hypothesis: Assumethe goods in transit will not be degenerated damaged; they do not take the working hours of the drivers into account, do not take the prevailing road conditions into account, regardless of rules and regulations of transportation, etc.

### 2.3. Symbol description and Model foundation

It is necessary to make the description of the symbols, as follows:

$\varphi=\{1,2...,K\}$ : The set of vehicle types, a total of $K$ types;

$m_k$: The number of vehicles type $k$, total number of vehicles is $M=\sum_{k=1}^{K} m_k$

$\phi_k$ : The set of vehicles type $k$, where $\phi_k=\{1,2,...m_k\}$

$Q_k$ : Capacity of vehicles, that is, maximum load of distribution vehicles;

$f_k$ : Fixed costs in traveling;

$v_k$ : Average cost of vehicle type $k$ per kilometer of distribution;

$L_k$ : The maximum travel distance of vehicle of type $k$

$kl$ : The $l$-th vehicle type $k$

$I$ : Total number of customers demand points

$V=I\cup\{0\}$: The set of the entire demand points and logistics center, node 0 stands for the logistic center;

$V'=\{1,2...,n\}$: The set of customers demand points

$D_{ij}$: Distance between customer $i$ and customer $j$,

$$D_{i,j}=\sqrt{(x_i-x_j)^2+(y_i-y_j)^2}$$

$d_i$: Distribution volume of customer $i(0\le i\le I)$ required, $(0\le d_i\le Q_k)$

$p_i$: Pick-upof customer $i(0 \le i \le I)$ required $(0 \le p_i \le Q_k)$

$U_{ikl}$: Load weight of after leaving the customers $i$, $(0 \le U_{ikl} \le Q_k)$

$n_{kl}$: The number of customers the $l$-th vehicle type $k$ served

$n_{klj}$: The $j$-th customer that the $l$-th vehicle type $k$ served, $(0 \le n_{klj} \le n_{kl})$ for example, $n_{423}$ represents the third customer that the second vehicle of type 4 served;

$$x_{ijkl} = \begin{cases} 1, \text{ the } l-\text{th vehicle of type k through node i to customer j} \\ 0, \text{ otherwise} \end{cases}$$

$$y_{ikl} = \begin{cases} 1, \text{ customer i was served by the } l\text{-th vehicle of type k, } i \in I, k \in \varphi, l \in \phi_k \\ 0, \text{ Otherwise} \end{cases}$$

This paper proposed the mathematized model of HFFVRPPD as follows:

$$\min F(x) = \sum_{k=1}^{K}\sum_{l=1}^{m_k}\left(f_k \cdot \sum_{i=1}^{I} x_{0ikl}\right) + \sum_{i=0}^{I}\sum_{j=0}^{I}\sum_{k=1}^{K}\sum_{l=1}^{m_k} x_{ijkl} \cdot D_{ij} \cdot v_k \quad (1)$$

$s.t.$

$$\sum_{k=1}^{K}\sum_{i=0}^{I}\sum_{l=1}^{m_k} x_{ijkl} = 1 \qquad \forall j \in V \quad (2)$$

$$\sum_{i=0}^{I} x_{ipkl} - \sum_{j=0}^{I} x_{pjkl} = 0 \qquad \forall p \in V, \forall k \in \varphi, \forall l \in \phi_k \quad (3)$$

$$\sum_{i=1}^{I} d_j y_{ikl} \le Q_k \qquad \forall k \in \varphi, \forall l \in \phi_k \quad (4)$$

$$\sum_{i=1}^{I} p_j y_{ikl} \le Q_k \qquad \forall k \in \varphi, \forall l \in \phi_k \quad (5)$$

$$0 \le U_{ikl} - \sum_{j=1}^{I} x_{ijkl}\left(d_i - p_i\right) \le Q_k \qquad \forall i \in V, \forall k \in \varphi, \forall l \in \phi_k \quad (6)$$

$$\sum_{i=0}^{I}\sum_{j=0}^{I} x_{ijkl} d_{ij} \le L_k \qquad \forall k \in \varphi, \forall l \in \phi_k \quad (7)$$

Equation (1) is the objective function, denoting the minimum total cost which is composed of fixed cost and travel cost. The constraint (2) is that every customer is visited by exactly one vehicle of one type, and the vehicle is used to serve at the most one route. The constraint (3) is the flowconservation formula, where every vehicle that arrives to a customer must leave that customer. The constraint (4) and (5) means the total recovery of all vehicles should not exceed capacity of the vehicle, and the total distribution of any one route delivery vehicles cannot exceed the total vehicle capacity. The constraint (6) explainsthat the pick-up volume of every vehicle must not exceed the capacity in any customer point. The constraint (7) ensures that the length of each delivery path does not exceed the maximum travelled distance of the vehicle.

# 3. Algorithm design

## 3.1. SA procedure

As HFFVRPPD is an NP-hard problem, an accurate algorithm, even if to the HFFVRP or VRPPD is almost impossible for larger-scale problems. So the heuristic algorithm was used to solve the problem. The basic principle of asimulated annealing algorithm comes from the fact that a solid will collapse from the solid structure into liquid structure if it is heated to a certain temperature, and by controlling its cooling process can make the molecules re-ranked as our expected steady state when they revert to the solid structure. Simulated annealing algorithm has been theoretically proven to be a probability of 1 converges to the global optimal solution of the global optimization algorithm, furthermore, because the best result is of less dependence than theinitial solution which used to research a new VRP.

The HFFVRPPD studied in this paper could be described by the simulated annealing algorithm as a solution of the problem and its objective function $f(i)$ is separately equivalent with a micro state $i$ and its energy $E(i)$.

Steps of the simulated annealing algorithm (takes minimizing the objective function) for example:

Step 1 Get an initial feasible solution $x_0$ randomly, set $t_0$ as the initial temperature, the current solution $x_i = x_0$, the current iteration step $k = 0$, the current temperature $t_k = t_0$

Step 2 If the temperature satisfies the loop stop condition, go to Step 3; otherwise, choose a neighborhood solution $x_j$ randomly from the neighborhood $N(x_i)$ and calculate $\Delta E_{ij} = E(x_j) - E(x_i)$. If $\Delta E_{ij} \le 0$, then $x_i = x_j$ otherwise, if $\exp(-\Delta E_{ij}/t) > rand(0,1)$ (a random number between 0 and 1), go to Step 2;

Step 3 $k = k+1$, $t_{k+1} = y(t_k)$ (temperature control function), If it meets the termination conditions, go to Step 4; Otherwise, go to Step 2;

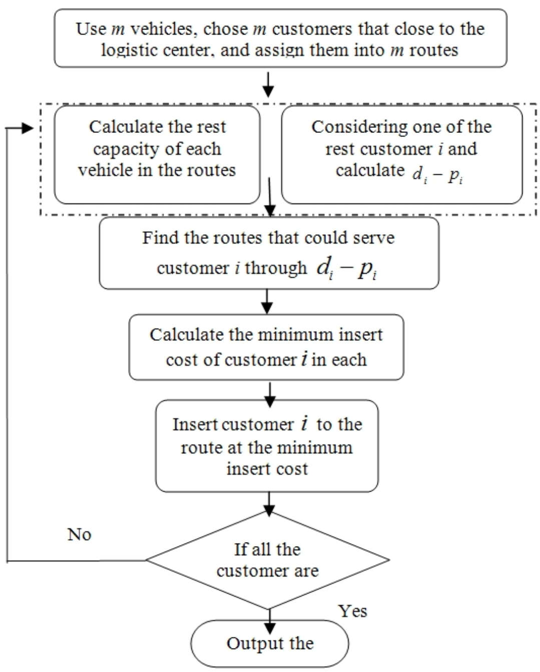
Step 4 Output the results, terminate the SA algorithm. Step two is the inside circle, which indicatesa random search at the same temperature. The outer loop includes the temperature decreased changes, the increase of iterations times, and the stopping criteria in Step 3.

## 3.2. Algorithm optimizing and algorithm flowchart

Considering the characteristics of Heterogeneous Fixed Fleet Vehicle Routing Problem with Pick-Up and Delivering, we optimized the conventional simulated annealing algorithm, and constructed a new simulated annealing algorithm for solving this problem. The improvements are as follows:

(1) Initial solution generation.

The solution is described as $R=(0\text{-}1\text{-}2\text{-}3\text{-}0\text{-}4\text{-}5\text{-}6\text{-}0\text{-}...\text{-}0)$, where 0 representsthe logistics center, a loop starts with 0 and ends with 0, in the middle of which are the customer nodes. We get the initial solution following the flow chart as follows:




<Figure 1> Algorithm of Initial Solution Generation

The minimum insert cost is calculated as follows:

$$c_i^k = \left(d_{i_1,i} + d_{i,i_2} - d_{i_1,i_2}\right) \times v_k \qquad (8)$$

Where $i_1$ and $i_2$ represent the customer before and after the insert location separately.

(2) Neighborhood and a feasible solution.

The traditional neighborhood operation method of the simulated annealing algorithm is $2-Opt$ ex-change. This exchange method is simple and exchange two nodes at one time, but the ability to search the solution space is not strong. So at each temperature, it takes a long time to search the solution space to ensure getting an optimal solution of the temperature. When the temperature slowly decreases, the number of the outer loop increases, the algorithm's time increase multiplies, leading the search time of the algorithm too long. To improve the ability of search solution space, we considered four operators: SWAP operator, the RELOCATE operator, the operator $2-Opt$ and the $2-Opt^*$ operator. Every time there is neighborhood operation, one of the four operators will be chosen to be usedrandomly.

Though this method didn't increase the number of solutions iterations, but the search solution space increases the scope of many, together with a memory array, which can assure of a more satisfactory optimal solution, so you can to a large extent, reduce the length of Markov chain, thereby saving time algorithm.

When it comes to the new solution, we judged if it is afeasible solution by following steps:

Step 1: Decode the new solution, calculate the total number of vehicles used $M'$.if $M'$ if bigger than $M$ , the number of vehicles that the logistics center has, turned to step 7, otherwise, turned to step 2.

Step 2: Calculate the total weight of pick-up $P_j$ and the total weight of delivery $D_j$, if $D_j > \max(Q_k)$ or $P_j > \max(Q_k)$, turned to step 7, otherwise, turned to step 3.

Step 3: Sort the total weight of delivering $D_j$ and the capacity of each vehicle types $Q_k$ by descend, and assigned the types of vehicle by capacity. If the number of routes which the total weight is more than the vehicles capacity is more than the number of vehicles with the capacity bigger than the given vehicles capacity, turned to step 7, otherwise, turned to step 4.

The total lengthof each route is $L_j$, if there is one or more route that the total length is larger than the maximum distance of the vehicle, use $L_k$, turned to step 7, otherwise, turned to step 5.

Step 5: Calculate the weight of each vehicle at every customer point. If there exist a situation that load weight of the vehicle is bigger than the maximum capacity of vehicle of this type, turned to step 7,

Step 6: If the solution is a feasible solution, calculate the value of the objective function, and the evaluated solution, if this satisfies the determinate principle, algorithm end,otherwise, turned to step 7.

Step 7: repeat the neighborhoodoperation, get a new solution , then turn to Step 1.

(3) Memory device design.

In the traditional simulated annealing algorithm, the output cannot guarantee to be the best solution. So we set a memory array S in the algorithm, which records the best solution as the algorithm progresses. At the beginning of the algorithm, S is initialized to be the initial solution, that is, $S=\Omega$, $E(S)=E(\Omega)$. When the next optimal solution $\Omega'$ is found, compare the objective functional value of $E(S)$ with $E(\Omega')$, and if $E(S)>E(\Omega')$, then $S=\Omega'$. Improving $S$ constantly, we can get the best solution of this search.

(4) The determination of the termination criteria.

Adopt mixed-stopping criterion, that is, when the temperature is belowa certain value, or the memory array of consecutive does not change for g times, the algorithm is terminated. With the initial temperature $t_0$, temperature coefficient $\alpha$, it is easy to control the number of iterative steps, and easy to get the global optimal solution. $g$ is used to eliminate unnecessary iteration in order to reduce the number of iterative steps and improve the algorithm efficiency. The value of g varies according to the size of the node, the larger the scale, the relatively larger is the value.

## 4. Computational experiments

Based on the data of the example in reference , assume that a single logistics center served 27customers, each customer need both of the pick-up and delivering service. We need to decide the types of vehicle must use, route of each vehicle to provide service to all the customers with the minimum cost. The information of customers is list in <Table 1>, the distance matrix of customers and logistics center are the same with the research result of Lin et al.(2009) and the information of the vehicles are list in <Table 2>.

<Table 1> Information of Customers' Pick-up and Delivering

| Id | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $d_i$ | - | 160 | 260 | 100 | 400 | 120 | 160 | 120 | 200 | 350 | 650 | 180 | 240 | 420 |
| $p_i$ | - | 130 | 270 | 125 | 333 | 112 | 185 | 157 | 226 | 433 | 467 | 118 | 130 | 222 |
| Id | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 |
| $d_i$ | 420 | 180 | 120 | 400 | 270 | 200 | 80 | 220 | 240 | 360 | 280 | 280 | 150 | 500 |
| $p_i$ | 400 | 109 | 235 | 406 | 238 | 124 | 106 | 150 | 160 | 305 | 229 | 151 | 109 | 640 |

<Table 2> Property of Distribution Vehicles

| Vehicle types | Number of vehicles | Load age/Kg | Fixed cost | Travel cost |
|---|---|---|---|---|
| A | 3 | 1100 | 100 | 1.2 |
| B | 2 | 1115 | 110 | 1.5 |
| C | 1 | 2000 | 95 | 1.1 |
| D | 1 | 850 | 100 | 1.6 |
| E | 4 | 990 | 80 | 1.3 |
| F | 1 | 900 | 120 | 1.4 |

Used the algorithmwe designed in this paper, we get the minimum cost is 912.1, and the details of the results are list in <Table 3>. To compare the cost of HFVRP and HFFVRPPD, we calculate the minimum cost of heterogeneous fixed fleet vehicle problem withthe single task only, and the results are compared in <Table 4>.

From <Table 4>, we find that compared to the heterogeneous fixed fleet vehicle problem with the single task only, total cost of HFFVRPPD was reduced by 46.7%. It greatly reduced the cost, so it is very useful to research the HFFVRPPD.

## 5. Conclusions

Vehicle routing problem forms an integral part of supply chain management, which plays a significant role for productivity improvement in organizations through efficient and effective delivery of goods/services to customers. In this paper, an attempt has been made to survey the recent developments in the vehicle routing problem (VRP) and its variants.

<Table 3> The Best Computational Results

| Number | Vehicle types | Load age | total weight of delivering | total weight of pick-up | Route distance | Route |
|---|---|---|---|---|---|---|
| 1 | C | 2000 | 1980 | 1891 | 179 | 0-12-14-9-8-10-16-0 |
| 2 | A | 1100 | 1040 | 982 | 163 | 0-3-1-2-5-6-22-0 |
| 3 | A | 1100 | 1010 | 1054 | 47 | 0-23-26-27-0 |
| 4 | A | 1100 | 1100 | 730 | 40 | 0-15-19-11-13-7-0 |
| 5 | E | 990 | 970 | 900 | 50 | 0-17-18-20-21-0 |
| 6 | E | 990 | 960 | 713 | 21 | 0-25-4-24-0 |
| Sum $d_i$ 7060 | | | | Fixed cost 555 | | |
| Sum $p_i$ 6270 | | | | Travel cost 357.1 | | |
| Total cost 912.1 | | | | | | |

<Table 4> Comparing the Results of HFVRP and HFFVRPPD

| | HFVRP | | HFFVRPPD |
|---|---|---|---|
| | Delivering | Pick-up | |
| A | 2 | 3 | 3 |
| B | 0 | 1 | 0 |
| C | 1 | 1 | 1 |
| D | 0 | 0 | 0 |
| E | 3 | 0 | 2 |
| F | 0 | 0 | 0 |
| Fixed cost | 535 | 505 | 555 |
| Travel cost | 332.1 | 337.9 | 357.1 |
| Total cost | 1710 | | 912.1 |

From the delivering practice, we have proposed a more realistic heterogeneous fixed fleetvehicle routing problem with pick-up and delivery (HFFVRPPD) model, considering vehicles have different capacity, largest travel distance, fixed cost and travel cost.

As Doris et al.(2008) appointedthat the HFFVRPPD is an NP-hardproblem , we used heuristic approach to get the solution. Based on the analysis of the advantages and disadvantages of the algorithm, we improve the algorithm:we designed the initial solution generation of HFFVRPPD, proposed the neighborhood operation with four operators selectedrandomly in each research, memory device andmulti- termination criteria are also included.

Computational experimentsshow that the model and algorithm are useful to the delivering company, it helps reduce the cost by reduce the number of vehicles need to use and the distance the vehicles travelled.

And we plan to take forward this research as follows.

1. Incorporate the findings of our analysis to an improved solution algorithm. Such a method may only allow splitting customers with the characteristics described in the previous section. This way, we have a good chance of still having the optimum solution within the feasible set, but not doubling the number of customers the method should work faster and have a better chance of finding good solutions.

2. Extend the scope of our analysis to the HFFVRPPD, allowing customers' delivery and pick-up needs to be served in several visits. In particular, we are interested in whether the theoretical properties of the VRPS observed carry over to the HFFVRPPD and It we will try our best to increasethe customer data in the further study.

3. Merging our lines of research in this paper, we wish toinvestigate the HFFVRPPD with Restricted Mixing. This model, forces customers to be served separately to avoid situations where there is a mixture of delivery and pickup goods on board but not enough space to have access to both kinds of goods.

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