

# An analytical solution for soil-lining interaction in a deep and circular tunnel

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## 원형터널에서 지반-라이닝 상호작용에 대한 수학적 해석해에 관한 연구

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**Abstract** This study deals with the analytical solution for soil-lining interaction in a deep and circular tunnel. Simple closed-form analytical solutions for thrust and moment in the circular tunnel lining due to static and seismic loadings are developed by considering the relations between displacement and interaction forces at the soil-lining interface. The interaction effect at the soil-lining interface is considered with new ratios (the normal and shear stiffness ratios). The effects of the ratios on the normalized thrust and the normalized moment are investigated.

**Keywords:** Analytical solution, tunnel, tunnel lining, soil-lining interaction

**요 지** 본 연구는 원형 터널에서 지반-라이닝 상호작용에 대한 수학적 모델에 관한 연구를 다룬다. 정적, 동적 하중으로 인하여 터널 라이닝에서 발생하는 축력과 모멘트를 구하는 간단한 수학적 해가 제시되었다. 수학적 해를 유도하기 위하여 지반-라이닝의 경계면에서의 힘-변위 관계식을 고려하였고, 경계면에서의 상호작용을 고려하고자 새로운 계수비들을 제시하였다. 축력과 모멘트에 대한 계수비의 영향이 조사되었다.

**주요어:** 원형터널, 터널 라이닝, 지반-라이닝 상호작용, 이론적 해석

## 1. Introduction

Prediction of stress resultants (thrust and moment) in the lining due to static and seismic loadings is needed to evaluate the structural stability of the lining (Lee et al., 2008; Ha et al., 2008; Choi et al., 2009). Many efforts have been made to develop simple closed-form analytical solutions for the thrust and moment of tunnel linings due to static and seismic loadings in circular rock or soil tunnel (Burns and Richard, 1964; Höeg, 1968; Kuesel, 1969; Peck et al., 1972; Einstein and Schwartz, 1979; Wang, 1993; Penzien and Wu, 1998; Park et al., 2009). The existing analytical solutions are limited to the following assumptions: (1)

full connection between soil and lining in the direction normal to the lining, (2) two extreme conditions (full-slip and no-slip conditions) in the tangential direction, (3) circular tunnel under plane strain condition, and (4) linear elastic behavior for soil and lining.

In any soil-structure interaction situation, relative movement of the structure with respect to the soil can occur. In numerical methods, interface or joint elements using the linear elastic and perfectly plastic constitutive model have been used to allow the differential movement of the soil and the structure at the soil-structure interface (Day and Potts, 1994).

This study deals with the analytical solutions for soil-lining interaction in a deep and circular tunnel. Simple closed-form analytical solutions for thrust and moment in the circular tunnel lining due to static and seismic loadings are developed by considering the relations

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between displacements and interaction forces at the soil-lining interface. In order to consider the differential movement of the soil and the lining at the soil-lining interface, new normal and shear stiffness ratios,  $K_n$  and  $K_s$ , are introduced. The plane strain analysis assumes the linear elastic behavior for the media of the soil, the lining and the soil-lining interface element. The effects of the ratios on the normalized thrust and the normalized moment are investigated.

## 2. Definition of the problem

Consider a circular tunnel of radius  $R$  located sufficiently below the ground surface and subjected to horizontal and vertical accelerations from an earthquake loading as shown in Fig. 1. The stress state of the soil around the tunnel can be expressed as

$$\sigma_v = (1 - k_v) \gamma H \quad (1)$$

$$\sigma_h = K(1 - k_v) \gamma H \quad (2)$$

$$\tau = -k_h \gamma H \quad (3)$$

where  $\sigma_v$  and  $\sigma_h$  are the normal stresses,  $\tau$  is the shear stress,  $k_v$  and  $k_h$  are the vertical and horizontal acceleration coefficients,  $K$  is the lateral earth pressure coefficient and  $\gamma$  is the unit weight of the soil over height  $H$ .

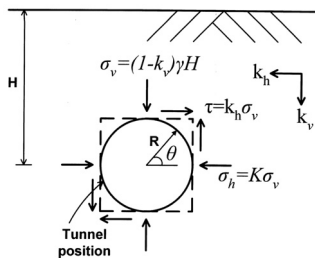


Fig. 1. A circular tunnel.

By neglecting the effect of vertical acceleration, the stress state of the soil around the tunnel during an earthquake can be considered by the sum of the static loading of  $\sigma_v$  and  $\sigma_h$ , and seismic loading of  $\tau$ , as shown in Fig. 2. Further the stress state due to static loading can be divided into isotropic  $\sigma_i$  and deviatoric  $\sigma_d$  parts. The seismic loading is equivalent to a compressive and a tensile free-field principal stresses at  $45^\circ$  with the direction of the pure shear. It is required to solve for the thrust and moment in the lining due to static and additional seismic loadings.

The shear stress can also be estimated as (Penzien and Wu, 1998),

$$\tau = \frac{E_s \gamma_c}{2(1 + \nu_s)} \quad (4)$$

where  $\gamma_c$  is the average free-field shear strain of the soil over the depth  $2R$ , which can be obtained from

$$\gamma_c = \frac{u(-R, t_c) - u(R, t_c)}{2R} \quad (5)$$

$u(y, t)$  is the horizontal free-field ground displacement with depth  $y$  and time  $t_c$  which produces the maximum shear-type deformation of the soil over the depth  $2R$  of the intended tunnel,  $E_s$  is the Young's modulus of the soil, and  $\nu_s$  is the Poisson's ratio of the soil.

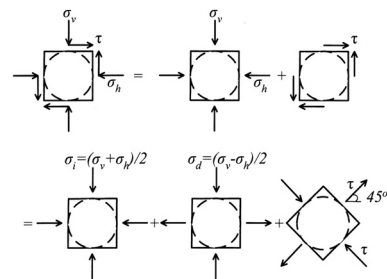


Fig. 2. State of stress during an earthquake.

### 3. Previous analytical solutions

The existing analytical solutions for thrust  $T_s$  and moment  $M_s$  due to static loading can be found for two extreme conditions (Einstein and Schwartz, 1979): for the full-slip condition at the soil-lining interface,

$$T_s = \frac{1}{2}(\sigma_v + \sigma_h)(1 + C_1)R + \frac{1}{2}(\sigma_v - \sigma_h)(1 - 2C_2)R \cos 2\theta \quad (6)$$

$$M_s = \frac{1}{2}(\sigma_v - \sigma_h)(1 - 2C_2)R^2 \cos 2\theta \quad (7)$$

where

$$C_1 = \frac{C^* F^* (1 - \nu_s)}{C^* + F^* + C^* F^* (1 - \nu_s)} \quad (8)$$

$$C_2 = \frac{(F^* + 6)(1 - \nu_s)}{2F^* (1 - \nu_s) + 6(5 - 6\nu_s)} \quad (9)$$

and for the no-slip condition at the soil-lining interface,

$$T_s = \frac{1}{2}(\sigma_v + \sigma_h)(1 + C_1)R + \frac{1}{2}(\sigma_v - \sigma_h)(1 + 2C_2)R \cos 2\theta \quad (10)$$

$$M_s = \frac{1}{4}(\sigma_v - \sigma_h)(1 - 2C_2 + 3C_3)R^2 \cos 2\theta \quad (11)$$

where

$$C_1 = \frac{C^* F^* (1 - \nu_s)}{C^* + F^* + C^* F^* (1 - \nu_s)} \quad (12)$$

$$\beta = \frac{(F^* + 6)C^* (1 - \nu_s) + 2F^* \nu_s}{3F^* + 3C^* + 2C^* F^* (1 - \nu_s)} \quad (13)$$

$$C_3 = \frac{C^* (1 - \nu_s)}{2[C^* (1 - \nu_s) + 4\nu_s - 6\beta - 3\beta C^* (1 - \nu_s)]} \quad (14)$$

$$C_2 = \beta C_3 \quad (15)$$

$C^*$  and  $F^*$  are compressibility and flexibility ratios, which are measures of the extensional and flexural stiffness, respectively, defined as

$$C^* = \frac{E_s R (1 - \nu_l^2)}{E_l A_l (1 - \nu_s^2)} \quad (16)$$

$$F^* = \frac{E_s R^3 (1 - \nu_l^2)}{E_l I_l (1 - \nu_s^2)} \quad (17)$$

$E_l$ ,  $\nu_l$ ,  $I_l$  and  $A_l$  are the Young's modulus, the Poisson's ratio, the moment of inertia and the cross-sectional area of the lining, respectively.

### 4. Present analytical solutions

#### 4.1 Static analysis

For convenience in analysis, the static loading condition in Fig. 2 will be separated into three cases: a circular cylindrical cavity subjected to excavation (Fig. 3(a)), a circular cylindrical lining subjected to contact stresses at the soil-lining interface (Fig. 3(b)), and a circular cylindrical cavity subjected to contact stresses at the soil-lining interface (Fig. 3(c)).

#### 4.1.1 Interaction force and displacement distribution

(1) A cylindrical cavity subjected to excavation

If a circular tunnel is subjected to the static loading of  $\alpha$  and  $\bar{\alpha}$  as shown in Fig. 3(a), the stresses around the cavity leads to the radial and circumferential displacement,  $u$  and  $v$ , respectively (Moore, 1994)

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{Bmatrix} u_i + u_d \cos 2\theta \\ v_d \sin 2\theta \end{Bmatrix} \quad (18)$$

where

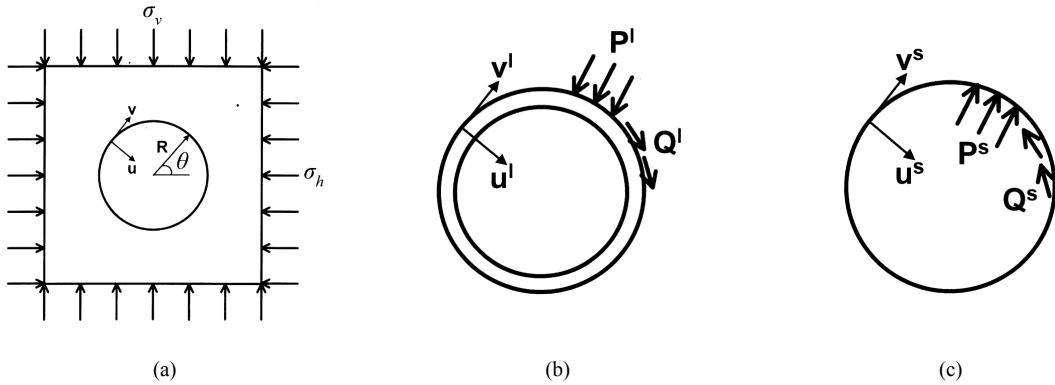


Fig. 3. Separation of static loading.

$$u_i = \frac{\sigma_i R}{2G_s} \quad (19)$$

$$\begin{Bmatrix} u_d \\ v_d \end{Bmatrix} = \frac{\sigma_d R(3-4\nu_s)}{2G_s} \begin{Bmatrix} 1 \\ -1 \end{Bmatrix} \quad (20)$$

$$\sigma_i = \frac{\sigma_v + \sigma_h}{2}, \quad \sigma_d = \frac{\sigma_v - \sigma_h}{2} \quad (21,22)$$

$G_s$  is the shear modulus of the soil. Subscripts  $i$  and  $d$  indicate isotropic and deviatoric components, respectively.

If the lining is considered in the cavity, then the radial and circumferential forces per unit circumference,  $P$  and  $Q$ , develop across the soil-lining interface. The soil experiences the forces  $P^s$  and  $Q^s$  which lead to radial and circumferential displacements  $u^s$  and  $v^s$ , while the lining is subjected to forces  $P^l$  and  $Q^l$  leading to displacements  $u^l$  and  $v^l$ .

(2) A cylindrical lining subjected to contact stresses

The stress-displacement relations for the lining can be expressed as (Flügge, 1966; Einstein and Schwartz, 1979),

$$\frac{d^2 v^l}{d\theta^2} + \frac{du^l}{d\theta} = -\frac{(1-\nu_l^2)R^2}{E_l A_l} Q^l \quad (23)$$

$$\begin{aligned} \frac{dv^l}{d\theta} + u^l + \frac{I_l}{A_l R^2} \left( \frac{d^4 u^l}{d\theta^4} + 2 \frac{d^2 u^l}{d\theta^2} + u^l \right) \\ = \frac{(1-\nu_l^2)R^2}{E_l A_l} P^l \end{aligned} \quad (24)$$

From Eq. (18) and using the following equations,

$$u^L = u^l \cos 2\theta \quad (25a)$$

$$v^L = v^l \sin 2\theta \quad (25b)$$

$$P^L = P^l \cos 2\theta \quad (25c)$$

$$Q^L = Q^l \sin 2\theta \quad (25d)$$

the relationship between displacements  $u^l$ ,  $v^l$  and interaction forces  $P^l$ ,  $Q^l$  in the lining can be expressed using the influence matrix  $I^l$ ,

$$u_i^l = I_i^l P_i^l \quad (26)$$

$$\begin{Bmatrix} u_d^l \\ v_d^l \end{Bmatrix} = I_d^l \begin{Bmatrix} P_d^l \\ Q_d^l \end{Bmatrix} \quad (27)$$

where

$$I_i^l = \frac{R^2(1-\nu_l^2)}{E_l A_l} \quad (28)$$

$$I_d^l = \begin{bmatrix} \frac{R^4(1-\nu_l^2)}{9E_l I_l} & -\frac{R^4(1-\nu_l^2)}{18E_l I_l} \\ -\frac{R^4(1-\nu_l^2)}{18E_l I_l} & \frac{R^4(1-\nu_l^2)}{36E_l I_l} + \frac{R^2(1-\nu_l^2)}{4E_l A_l} \end{bmatrix} \quad (29)$$

(3) A cylindrical cavity subjected to contact stresses

The response of a cylindrical cavity to a continuous cylindrical lining load can be obtained using the plane strain flexibility matrix  $I^s$ , (Moore, 1994)

$$u_i^s = I_i^s P_i^s \quad (30)$$

$$\begin{Bmatrix} u_d^s \\ v_d^s \end{Bmatrix} = I_d^s \begin{Bmatrix} P_d^s \\ Q_d^s \end{Bmatrix} \quad (31)$$

where

$$I_i^s = \frac{R}{2G_s} \quad (32)$$

$$I_d^s = \frac{R}{G_s} \begin{bmatrix} \frac{5-6\nu_s}{6} & -\frac{2-3\nu_s}{3} \\ -\frac{2-3\nu_s}{3} & \frac{5-6\nu_s}{6} \end{bmatrix} \quad (33)$$

#### 4.1.2 Soil–lining interaction

In numerical methods, interface elements have been used to allow the differential movement of the soil and the structure at the soil-structure interface (Day and Potts, 1994). In this study, elastic normal and shear stiffnesses,  $k_n$  and  $k_s$ , are used to simulate the differential movement at the soil-lining interface. Then, using the flexibility equations for soil and lining and introducing normal and shear stiffness ratios,  $K_n$  and  $K_s$ , the interaction forces can be evaluated for interface condition.

(a) Uniform response

For the uniform response of the soil-lining system, the following equilibrium of interaction forces and

compatibility of displacements can be considered:

$$P_i^s + P_i^l = 0 \quad (34)$$

$$u_i = u_i^l - u_i^s + \frac{1}{k_n} P_i^l \quad (35)$$

By substituting the appropriate equations in (18)~(33) into (34)~(35), one can obtain

$$P_i^l = \frac{1}{\left\{ \frac{R^2(1-\nu_l^2)}{E_l A_l} + \frac{R}{2G_s} + \frac{1}{k_n} \right\}} \frac{\sigma_i R}{2G_s} \quad (36)$$

Using the compressibility ratio  $C$  and the normal stiffness ratio  $K_n$ , defined as

$$C = \frac{E_s(1-\nu_s^2)R}{E_l A_l(1+\nu_s)(1-2\nu_s)} \quad (37)$$

$$K_n = \frac{E_s}{2(1+\nu_s)(1-2\nu_s)Rk_n} \quad (38)$$

Eq. (36) can be rewritten as

$$P_i^l = \frac{1}{\{(C + 2K_n)(1-2\nu_s) + 1\}} \sigma_i \quad (39)$$

(b) Deviatoric response

Consider the following equilibrium of interaction forces and compatibility of displacement for the deviatoric response of the soil-lining system,

$$P_d^s + P_d^l = 0 \quad (40)$$

$$Q_d^s + Q_d^l = 0 \quad (41)$$

$$\begin{Bmatrix} u_d \\ v_d \end{Bmatrix} = \begin{Bmatrix} u_d^l \\ v_d^l \end{Bmatrix} - \begin{Bmatrix} u_d^s \\ v_d^s \end{Bmatrix} + \begin{bmatrix} \frac{1}{k_n} & 0 \\ 0 & \frac{1}{k_s} \end{bmatrix} \begin{Bmatrix} P_d^l \\ Q_d^l \end{Bmatrix} \quad (42)$$

When  $k_n \rightarrow \infty$  and  $k_s \rightarrow \infty$ , Eqs. (40)~(42) represent the no-slip interface condition. When  $k_n \rightarrow \infty$  and  $k_s \rightarrow 0$ ,  $Q_d^s$  and  $Q_d^l$  vanish, which means the lining is subjected to sliding along the interface.

By substituting the appropriate equations in (18)~(33) into (40)~(42), one can obtain

$$\begin{cases} P_d^l \\ Q_d^l \end{cases} = \frac{\sigma_d R(3-4\nu_s)}{2G_s \Delta} \begin{cases} \left[ \frac{R^2(1-\nu_l^2)}{4E_l A_l} - \frac{R^4(1-\nu_l^2)}{36E_l I_l} + \frac{R}{6G_s} + \frac{1}{k_s} \right] \\ \left[ -\frac{R^4(1-\nu_l^2)}{18E_l I_l} - \frac{R}{6G_s} - \frac{1}{k_n} \right] \end{cases} \quad (43)$$

where

$$\begin{aligned} \Delta = & \frac{R^2(1-\nu_l^2)}{4E_l A_l} \left\{ \frac{R^4(1-\nu_l^2)}{9E_l I_l} + \frac{R(5-6\nu_s)}{6G_s} \right\} + \frac{R^5(1-\nu_l^2)(3-2\nu_s)}{72E_l I_l G_s} \\ & + \left\{ \frac{R(3-4\nu_s)}{12G_s} \right\}^2 + \frac{1}{k_n} \left\{ \frac{R^4(1-\nu_l^2)}{36E_l I_l} + \frac{R^2(1-\nu_l^2)}{4E_l A_l} + \frac{R(5-6\nu_s)}{6G_s} \right\} \\ & + \frac{1}{k_s} \left\{ \frac{R^4(1-\nu_l^2)}{9E_l I_l} + \frac{R(5-6\nu_s)}{6G_s} \right\} + \frac{1}{k_n k_s} \end{aligned} \quad (44)$$

Using the flexibility ratio  $F$  and the shear stiffness ratio  $K_s$ , defined as

$$F = \frac{E_s(1-\nu_l^2)R^3}{6E_l I_l(1+\nu_s)} \quad (45)$$

$$K_s = \frac{E_s}{2(1+\nu_s)Rk_s} \quad (46)$$

Eqs. (43) and (44) can be rewritten as

$$\begin{cases} P_d^l \\ Q_d^l \end{cases} = \frac{\sigma_d(3-4\nu_s)}{\Delta'} \begin{cases} -F + 1.5(1-2\nu_s)C + 2 + 12K_s \\ -2(F+1) - 12(1-2\nu_s)K_n \end{cases} \quad (47)$$

where

$$\begin{aligned} \Delta' = & \{(3-2\nu_s) + (1-2\nu_s)C\}F + (2.5-8\nu_s + 6\nu_s^2)C + 6-8\nu_s \\ & + K_n(1-2\nu_s)\{2F + 3(1-2\nu_s)C + 4(5-6\nu_s)\} \\ & + 4K_s\{2F + 5-6\nu_s\} + 24(1-2\nu_s)K_n K_s \end{aligned} \quad (48)$$

Note that when  $K_n=K_s=0$ , Eqs. (47) and (48) represent the no-slip interface condition, whereas the case of

$K_n=0$  and  $K_s \rightarrow \infty$  indicates the full-slip interface condition.

Then, the thrust and the moment in the lining due to static loading of  $\sigma_i$  and  $\sigma_d$  can be obtained:

$$T_s = T_l + T_d \cos 2\theta \quad (49)$$

$$M_s = M_d \cos 2\theta \quad (50)$$

where

$$T_l = P_l^l R \quad (51)$$

$$T_d = \frac{R}{3}(-P_d^l + 2Q_d^l) \quad (52)$$

$$M_d = \frac{R^2}{3} \left( -P_d^l + \frac{Q_d^l}{2} \right) \quad (53)$$

## 4.2 Seismic analysis

As mentioned in Section 2, the shear stress due to additional seismic loading is equivalent to a compressive and a tensile free-field principal stresses at  $45^\circ$  with the direction of the pure shear. So the thrust and moment due to additional seismic loading can be obtained by simply substituting  $\tau$  for  $\sigma_d$  and  $\cos 2(\theta + \pi/4)$  for  $\cos 2\theta$  in Eqs. (49)~(53). However, since the shear stress is generated after the cavity is opened, the following  $\sigma_d$  is considered:

$$\sigma_d = \frac{2(1-\nu_s)}{(3-4\nu_s)}(\sigma_v - \sigma_h) = \frac{4(1-\nu_s)}{(3-4\nu_s)}\tau \quad (54)$$

Therefore, the thrust and the moment due to additional seismic loading can be obtained:

$$T_e = T_d \cos 2 \left( \theta + \frac{\pi}{4} \right) \quad (55)$$

$$M_e = M_d \cos 2 \left( \theta + \frac{\pi}{4} \right) \quad (56)$$

where subscript *e* indicates the solution for additional seismic loading.

### 5. Definition of stiffness ratios for interface element

In the previous researches (Burns and Richard, 1964; Höeg, 1968; Peck et al., 1972), the relative stiffness of the lining and the soil is characterized by two ratios (the compressibility ratio and the flexibility ratio). The compressibility ratio *C* is a measure of the extensional stiffness of the soil relative to that of the lining, while the flexibility ratio *F* is a measure of the flexural stiffness of the soil relative to that of the lining: (Peck et al., 1972)

$$C = \frac{\left(\frac{P}{\Delta D/D}\right)_{soil}}{\left(\frac{P}{\Delta D/D}\right)_{lining}} = \frac{E_s}{(1+\nu_s)(1-2\nu_s)} \cdot \frac{R(1-\nu_l^2)}{E_l A_l} \tag{57}$$

$$F = \frac{\left(\frac{P}{\Delta D/D}\right)_{soil}}{\left(\frac{P}{\Delta D/D}\right)_{lining}} = \frac{E_s}{(1+\nu_s)} \cdot \frac{R^3(1-\nu_l^2)}{6E_l I_l} \tag{58}$$

In a similar way, the normal stiffness ratio and the shear stiffness ratio are introduced to characterize the

relative stiffness of the soil and the interface element. The normal stiffness ratio *K<sub>n</sub>* is obtained for a uniform external pressure,

$$K_n = \frac{\left(\frac{P}{\Delta D/D}\right)_{soil}}{\left(\frac{P}{\Delta D/D}\right)_{interface}} = \frac{E_s}{(1+\nu_s)(1-2\nu_s)} \cdot \frac{1}{2Rk_n} \tag{59}$$

and the shear stiffness ratio *K<sub>s</sub>* is obtained for a pure shear,

$$K_s = \frac{\left(\frac{P}{\Delta D/D}\right)_{soil}}{\left(\frac{P}{\Delta D/D}\right)_{interface}} = \frac{E_s}{(1+\nu_s)} \cdot \frac{1}{2Rk_s} \tag{60}$$

### 6. Effects of normal and shear stiffness ratios

The interaction effect at the soil-lining interface is included using the normal and shear stiffness ratios. The effects of these ratios on the normalized thrust and the normalized moment are investigated using the values of  $\nu_s=0.35, \nu_l=0.15, C=1.0, F=1.0$ .

Figs. 4 and 5 show the results of normalized thrust and moment distributions with respect to *K<sub>s</sub>* for *K<sub>n</sub>*=0.5 and *K<sub>n</sub>*=0, while the results for *K<sub>s</sub>*=2.0 are shown

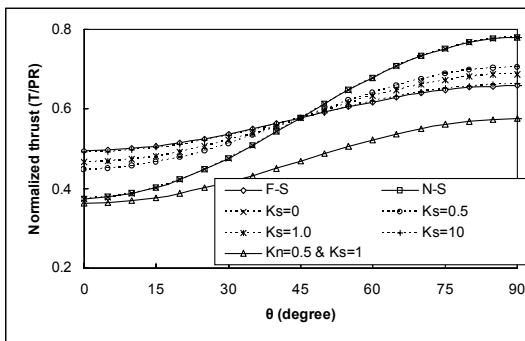


Fig. 4. Normalized thrust (*K*=0.5, *K<sub>n</sub>*=0).

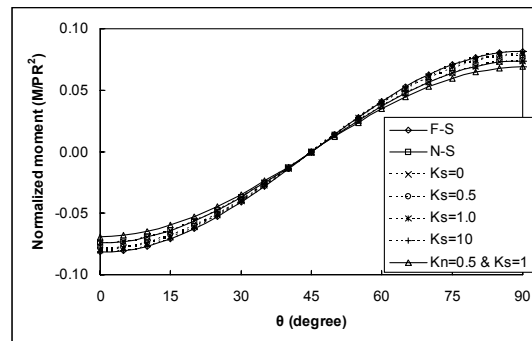


Fig. 5. Normalized moment (*K*=0.5, *K<sub>n</sub>*=0).

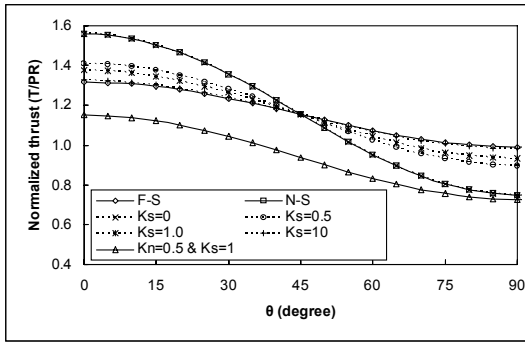


Fig. 6. Normalized thrust ( $K=2.0$ ,  $K_n=0$ ).

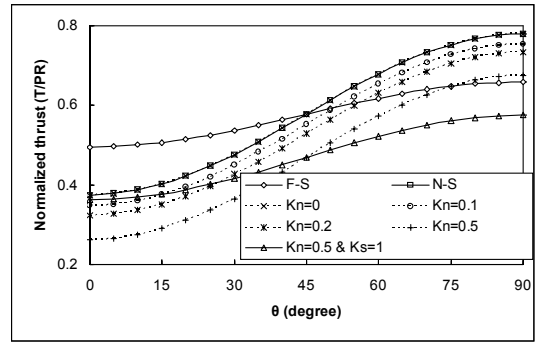


Fig. 8. Normalized thrust ( $K=0.5$ ,  $K_s=0$ ).

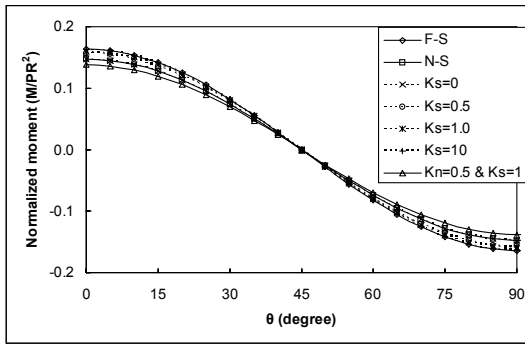


Fig. 7. Normalized moment ( $K=2.0$ ,  $K_n=0$ ).

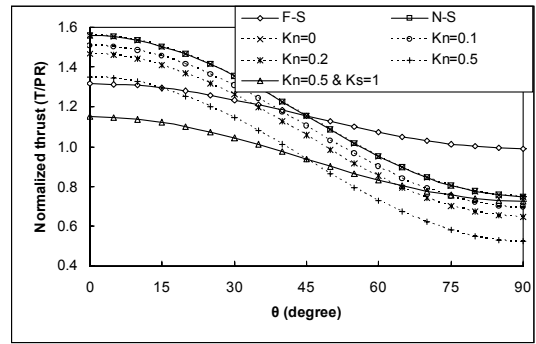


Fig. 9. Normalized thrust ( $K=2.0$ ,  $K_s=0$ ).

in Figs. 6 and 7. For comparison, the results for full-slip (F-S) and no-slip (N-S) conditions, and the case of  $K_n=0.5$  and  $K_s=1$  are also shown in the figures.  $\theta=90^\circ$  indicates the tunnel crown, while  $\theta=0$  indicates the tunnel springline. As expected, the case of  $K_n=K_s=0$  gives the same results as those of no-slip interface condition. The normalized thrust increases with the decrease of  $K_s$  from the full-slip condition to the no-slip condition. The normalized moment decreases as  $K_s$  decreases from the full-slip condition to the no-slip condition. While the shear stiffness ratio  $K_s$  can theoretically vary from zero to infinity, Figs. 4 to 7 show that the shear stiffness ratio seems to be limited to a certain range.

Figs. 8 and 9 show the results of normalized thrust

with respect to  $K_n$  for  $K=0.5$  and  $2.0$  respectively, with  $K_s=0$ . The normalized thrust increases with the decrease of  $K_n$ . The normalized moment decreases as  $K_n$  increases.

## 7. Conclusions

Simple closed-form analytical solutions for thrust and moment in the lining of a deep and circular tunnel due to static and seismic loadings have been presented by considering the relations between displacements and interaction forces at the soil-lining interface. The normal stiffness ratio and the shear stiffness ratio are introduced as a means of defining interface element stiffness. Unlike the existing analytical solutions, which consider only two extreme conditions (the full-slip



interface condition and the no-slip interface condition), new solutions allow the differential movement of the soil and the lining at the soil-lining interface.

The no-slip interface condition is one extreme case of  $K_n=K_s=0$ , whereas the full-slip interface condition is the other extreme case of  $K_n=0$  and  $K_s \rightarrow \infty$ . The normalized thrust increases with the decrease of  $K_n$  and  $K_s$ . The normalized moment decreases with the decrease of  $K_s$  and the increase of  $K_n$ .

## References

1. Burns, J.Q. and Richard, R.M. (1964), Attenuation of stresses for buried cylinders. In Proceedings of the Symposium on Soil-Structure Interaction, University of Arizona, AZ, pp. 378-392.
2. Choi, S.H., Park, I.J. and Kim, S.H. 2009. Aseismic analysis for large underground structure. *Tunnelling Technology*, 11(2), pp. 163-174.
3. Day, R.A. and Potts, D.M. 1994. Zero thickness interface elements-numerical stability and application. *International Journal for Numerical and Analytical Methods in Geomechanics*, 18, pp. 689-708.
4. Einstein, H.H. and Schwartz, C.W. 1979. Simplified analysis for tunnel supports. *Journal of the Geotechnical Engineering Division*, 105, pp. 499-518.
5. Flügge, W. 1966. *Stresses in shells*, Springer-Verlag, Inc., New York, N.Y.
6. Ha, T.W., Kim, D.Y., Shin, Y.W. and Yang, H.S. 2008. Evaluation methods of shotcrete lining stresses considering steel rib capacities by two-dimensional numerical analysis. *Tunnelling Technology*, 10(3), pp. 269-282.
7. Höeg, K. 1968. Stresses against underground structural cylinders. *Journal of the Soil Mechanics and Foundations Division*, 94, pp. 833-858.
8. Kuesel, T.R. 1969. Earthquake design criteria for subways. *Journal of the Structural Engineering Division*, 95(6), 1213-1231.
9. Lee, G.P., Lee, S.W., Shin, H.S. and Hwang, J.H. 2008. Mechanical behavior of tunnel liner using precast segment reinforced by rib. *Tunnelling Technology*, 10(3), 295-302.
10. Moore, I.D. 1994. Analysis of rib supports for circular tunnels in elastic ground. *Rock Mechanics and Rock Engineering*, 27, pp. 155-172.
11. Park, K.H., Tantayopin, K., Tontavanich, B. and Owatsirwong, A. 2009. Analytical solution for seismic-induced ovaling of circular tunnel lining under no-slip interface conditions: a revisit. *Tunneling and Underground Space Technology*, 24, pp. 231-235.
12. Peck, R.B., Hendron, A.J. and Mohraz, B. 1972. State of the art in soft ground tunneling. In Proceedings of the First Rapid Excavation and Tunneling Conference, American Institute of Mining, Metallurgical, and Petroleum Engineers, NY, pp. 259-286.
13. Penzien, J. and Wu, C.L. 1998. Stresses in linings of bored tunnels. *Earthquake Engineering and Structural Dynamics*, 27, pp. 283-300.
14. Wang, J.N. 1993. *Seismic design of tunnels*. Parsons Brinckerhoff Quade & Douglas, Inc., NY, Monograph 7.

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