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Model Equivalence in Covariance Structure Modeling: Definition and Implications*

Soonmook Lee
Fordham University

Testing a causal model, based on empirical data, through covariance structure modeling, often leads to the observation that any particular hypothesized model may have equivalent models. Model equivalence occurs when two or more covariance structure models generate identical covariance matrices. These covariance matrices, commonly referred to as model estimates of covariance matrices or reproduced covariance matrices, must be distinguished from empirically observed or sample covariance data. When two or more models are equivalent, the result is that they are equally fit to any observed data and thus are not distinguishable by data analysis. In proposing a model which supports the hypotheses of interest, an investigator is obliged to rule out the equivalent models by substantive interpretation.

This study is concerned with introducing the issue of model equivalence in covariance structure modeling(CSM) to empirical researchers. Empirical researchers should recognize that testing a model with empirical data by covariance structure modeling does not guarantee a solution unique to that model. That is, models equivalent to a particular hypothesized model may exist. Two or more covariance structure models are equivalent when they generate identical covariance matrices(Joreskog and Sorbom, 1984; Stelzl, 1986), and by implication, the equivalent models represent identical covariance matrices in the population. These covariacne matrices would be generated by the observed variables in the population whose relationships are hypothesized by the model. Thus, they are often called model estimates of covariance matrices, estimated covariance matrices, or reproduced covariance matrices. The model estimate of a covariance matrix must be distinguished from empirically observed or sample covariance matrices to which the models are fitted. Although a model may be correct, the empirically observed covariance data will be different from the model estimate because of sampling error. The degree of discrepancy between a model estimate and empirical covariance matrix determines the

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overall goodness of fit of a model to an empirical data set; overall fit measures are computed based on this discrepancy. When two or more models are equivalent, meaning that they generate identical model estimates of covariance matrices, these covariance matrices will be equally discrepant from any empirical data set to which the models are fitted. This equal discrepancy will result in identical values on the overall fit measure. Thus, equivalant models are not distinguishable by data analysis, in which fit measures are used as the criteria of evaluating goodness of models. While model equivalence does add difficulty to the testing of hypothesized models, it also highlights an important issue often neglected by empirical researchers. In proposing a model which supports the hypotheses of interest, an investigator is obliged to rule out models which explain the phenomena of interest equally well. How to determine the equivalent models and how to evaluate them are essential in applying CSM. In this paper, the focus of discussion will be the definition and implications of model equivalence. Details are provided in the following sections: (a) occurrence of equivalent models in CSM, (b) overview of CSM, (c) model equivalence in CSM, and (d) demonstration of equivalent models and implications of model equivalence.

Occurrence of Equivalent Models in CSM

During the last ten years, CSM has emerged as a powerful technique to investigate structural relationships among variables. Aided by the development of the computer program LISREL(Joreskog and Sorbom, 1984), CSM has been widely used in the behavioral and social sciences to develop and test theories. From the outset of its conceptualization, theorists of CSM have pointed out that for any given model, there may be one or more alternative models that are equally valid in a mathematical and statistical sense, when a covariance structure is analyzed. The existence of equivalent models will become obvious by understanding the nature of modeling in behavioral and social sciences.

In practicing science, it is desirable that an investigator be able to control the variables of interest and observe the effects of such variation on some external criteria, i.e., dependent variables. Natural science is in a better situation to control variables in an experimental laboratory. Comparable control in the social and behavioral sciences is simply not accessible in most situations. In most non-experimental studies, what we can do is observe the resultant of relationships among variables, which resultant is sampled form the population as a form of sample covariance(correlation) matrix. In the process of investigating the structure underlying the covariance (correlation) data, we apply an educated guess about the plausible relationships which would exist in the poupulation. This educated guess involves modeling or constructing hypotheses.

Since correlation data is a special form of covariance data when the variables are standardized, covariances and correlations will be interchangeably used in this paper. Covariance data is the common metric used in modeling possible relationships among variables. CSM is the most popular technique in specifying and testing a model's fit to a covariance data set. Although models may be logically distinct, some of them may generate identical covariance matrices. The covariance matrix which would be generated by population-based variables whose relationships are described by a model is called a model estimate of covariance matrix, estimated covariance matrix, or reproduced covariance matrix in CSM. An estimated covariance matrix is almost always different from a sample covariance matrix. There are two sources causing discrepancy between the sample and estimated covariance matrix: Sampling error due to the nature of the smple and modeling error due to misspecifications in the model. Regardless of different sources and size of this discrepancy, an estimated covariance matrix can be generated by two or more logically different models. In this case, fitting any one of those models to a sample covariance data is equally good, in terms of the likelihood that the empirical data might have been sampled from a population where the fitted model would exit. The equally good likelihood is expressed as equally good fit measures. Thus, for any given sample covariance data, the structure among the variables may be modelled in several different ways which are equally good in mathematical and statistical sense. The term "equivalent model" is named after this mathmatical/statistical equivalence. The formal definition of equivalent models will be given later. Among these equally good or equivalent models, the most plausible model may be determined by interpreting the substantive logic underlying each model. Although investigators search for the best model by analyzing sample covariance data, the model achieved by data analysis may be nothing but an equivalent model to the best model. Since the true model is never known, the term "best model" will be used to represent the most plausible model that would exist in the population.

More than one equivalent model may be observed in empirical data analysis and/or hypothetical model construction. Faced with multiple equivalent models, substantive interpretation is needed to distinguish them. Given the existence of equivalent models to the best model, data analysis does not necessarily lead us to the best model. Without allowing for the possibility of model equivalence, the effectiveness of applying CSM is significantly limited. Despite the importance of model equivalence, the discussion concerning it has not taken place in the literature. Although it has never been questioned that the same set of data may be fitted equally well by quite different models in logic, model equivalence has rarely been subjected to systematic investigation(Stelzl, 1986). Few expository papers have been published on the implications of equivalent models for theory construction and development for applied researchers. Equivalence in CSM has a long history of recognition, but virtually no history of examination. Before detailed discussion of model equivalence, overview of CSM will be presented.

Overview of Covariance Structure Modeling

CSM is well known by other names such as path analysis, simultaneous equation modeling, linear structural relations(LISREL) modeling, or causal modeling. All of these different names represent techniques concerned with hypothesizing, testing, modifying, and cross-validating models to analyse empirically observed covariance data. In scientific research, it is desirable to secure data which will allow cause and effect inference. Usually data appropriate for causal inference can be obtained by controlling independent variables and observing the outcome of dependent variables. However, many situations do not allow researchers to control variables, in which case only the covariances among the variables, and not cause and effect relationships, can be stated. Covariance data simply describes the degree of covariation among a set of variables without explicitly indicating the complex relationships which might underlie them. In order to reveal these complex relationships, various models must be proposed and tested. CSM is a quantitative implementation of analyzing covaricance structures by modeling and testing the network of relationships.

Historically, CSM is an outgrowth of path analysis in biometrics and factor analysis in psychometrics. Path analysis is concerned with the network of measured variables that are explicitly observed. Factor analysis involves extracting factors which are latent in a set of measured variables. Measured variables are indicators of a factor. A factor may be called a latent variable. A measured variable (MV) is a variable that is directly observed and measured. A latent variable(LV) is a hypothetical construct that is not directly measurable, but is approximated by using valid and reliable MVs as indicators.

Models are constructed in CSM based upon the relationship between the LVs and MVs, referred to as the measurement part of CSM, and are constructed based upon the causal relationship among the LVs, referred to as the structural part of CSM. Thus, a covariance strusture model generally consists of two submodels: Measurement model for measurement part and structural model for structural part. For a given set of LVs and MVs, a covariance structure model is defined as a hypothetical pattern of relationships among LVs and MVs. Then, CSM is a general method of modeling and testing the relationships among measured variables and latent variables. These relationships are represented by known or unknow parameters in the model. Given the sample covariance data of MVs, one can estimate the unknown parameters and evaluate the goodness of fit of the model. Depending on the fit measures, model modification is considered (Long, 1983).

A brief review of the mathematical framework of CSM is presented to define the equations, terms and notations which are used throughout this study. Because LISREL (Joreskog and Sorbom, 1984) is the most widely used computer program to solve CSM, the mathematical representation employed in LISREL is presented here.

LISREL Model

Measurement Model: $X=\Lambda_x \xi + \delta$ $\quad Y=\Lambda_y \eta + \varepsilon$

Structural Model: $\eta = B\eta + \Gamma\xi + \zeta$

Measurement model: relationships between MVs and LVs

Structural model: relationships among LVs

The symbols used in the measurement model and the structural model will be described. Each symbol stands for a vector or a matrix.

x : independent MV

y : dependent MV

ξ : independent LV

η : dependent LV

δ : error of independent MV

ε : error of dependent MV

ζ : equation error or residual

Λ_x: factor loading of x on ξ

Λ_y: factor loading of y on η

Γ : path coefficient between independent LV and dependent LV

B: path coefficient between dependent LVs

Assumptions:

1) All variables are measured from their means.

2) None of the structural equations are redundant.

3) The measurement error and the equation error are uncorrelated.

4) The LVs and measurement errors are uncorrelated.

5) The independent LV and the equation error are uncorrelated. For this study, a simpler representation of CSM was developed, which will be called the four-parameter model in contrast to the LISREL model, which uses eight parameter matrices for model specification. In the four-parameter model, different notations to distinguish between independent and dpendent variables in MVs and LVs are not used, although conceptually they are distinguished. This is for simplifying masthematical demonstrations. The differences in notations between LISREL and four-parameter model will be shown.

LISREL	Four-Parameter Model
x, y	y
δ, ε	ε
ξ, η	η
Γ, B	B

Four-Parameter Model

Simple definitions of model will be used.

Measurement Model: $y=\Lambda\eta + \varepsilon$

Structural Model: $\eta = B\eta + \zeta$

Notations are defined as following :

y=vector of MV

Λ=matrix of factor loadings of y on η

η=vector of LV

B=matrix of path coefficients between LVs

ε=vector of measurement error

ζ=vector of equation error or residual

The assumptions are the same as those in LISREL model.

Measurement model and covariance matrix of MVs

The covariance matirx of MVs can be derived as follows from the measurement model. Because all variables are measured from their means, the expectation of an outer product of two vectors of variables will be a covariance matrix of the variables.

$$y=\Lambda\eta+\varepsilon$$
$$y'=\eta'\Lambda'+\varepsilon'$$
$$E(yy')=\Lambda\cdot E(\eta\eta')\cdot\Lambda'+E(\varepsilon\varepsilon')$$
$$\Sigma=\Lambda\omega\Lambda'+\theta \qquad (1)$$

Σ : covariance matrix of MVs

ω : covariance matrix of LVs

θ : covariance matrix of measurement errors

Structural model and covariance matrix of MVs

The relationship between the structural model and the covariance matrix of observed variables is shown below :

$$\eta=B\eta+\zeta$$
$$(I-B)\eta=\zeta$$
$$\eta=(I-B)^{-1}\cdot\zeta$$
$$\eta=\ddot{B}^{-1}\zeta$$
$$\eta'=\zeta'\ddot{B}^{-T}(\ddot{B}^{-T} \text{ stands for the transposition of } \ddot{B}^{-1})$$
$$E(\eta\eta')=\ddot{B}^{-1}(\zeta\zeta')\ddot{B}^{-T}$$
$$\omega=\ddot{B}^{-1}\Psi\ddot{B}^{-T} \qquad (2)$$

Ψ : covariance matrix of ζ

In equation 2, the covariance matrix of LVs or ω is expressed as a function of the parameters in a structural model. By substituting the matrix ω in equation 1 with the right hand side of equation 2, the structural model is represented as a part of the total formation of the covariance matrix of MVs. From equation 1 and equation 2,

$$\Sigma=\Lambda(\ddot{B}^{-1}\Psi\ddot{B}^{-T})\Lambda'+\theta \qquad (3)$$
$$\text{or } \Sigma=\Lambda(I-B)^{-1}\Psi(I-B)^{-T}\Lambda'+\theta \qquad (4)$$

Model specification and parameter estimation

In the four-parameter model, Σ is expressed as a function of four parameter matrices Λ,B,Ψ, and θ (see equation 4). Each parameter matrix of these represents factor loadings(Λ), path coefficients(B), and variances/covariances for error terms (Ψ for ζ and θ for ε) in a path diagram. A path diagram is a structural network of all the variables in a CSM. A simple example of a path diagram is shown in Figure 1 for illustration.

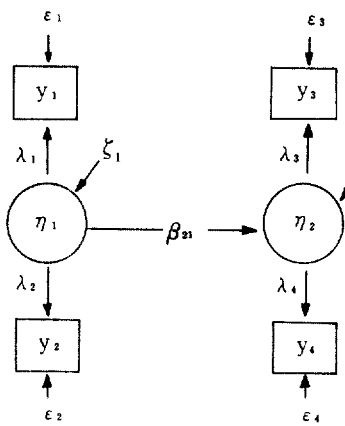

Figure 1. A Simple Diagram of a Covariance Structure Model

Conventionally, circles denote LVs, squares denote MVs, and an arrow indicates direct effect and its direction. Arrows between LVs and MVs indicate factor loadings of MVs on LVs. An arrow between two LVs is called a "path." The "λ" is the element of Λ, "β" is the element of B, the variance/covariance of "ε" is the element of θ, and the variance/covariance of "ζ" is the element of Ψ. Each element in a parameter matrix can be classified as a fixed parameter or free parameter. By fixing or freeing each parameter, the parameter matrices specify the appearance of the path diagram. Fixed parameters are the known coefficients assigned to the paths, factor loadings, or variance/covariance of error terms. If a parameter is fixed to zero, the path, factor loading or error represented by that parameter is eliminated in the path diagram. A path whose coefficient is fixed to zero may be called a "zero path." Free parameters are the unknown path coefficients, factor loadings, or error terms which are estimated by solving the CSM. A path where a free parameter is assigned may be called a "non-zero path."

Solving the CSM in order to estimate free parameters is accomplished by solving the sample version of equation 3:

$$S = \Lambda(\ddot{B}^{-1}\Psi\ddot{B}^{-T})\Lambda' + \theta.$$

Estimating free parameters from the sample covariance matrix S includes two steps: first, extending the right hand side in equation 3 to derive separate equations and second, relating each element in S to a corresponding equation from the first step. The equations obtained from the second step are called normal equations(James, Mulaik, and Brett, 1982). If the number of normal equations is the same as the number of free parameters, the whole model may be just-identified. When the model is just-identified, the numerical values of parameters are determined in only one way. If the number of normal equations is less than the number of free parameters, the model is underidentified and the values of underidentified parameters are not unique and not reliable for interpretation. If the number of normal equations is more than the number of free parameters, the model is almost always overidentified, and the parameters are estimated by unweighted least square, generalized least square, or maximum likelihood methods.

Underidentification is not interesting to investigators because of the indeterminate value of the parameters. just-identification of a whole model does not have scientific parsimony, and therefore provides little assistance in understanding the relationships among the variables. A just-identification model shows a perfect fit to any data set and cannot be statistically tested. Overidentification of a model means that the model is parsimonious but shows less than perfect fit to collected data, Instead of meaningless perfect fit, scientific parsimony is obtained by specifying an overidentified model with a reasonable degree of fit to the data.

Overall fit measures

Once the parameter estimates(Λ, B, Ψ, θ) are obtained by solving the sample version of equation 3, they are substituted into the right hand side of equation 3 to obtain $\hat{\Sigma}$, the model estimate of covariance matrix or reproduced covariance matrix. If the procedure of obtaining parameter estimates is expressed as "S⇨f(parameters)", the procedure of obtaining $\hat{\Sigma}$ is "$\hat{\Sigma}$⇦f(parameters)." If a model is just-identified, parameter values are determined in only one way from S⇨f(parameters) and will be exactly the same as S from S⇦f(determined parameters). However, if a model is overidentified, parameters are estimated optimally satisfying the restrictions given by more equations than are needed to determine the parameter values in only one way. Therefore, there is a discrepancy between S and $\hat{\Sigma}$ obtained by $\hat{\Sigma}$⇦f(estimated parameters).

The discrepancy between $\hat{\Sigma}$ and S represents the lack of fit of the model caused by sampling error and modeling error, and is used to compute overall fit measures, The most commonly used fit index is the χ^2

value and its corresponding probability level (P-value) with which χ^2 is larger than the obtained value from fitting the hypothesized model to the data. The P-value is determined for a particular value and its degrees of freedom. the degrees of freedom will be determined by substracting the number of free parameters from the number of normal equations(i.e., the number of variances and covariances of the MV_s). If the χ^2 value is very large compared to its degrees of freedom, the P-value becomes very small, indicating that the discrepancy between $\hat{\Sigma}$ and S is significantly high and considered to contain more than random sampling error. A large discrepancy between $\hat{\Sigma}$ and S reflects modeling error in addition to sampling error. Then, it is likely that S may have been sampled from a population where the relations of variables can be described by a model other than the initially hypothesized model. The initial model may be modified to make it fit better to the data. However, the χ^2 value and its probability level are obtained under the full information maximum likelihood method, and are therefore sensitive to distribution and sample size. The root mean square residual (RMR) (Joreskog and Sorbom, 1984) is easier to interpret without statistical knowledge because it measures the average residual in relation to the size of the observed variance and covariance in a S matrix(Joreskog and Sorbom, 1984).

$$RMR=\sqrt{[2\Sigma\Sigma(s_{ij}-\hat{\sigma}_{ij})^2]/k(k+1)}$$

where s_{ij} and $\hat{\sigma}_{ij}$ are typical elements of S and $\hat{\Sigma}$ respectively, k is the number of measured variables, and $k(k+1)/2$ is the total number of covariance/variance elements ; $\binom{k}{2}+k=k(k-1)/2+k=k(k+1)/2$. When RMR is small, a model is considered to be a good fit to a dat set. When the input data is a correlation matrix, an RMR value less than .05 is considered to indicate a good fit.

Model Equivalence in CSM

Model Equivalence : Its Definition and Result

When two models are represented by distinct path diagrams, they appear to be different models. However, if the two models generate the identical covariance matrices, i.e. $\hat{\Sigma}$ matrices, they are "equivalent." To emphasize the relationship between the path diagram and the concept of equivalence, equivalence in CSM is defined as the property that models with distinct path diagrams generate identical estimated covariance matrices(Joreskog and Sorbom, 1984 ; Stelzl, 1986). In a mathematical expression, equation $\hat{\Sigma}_1=\hat{\Sigma}_2$ defines the equivalence of model 1 and model 2. By this definition, the same number of MVs in both models are necessary for models to be equivalent because the estimated covariance matrices should be identical elementwise. The number of LVs may or not may be the same because LVs are indirectly estimated by MVs. The same number of LVs between two equivalent models will be assumed for the convenience of illustration in the present study.

Because of sampling error and modeling error, it is expected that there is discrepancy between a sample covariance matrix S, and a model estimate of covariance matrix, $\hat{\Sigma}$. This discrepancy between S and $\hat{\Sigma}$ is the basic component in computing the degree of fit of the model to the given data, i.e. S. When this discrepancy is small, it is attributed to random sampling error. In this situation, the hypothesis that the model may exist in the population is "well supported" by the empirical data. Any two models with identical $\hat{\Sigma}$ will show an equal degree of discrepancy between S and $\hat{\Sigma}$. Thus, it follows that equivalent models, when they are fitted to an empirical data set, necessarily show identical values on the fit measure. That is, equal degree of fit is a necessary result of model equivalence. When the goodness of fit indices are used to determine the best fitting model, as is ordinarilly done in applications of CSM, equivalent models are not distinguishable.

It should be emphasized that the term model equivalence reflects mathematical and statistical equivalence of two models with distinct path diagrams, regardless of difference in substantive logic underlying the models. If equivalent models represent equivalent hypotheses, the issue of model equivalence would not be of any scientific interest. Model equivalence becomes an important issue because any number of models generating identical $\hat{\Sigma}$s are not differentiated by mathematical/statistical ways in spite of different theoretical structures underlying the models.

Equally valid alternatives in modeling covariance structure

Because the true structural relationship is not observable and only the degree of covariation between the variables can be determined from data, there are many alternatives in modeling the structure underlying the empirical data. Unless the correlation between observed variables, X and Y, is caused by an accident(see Kenny, 1987, p.122 for example), the covariation is modelled by one or more of the three effects specifying cause-effect relationship : Direct effect, indirect effect, or spurious effect(Kenny, 1979). A direct effect is X→Y or X←Y, where there is no intervening variable between X and Y. An indirect effect may be expressed by as X→Z→Y or X←Z←Y, where Z represents one or more intervening variable(s). A spurious effect occurs when two variables are correlated only because they are caused by a common cause ; in X←Z→Y, there is no direct or indirect effect between X and Y, rather, X and Y are correlated because of the common cause Z. Thus, based on one correlation between X and Y, one can hypothesize causal relationships in five different ways: X→Y, X←Y, X→Z←Y, X←Z←Y, or X←Z→Y. Any one from of the five is equally valid in modeling the covariance structure between X and Y. When latent variables are involved, the number of modeling alternatives will increase geometrically.

Although the existence of a significant correlation between X and Y implies causality most of the time, correlation alone does not indicate the exact form of causation(Kenny, 1987 ; Mulaik, 1987). Given a correlation between two variables, which one form of causation occurs is uncertain and ambiguous. When variables believed to be causally related are not measured directly but rather are only indicated by measured variables, as in the case of latent variables, hypothesizing structural relations may become more uncertain and ambiguous than when cause-and-effect variables are directly observed(Mulaik, 1987). Mulaik argues that even experimental studies are not free from the uncertainty and ambiguity of causal inference, although they may have relatively less degree of uncertainty and ambiguity than non-experimental studies(see Mulaik, 1987, p.20 for detailed discussion).

Examples of alternative but equivalent models will be shown, using three variables that are assumed to be directly observed. Suppose Model A, based upon the intercorrelation among three variables, is hypothesized as follows: weather condition→psychological state→productivity. In this model, it is implied that one's productivity is directly influenced by his/her psychological state, and that latter is influenced by the weather condition. After collecting data, this model might yield a fitness fa to the data. What happens when an alternative, Model B- "weather condition←psychological state→productivity", - is fit to the same data? This model might appear silly in termsof substantive logic because it is very unlikely that one's psychological state has a direct effect on the weather condition. However, the two models are mathematically equivalent and thus, data analysis to fit Model B will always show the same fitness fa as Model A does. This property, that the same fitness fa is obtained by hypothesizing conceptually different models, is alluded to in many texts on CSM(Pedhazur, 1982 ; Cohen and Cohen, 1983 ; Duncan, 1975 ; Heise, 1975 ; Dwyer, 1983 ; Kenny, 1979 ; Van de Geer, 1971). As for the very good fit of a model, many authors warn not to interpret it as a proof of the model in the population

(Bentler, 1980; Joreskog and Sorbom, 1984; Duncan, 1975 Saris & Stronghorst, 1984). Thus, even with a good fitting model the investigator is obliged to rule out alternative equally good fitting models by a non-quantitative way. Thus, evaluation of multiple equivalent models will be in order.

Evaluation of equivalent models

When data analysis cannot distinguish Model A from Model B, a researcher may be able to determine which is the more plausible based on a substantive interpretation of the equivalent models. In the above example Model B looks odd, making it easy to choose Model A. However, if the two equivalent models C and D are as follows, both are meaningful, so that the choice between the two is very difficult.

Model C: task difficulty→motivation→performance
Model D: task difficulty←motivation→performance

It is possible that task difficulty motivates the worker differently, as in Model C. Inversely, motivation may be a factor in perceiving task difficulty differently , as in Model D. If a researcher hypothesizes a model, he/she should not ignore the existence of competing equivalent a priori models. If the multiple equivalent models are meaningful, as is the case with Models C and D, they can be distinguished only by suitable interpretation because they are equally valid in terms of mathematical and statistical fit. This issue will be discussed again in the implications of model equivalence later.

Demonstration and Implications of Equivalent Models

Demonstration of equivalent models

A numerical example of equivalent models will now be demonstrated. Model 1 "$\eta_1 \rightarrow \eta_2 \rightarrow \eta_3$" and Model 2 "$\eta_1 \leftarrow \eta_2 \rightarrow \eta_3$" are equivalent. To demonstrate their equivalence with respect to numerical data, two MVs are observed for each LV, The complete form of each model is shown in the path diagrams in Figure 2 and Figure 3. Suppose the sample covariance matrix of MVs is as shown in Table

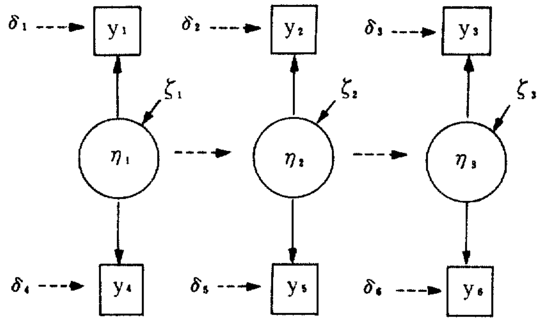

Figure 2. Model I

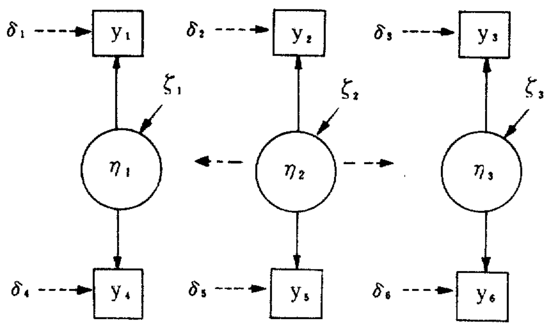

Figure 3. Model 2

Table I Coveriance Matrix of MV

$s=$ y_1	.5					
y_2	.1	.3				
y_3	.01	.01	.6			
y_4	.18	.02	.01	.4		
y_5	.13	.18	.01	.11	.57	
y_6	.05	.01	.20	.01	.01	.7

When S is analysed by LISREL VI, both models produce the same $\hat{\Sigma}$s as shown in Table 2. The $\hat{\Sigma}$ is the "fitted moment matrix" in the LISREL output. The moment represents covariance or correlation.

Table 2 Estimated Covariance Matrix of MV

$\Sigma_1 = \Sigma_2 =$	y_1	.5					
	y_2	.091	.3				
	y_3	.007	.009	.6			
	y_4	.18	.051	.004	.4		
	y_5	.141	.18	.013	.079	.57	
	y_6	.010	.013	.20	.006	.02	.7

The goodness of fit index, RMR is .014 for both models 1 and 2. This fit is very good but not perfect. Regardless of the goodness of fit, model 1 and model 2 produce the same $\hat{\Sigma}$s with any empirical data S, thus indicating that two models are equivalent.

Empirical Implication

At the data analysis stage, a model may be modified by changing its specifications. Changing the specification in hopes of improving parsimony and/or fit of the model to the data is referred to as a specification search(Leamer, 1978 ; Long, 1983 ; MacCallum, 1986). Empirically, model equivalence may occur in the process of specification searches. There are two things to note in this regard. First, in the process of a specification search, a researcher is encountered with a chance of generating equivalent models. Second, given the existence of the best model, specification search may lead to a model which is equivalent to the best model.

Generating equivalent models. In the process of model modification, the researcher may arrive at a point where equivalent modifications can be made, a point where different modifications result in equivalent models. LISREL program provides modification indices (MI) indicating the improvement of fit if a fixed parameter is freed. If the same numerical values of MIs are shown on different parameters, fit will be equally improved regardless of which parameter is freed among those MIs of identical values. The number of MIs that have identical values is the number of models that can be generated by this type of modification ; these models necessarily show equal degree of fit. Although equal fit is not equalized with model equivalence, equivalent $\hat{\Sigma}$s are always observed among these equally fitting models. Therefore, researchers are empirically encountered with a chance of generating equivalent models when two or more MIs show identical values. The issue of model equivalence confronts the investigator, and initiates a decision making as to which parameters should be logically freed.

Arrival at a model equivalent to the best model. After data analysis is done and a model is finally chosen, the model may have equivalent models that have alternative theoretical structures with equal goodness in terms of mathematical and statistical fit. The chosen model may be more plausible than any other of its equivalent models. However, the best model is not always selected. Given the possible existence of equivalent models to a particular model, it is always conceivable that investigators may arrive at one of the equivalent models rather than the best model. Thus, model searches relying on data analysis is not complete in itself because it does not distinguish equivalent models from the best model.

Theoretical Implication

Theoretically, equivalent models can be found a priori by applying some rules (e.g., Stelzl, 1986), although applicability of the rules is limited. When formulating competing models before data analysis, it is possible to hypothesize competing models which would generate identical $\hat{\Sigma}$s if they are fitted to any sample covariance data, S. That is, equivalent models may be constructed at the hypothesis generation stage. The a priori rules to test whether such competing models are equivalent will be discussed later. If the competing models do happen to be equivalent, they cannot be tested against each other. Thus, multiple equivalent models may be constructed a priori, as well as be empirically generated during data analysis. In either case, substantive interpretation is the only way to distinguish them. If some of the equivalent models are theoretically meaningful but not known to the investigator, a serious limitation to theory development exists. The following theoretical implications will be discussed in turn: caution in constructing competing models a priori, the need of substantive interpretation, and the unknown meaningful equivalent models.

Caution in constructing competing models a priori. At the hypothesis generating stage, one model may be constructed to test its fit to the data. The investigator may construct alternative models a priori to compete with the initial model. In terms of the confirmatory use of CSM, this strategy is preferrable to testing only the initial model and continuously modifying it.

Given the possible existence of a set of equivalent models to a hypothesized model, the competing models may be members of this set. If this is the case, it is impossible to test the competing models against the initial model because they will equally fit the data. In this case, data analysis does not provide a way to test competing models against the initial model. Suppose an initial model is "$\eta_1 \rightarrow \eta_2 \rightarrow \eta_3$. " If another model "$\eta_1 \leftarrow \eta_2 \rightarrow \eta_3$" is constructed as a competing model a priori, it would be impossible to test these two models since they are equivalent models.

Need of substantive theoretical interpretation. Multiple equivalent models can be found at the hypothesis generating stage or at the chance of empirically generating equivalent models. When a particular model is chosen at the end of data analysis, one or more equivalent models to the particular model can be generated by applying some rules(e.g., Stelzl, 1986). Equivalent models are equally good in terms of fit. Given that overall fit measures are used as criteria for determining the goodness of a model, equivalent models cannot be distinguished by the fit measures which are mathematical and statisical indices. Rather they can be distinguished by comparing the substantive logic underlying their distinct path diagrams. The value of a given model is determined as much by the logic underlying its structure, as by the empirical demonstration of the goodness of fit to the collected data(Cohen & Cohen, 1983). when a set of models shows an equal degree of fit as a result of model equivalence, empirical data does not provide any information to evaluate the equivalent models.

In order to determine the most plausible among the equivalent models, substantive interpretation of path diagrams is the only way. Since equivalent models may be distinguished by their path diagrams, comparisons of the path diagrams in terms of what they substantively hypothesize will allow the investigators to evaluate equivalent models. Thus, encountered with equivalent models, it is required that one's theory be comprehensive and robust enough to rule out less plausible but mathematically equivalent models. In sum, in order to support the plausible structure of a model, alternative equivalent models are to be ruled out by logical and substantive arguments.

Unknown meaningful equivalent models. In order to obtain equivalent models, some of the directions of the paths of the hypothesized model may be reversed. Inverting the direction of a path means a change in the structural hypotheses, and this usually will bring about a remarkably different interpretation and provide a different perspective in theory development. Thus, if a meaningful equivalent model is not known to the investigator, a serious limitation to theory construction has occurred. By proposing one model without taking into account the possible existence of meaningful equivalent models, theoretical constructions are incomplete (Stelzl, 1986). At any decision stage, to stop a specification search and select a model as the best one is a risky affair, since the possibility exist that there may be more than one meaningful equivalent model to the selected model.

Conclusion

In addition to the important implications of model equivalence, it should be emphasized that a particular model may have equivalent models, regardless of the source of the model and regardless of its degree of fit to the data. Whether the model is initially hypothesized and found to fit the data well, or whether it results from modification of an initial model, there will still exist alternative models that are equivalent and thus will fit equally well, The question of whether the initial or the modified model fits the data well or poorly does not affect the issue of model equivalence. Thus, arguments in terms of the number of modification made to an initial model or degree of fit do not in any way free the researcher from the responsibility of dealing with the issue of equivalent alternative models. Because at least one equivalent model exists to any given model (Lee, 1987), a researcher must evalute and rule out the equivalent model(s) to complete his/her work in proposing a model.

Despite the general awareness of the existence of equivalent alternative models, the issue is seldom, if ever, explicitly dealt with in applied research. It could be that investigators just do not report alternative models that are deemed implausible or perhaps they do not search for models that conflict with their hypotheses. At least one major reason for this is that very little is known about how to identify alternative models which are equivalent to a given model. Stelzl(1986) developed four rules that can be used to check model equivalence or develop equivalent models. Applicability of her rules is limited to recursive structural models, where path directions in the structural part of the CSM is unidirectional. A simplified rule accommodating Stelzl's four rules has been developed by Lee(1988). Rules that are applicable to non-recursive models as well as to recursive models are developed by Lee and MacCallum(1988). However, these rules to discover equivalent models are not exhausitive and not as yet widely introduced. For future research, it would be very desirable to develop an efficient procedure for determining equivalent models, a procedure that would allow researchers not only to discover models equivalent to a given model, but also to explicitly evaluate them. Once equivalent models are determined and evaluated, they can yield increased support for a given model if the alternatives were shown to be implausible, or they could reveal plausible alternative models previously unrecognized.

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공변량 구조모델의 모형등치성

이 순 묵
Fordham University

공변량 구조모델은 현재 행동과학 및 사회과학 분야에서 비실험데이타를 분석하는 데 가장 인기있고 위력있는 다변량 분석 테크닉이다. 공변량 구조모델의 테크닉을 사용해서 모델을 경험데이타에 적용시키는 경우, 어느 특정모델이건 수리통계학적으로 "동질"의 "다른" 모델을 가지게 된다. 공변량 구조모델에서 두 모델이 "다르다"함은 변수간 구조모형(Path Diagram)이 두 모델간에 서로 다름을 의미하며, 두 모델이 수리통계학적으로 "동질"이라 함은 두 모델을 경험데이타에 적용시킬 경우 수리통계학적으로 똑 같은 값의 적용수치(Fit Measure)가 나옴을 의미한다. 변수간 구조모형이 다름에도 불구하고 수리통계학적으로 똑같은 값의 적용 수치를 보이는 모델들을 동류모델들(Equivalent Models)이라 한다. 공변량 구조모델을 이용한 연구결과 도달한 어느 한 모델이 과연 眞 모델(True Model)인지, 또는 眞 모델의 동류모델들중의 하나인지는 데이타 분석에 의해 결정될 수 없고 연구 주제에 기초한 理論的 定性分析에 의해서만 결정된다. 이들 동류모델의 정의와 그들의 존재가 가져오는 제반 문제가 이 논문에서 토의되고 있다.