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CONVERGENCE AND ALMOST STABILITY OF ISHIKAWA ITERATION METHOD WITH ERRORS FOR STRICTLY HEMI-CONTRACTIVE OPERATORS IN BANACH SPACES

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ABSTRACT. Let K be a nonempty convex subset of an arbitrary Banach space X and $T : K \to K$ be a uniformly continuous strictly hemi-contractive operator with bounded range. We prove that certain Ishikawa iteration scheme with errors both converges strongly to a unique fixed point of T and is almost T-stable on K. We also establish similar convergence and almost stability results for strictly hemi-contractive operator $T : K \to K$, where K is a nonempty convex subset of arbitrary uniformly smooth Banach space X. The convergence results presented in this paper extend, improve and unify the corresponding results in Chang [1], Chang, Cho, Lee & Kang [2], Chidume [3, 4, 5, 6, 7, 8], Chidume & Osilike [9, 10, 11, 12], Liu [19], Schu [25], Tan & Xu [26], Xu [28], Zhou [29], Zhou & Jia [30] and others.

1. INTRODUCTION

Let X be an arbitrary Banach, X^* be its dual space and $\langle x, f\rangle$ be the generalized duality pairing between $x \in X$ and $f \in X^*$. The mapping $J : X \to 2^{X^*}$ defined by

$$J(x) = \{f \in X^* : \mathrm{Re}\langle x, f\rangle = \|x\|^2 = \|f\|^2\}, \quad x \in X,$$

is called the normalized duality mapping. It is known that X is uniformly smooth if and only if X^* is uniformly convex. The symbols $D(T)$, $R(T)$ and $F(T)$ denote the domain, the range and the set of fixed points of an operator T, respectively.

Definition 1.1 (Chidume & Osilike [9], Weng [27]). Let X be an arbitrary normed linear space and $T : D(T) \subseteq X \to X$ be an operator.

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(i) T is said to be *strongly pseudocontractive* if there exists $t > 1$ such that for each $x, y \in D(T)$ and $r > 0$

$$\|x - y\| \leq \|(1 + r)(x - y) - rt(Tx - Ty)\|. \tag{1.1}$$

(ii) T is said to be *local strongly pseudocontractive* if for each $x \in D(T)$ there exists $t_x > 1$ such that for all $y \in D(T)$ and $r > 0$

$$\|x - y\| \leq \|(1 + r)(x - y) - rt_x(Tx - Ty)\|. \tag{1.2}$$

(iii) T is said to be *strictly hemi-contractive* if $F(T) \neq \varnothing$ and if there exists $t > 1$ such that for all $x \in D(T)$, $q \in F(T)$ and $r > 0$,

$$\|x - q\| \leq \|(1 + r)(x - q) - rt(Tx - q)\|. \tag{1.3}$$

Clearly, each strongly pseudocontractive operator is local strongly pseudocontractive.

Let K be a nonempty convex subset of an arbitrary normed linear space X and $T : K \to K$ be an operator. Assume that $x_0 \in K$ and $x_{n+1} = f(T, x_n)$ defines an iteration scheme which produces a sequence $\{x_n\}_{n=0}^{\infty} \subset K$. Suppose, furthermore, that $\{x_n\}_{n=0}^{\infty}$ converges strongly to $q \in F(T) \neq \varnothing$. Let $\{y_n\}_{n=0}^{\infty}$ be any bounded sequence in K and put $\varepsilon_n = \|y_{n+1} - f(T, y_n)\|$.

Definition 1.2. (i) The iteration scheme $\{x_n\}_{n=0}^{\infty}$ defined by $x_{n+1} = f(T, x_n)$ is said to be T-stable on K if $\lim_{n\to\infty} \varepsilon_n = 0$ implies that $\lim_{n\to\infty} y_n = q$;

(ii) The iteration scheme $\{x_n\}_{n=0}^{\infty}$ defined by $x_{n+1} = f(T, x_n)$ is said to be *almost T-stable* on K if $\sum_{n=0}^{\infty} \varepsilon_n < \infty$ implies $\lim_{n\to\infty} y_n = q$.

It is easy to see that an iteration scheme $\{x_n\}_{n=0}^{\infty}$ which is T-stable on K is almost T-stable on K. Osilike [23] proved that an iteration scheme which is almost T-stable on X may fail to be T-stable on X.

Let us recall the following three iteration processes due to Mann [20], Ishikawa [16] and Xu [28], respectively.

Let K be a nonempty convex subset of an arbitrary normed linear space X and $T : K \to K$ be an operator.

(i) For any given $x_0 \in K$ the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_{n+1} = (1 - a_n)x_n + a_nTy_n, \quad y_n = (1 - b_n)x_n + b_nTx_n, \quad n \geq 0,$$

is called the *Ishikawa iteration sequence*, where $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ are real sequences in $[0, 1]$ satisfying appropriate conditions.

(ii) In particular, if $b_n = 0$ for all $n \geq 0$, then the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_0 \in K, \ x_{n+1} = (1 - a_n)x_n + a_n T x_n, \quad n \geq 0,$$

is called the *Mann iteration sequence.*

(iii) For any given $x_0 \in K$ the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_{n+1} = a_n x_n + b_n T y_n + c_n u_n, \ y_n = a'_n x_n + b'_n T x_n + c'_n v_n, \quad n \geq 0,$$

where $\{u_n\}_{n=0}^{\infty}, \{v_n\}_{n=0}^{\infty}$ are arbitrary bounded sequences in K and $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$ and $\{c'_n\}_{n=0}^{\infty}$ are real sequences in $[0,1]$ such that $a_n + b_n + c_n = a'_n + b'_n + c'_n = 1$ for all $n \geq 0$, is called the *Ishikawa iteration sequence with errors.*

(iv) If, with the same notations and definitions as in (iii), $b'_n = c'_n = 0$ for all $n \geq 0$, then the sequence $\{x_n\}_{n=0}^{\infty}$ now defined by

$$x_0 \in K, \ x_{n+1} = a_n x_n + b_n T x_n + c_n u_n, \quad n \geq 0,$$

is called the *Mann iteration sequence with errors.*

It is clear that the Ishikawa and Mann iteration sequences are all special cases of the Ishikawa and Mann iteration sequences with errors, respectively.

Chidume [3] proved that if $X = L_p$ (or l_p) for $p \geq 2$, K is a nonempty bounded closed convex subset of X and $T : K \to K$ is a Lipschitz strongly pseudocontractive mapping, then the Mann iteration sequence converges strongly to the unique fixed point of T. Afterwards, several authors extended the result of Chidume in various directions (see *e. g.*, Chang [1], Chang, Cho, Lee & Kang [2], Chidume [4, 5, 6, 7, 8], Chidume & Osilike [9, 10, 11, 12], Liu [19], Schu [25], Tan & Xu [26], Xu [28], Zhou [29] and Zhou & Jia [30]). Chidume [4] obtained that the Ishikawa iteration process can be used to approximate the fixed point of the Lipschitz strongly pseudocontractive mapping $T : K \to K$, where K is a nonempty bounded closed convex subset of a real uniformly smooth Banach space X. Xu [28] extended the results of Chidume in Chidume [3] and Chidume [4] to both the Ishikawa iteration method with errors and without the Lipschitz assumption. Chidume & Osilike [9] improved the result of Chidume [3] to strictly hemi-contractive mappings and real uniformly smooth Banach spaces. Chidume [9] generalized the results in Chidume [3, 4] and Xu [28] to both real Banach spaces, the Ishikawa iteration method with errors and uniformly continuous strongly pseudocontractive mappings.

A few stability results for certain classes of nonlinear mappings have been established by several authors (see *e. g.*, Harder [13, 14, 15], Osilike [21, 22, 23]). Rhoades

[24] proved that the Mann and Ishikawa iteration methods may exhibit different behaviors for different classes of nonlinear mappings. Harder & Hicks [15] revealed that the importance of investigating the stability of various iteration procedures for various classes of nonlinear mappings. Harder [13] established applications of stability results to first order differential equations. Osilike [21, 22] obtained that certain Mann and Ishikawa iteration methods are T-stable on X when T is a Lipschitz strongly pseudocontractive operators in real q-uniformly smooth Banach spaces or real Banach spaces, respectively.

Let K be a nonempty closed convex subset of an arbitrary Banach space X and $T : K \to K$ be a uniformly continuous strictly hemi-contractive operator with bounded range. In this paper, we prove that certain Ishikawa iteration scheme with errors both converges strongly to a unique fixed point of T and is almost T-stable on K. Furthermore, we also establish similar convergence and almost stability results for strictly hemi-contractive operator $T : K \to K$, where K is a nonempty convex subset of arbitrary uniformly smooth Banach space X. The convergence results presented in this paper extend, improve and unify the corresponding results in Chang [1], Chang, Cho, Lee & Kang [2], Chidume [3, 4, 5, 6, 7, 8], Chidume & Osilike [9, 10, 11, 12], Liu [19], Schu [25], Tan & Xu [26], Xu [28], Zhou [29], Zhou & Jia [30] and others.

2. Preliminaries

We need the following Lemmas which play crucial roles in the proofs of our results.

Lemma 2.1 (Kato [17]). *Let X be an arbitrary Banach space and $x, y \in X$. Then $\|x\| \leq \|x+ry\|$ for every $r > 0$ if and only if there is $j \in J(x)$ such that $Re\langle y, j\rangle \geq 0$.*

Lemma 2.2 (Liu [18]). *Suppose that $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$, $\{\gamma\}_{n=0}^{\infty}$ and $\{\omega_n\}_{n=0}^{\infty}$ are nonnegative sequences such that*

$$\alpha_{n+1} \leq (1-\omega_n)\alpha_n + \beta_n\omega_n + \gamma_n, \quad n \geq 0,$$

with $\{\omega_n\}_{n=0}^{\infty} \subset [0,1]$, $\sum_{n=0}^{\infty}\omega_n = \infty$, $\lim_{n\to\infty}\beta_n = 0$ and $\sum_{n=0}^{\infty}\gamma_n < \infty$. Then $\lim_{n\to\infty}\alpha_n = 0$.

Lemma 2.3 (Chidume & Osilike [9]). *Let X be a Banach space and $T : D(T) \subseteq X \to X$ be an operator with $F(T) \neq \varnothing$. Then T is strictly hemi-contractive if and only if there exists $t > 1$ such that for each $x \in D(T)$ and $q \in F(T)$, there exists*

$j \in J(x-q)$ satisfying

$$\operatorname{Re}\langle x - Tx, j\rangle \geq \left(1 - \frac{1}{t}\right)\|x-q\|^2. \tag{2.1}$$

Lemma 2.4. *Let X be an arbitrary normed linear space and $T : D(T) \subseteq X \to X$ be an operator.*

(i) *If T is a local strongly pseudocontractive operator and $F(T) \neq \varnothing$, then $F(T)$ is a singleton and T is strictly hemi-contractive;*
(ii) *If T is strictly hemi-contractive, then $F(T)$ is a singleton.*

Proof. Suppose that $F(T) \neq \varnothing$ and T is a local strongly pseudocontractive operator. We assert first of all that $F(T)$ is a singleton. Otherwise there exist distinct elements $p, q \in F(T)$. Since T is local strongly pseudocontractive, then there exists $t_p > 1$ such that for all $y \in D(T)$ and $r > 0$,

$$\|p - y\| \leq \|(1+r)(p-y) - rt_p(Tp - Ty)\|. \tag{2.2}$$

Set $y = q \in F(T) \subseteq D(T)$ and $r = \frac{1}{2(t_p-1)}$. It follows from (2.2) that

$$0 < \|p-q\| = |1 + r(1-t_p)| \cdot \|p-q\| = \frac{1}{2}|p-q\|,$$

which is a contradiction. Hence $F(T) = \{q\}$ for some $q \in D(T)$.

Next we show that T is strictly hemi-contractive. Note that T is a local strongly pseudocontractive operator and $F(T) = \{q\}$. Put $t = t_q$. Then (1.2) ensures that

$$\|q - y\| \leq \|(1+r)(q-y) - rt(q - Ty)\|$$

for all $y \in D(T)$ and $r > 0$. That is, T is strictly hemi-contractive.

The proof of (ii) now follows exactly as in the first part of the proof of (i). This completes the proof. □

Lemma 2.5 (Chang, Cho, Lee & Kang [2]). *Let X be a Banach space. Then X is a uniformly smooth if and only if J is single valued and uniformly continuous on any bounded subset of X.*

In the sequel, we shall denote the single valued normalized duality mapping by j.

Lemma 2.6 (Xu [28]). *Let X be a uniformly smooth Banach space and let $J : X \to 2^{X^*}$ be the normalized duality mapping. Then*

$$\|x+y\|^2 \leq \|x\|^2 + 2\operatorname{Re}\langle y, j(x+y)\rangle, \quad x, y \in X.$$

3. Main results

In this section, I denotes the identity mapping on X, $d_n = b_n+c_n$ and $d'_n = b'_n+c'_n$ for all $n \geq 0$ and $k = \frac{t-1}{t} \in (0,1)$, where t is the constant appearing in (1,3) or (2.1). Our main results are as follows.

Theorem 3.1. *Let K be a nonempty convex subset of an arbitrary Banach space X and let $T : K \to K$ be a uniformly continuous and strictly hemi-contractive operator wiht $R(T)$ bounded. Suppose that $\{u_n\}_{n=0}^{\infty}$ and $\{v_n\}_{n=0}^{\infty}$ are arbitrary bounded sequences in K and $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$, $\{c'_n\}_{n=0}^{\infty}$ and $\{r_n\}_{n=0}^{\infty}$ are any sequences in $[0,1]$ satisfying*

$$a_n + b_n + c_n = a'_n + b'_n + c'_n = 1, \quad n \geq 0; \tag{3.1}$$

$$c_n = r_n b_n, \quad n \geq 0; \tag{3.2}$$

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} r_n = \lim_{n\to\infty} b'_n = \lim_{n\to\infty} c'_n = 0; \tag{3.3}$$

$$\sum_{n=0}^{\infty} b_n = \infty. \tag{3.4}$$

Suppose that $\{x_n\}_{n=0}^{\infty}$ is the sequence generated from an arbitrary $x_0 \in K$ by

$$z_n = a'_n x_n + b'_n T x_n + c'_n v_n, \ x_{n+1} = a_n x_n + b_n T z_n + c_n u_n, \quad n \geq 0. \tag{3.5}$$

Let $\{y_n\}_{n=0}^{\infty}$ be any bounded sequence in K and define $\{\varepsilon_n\}_{n=0}^{\infty}$ by

$$w_n = a'_n y_n + b'_n T y_n + c'_n v_n, \ \varepsilon_n = \|y_{n+1} - p_n\|, \quad n \geq 0, \tag{3.6}$$

where $p_n = a_n y_n + b_n T w_n + c_n u_n$. Then there exist nonnegative sequences $\{s_n\}_{n=0}^{\infty}$ and $\{t_n\}_{n=0}^{\infty}$ such that

(i) *the sequence $\{x_n\}_{n=0}^{\infty}$ converges strongly to the unique fixed point q of T and*

$$\|x_{n+1} - q\| \leq \Big(1 - kb_n\Big)\|x_n - q\| + k^{-1}b_n s_n + k^{-1}c_n\|u_n - q\|, \quad n \geq 0;$$

(ii) $\|y_{n+1} - q\| \leq (1 - kb_n)\|y_n - q\| + k^{-1}b_n t_n + k^{-1}c_n\|u_n - q\| + \varepsilon_n, \quad n \geq 0;$

(iii) $\sum_{n=0}^{\infty} \varepsilon_n < \infty$ *implies that* $\lim_{n\to\infty} y_n = q$, *so that $\{x_n\}_{n=0}^{\infty}$ is almost T-stable on K;*

(iv) $\lim_{n\to\infty} y_n = q$ *implies that* $\lim_{n\to\infty} \varepsilon_n = 0$.

(v) $\lim_{n\to\infty} s_n = \lim_{n\to\infty} t_n = 0$.

Proof. Lemma 2.4 ensures that $F(T)$ is a singleton. That is, $F(T) = \{q\}$ for some $q \in K$. Let $s_n = \|Tx_{n+1} - Tz_n\|$, $t_n = \|Tp_n - Tw_n\|$ for all $n \geq 0$ and

$$M = 1 + \|x_0 - q\| + \sup\{\|Tx - q\| : x \in K\} + \sup\{\max\{\|u_n - q\|, \|v_n - q\|, \|y_n - q\| : n \geq 0\}\}.$$

It is easy to verify that

$$\max\{\|x_n - q\|, \|z_n - q\|, \|w_n - q\|, \|p_n - q\|\} \leq M, \quad n \geq 0. \tag{3.7}$$

Note that

$$\|x_{n+1} - z_n\| \leq b_n\|x_n - Tz_n\| + c_n\|x_n - u_n\| + b'_n\|x_n - Tx_n\| + c'_n\|x_n - v_n\| \leq 2M(d_n + d'_n) \to 0$$

and

$$\|p_n - w_n\| \leq b_n\|y_n - Tw_n\| + c_n\|y_n - u_n\| + b'_n\|y_n - Ty_n\| + c'_n\|y_n - v_n\| \leq 2M(d_n + d'_n) \to 0$$

as $n \to \infty$. Since T is uniform continuity, it follows that

$$\lim_{n\to\infty} s_n = \lim_{n\to\infty} t_n = 0. \tag{3.8}$$

Since T is strictly hemi-contractive, it follows from Lemma 2.3 that

$$\mathrm{Re}\langle x - Tx, j(x - q)\rangle \geq k\|x - q\|^2, \quad x \in K,$$

which implies that

$$\mathrm{Re}\langle (I - T - kI)x - (I - T - kI)q, j(x - q)\rangle \geq 0, \quad x \in K.$$

In view of Lemma 2.1, we have

$$\|x - q\| \leq \|x - q + r((I - T - kI)x - (I - T - kI)q)\|, \quad x \in K,\ r > 0. \tag{3.9}$$

It follows from (3.1) and (3.5) that for all $n \geq 0$,

$$\begin{aligned}(1 - d_n)x_n &= x_{n+1} - b_nTz_n - c_nu_n \\ &= (1 - (1 - k)b_n)x_{n+1} + b_n((1 - k)I - T)x_{n+1} \\ &\quad + b_n(Tx_{n+1} - Tz_n) - c_nu_n,\end{aligned} \tag{3.10}$$

and

$$(1 - d_n)q = (1 - (1 - k)b_n)q + b_n((1 - k)I - T)q - c_nq. \tag{3.11}$$

By virtue of (3.9)–(3.11), we infer that for any $n \geq 0$,

$$\begin{aligned}(1-d_n)\|x_n - q\| &\geq (1-(1-k)b_n)\|x_{n+1} - q + \frac{b_n}{1-(1-k)b_n}[((1-k)I-T)x_{n+1} \\ &\quad - ((1-k)I-T)q]\| - b_n\|Tx_{n+1} - Tz_n\| - c_n\|u_n - q\| \\ &\geq (1-(1-k)b_n)\|x_{n+1} - q\| - b_n\|Tx_{n+1} - Tz_n\| - c_n\|u_n - q\|,\end{aligned}$$

which implies that for all $n \geq 0$,

$$\begin{aligned}\|x_{n+1} - q\| &\leq \frac{1-d_n}{1-(1-k)b_n}\|x_n - q\| + \frac{b_n}{1-(1-k)b_n}\|Tx_{n+1} - Tz_n\| \\ &\quad + \frac{c_n}{1-(1-k)b_n}\|u_n - q\| \\ &\leq (1 - \frac{kb_n + c_n}{1-(1-k)b_n})\|x_n - q\| + k^{-1}b_n s_n + k^{-1}c_n\|u_n - q\| \\ &\leq (1-kb_n)\|x_n - q\| + k^{-1}b_n s_n + k^{-1}c_n\|u_n - q\|. \qquad (3.12)\end{aligned}$$

Put $\alpha_n = \|x_n - q\|$, $\omega_n = kb_n$, $\beta_n = k^{-2}(s_n + r_n\|u_n - q\|)$ and $\gamma_n = 0$ for each $n \geq 0$. Using (3.2) and (3.12), we have

$$\alpha_{n+1} \leq (1-\omega_n)\alpha_n + \omega_n\beta_n + \gamma_n, \quad n \geq 0.$$

Observe that $\sum_{n=0}^{\infty}\omega_n = \infty$, $\omega_n \in [0,1]$, $\sum_{n=0}^{\infty}\gamma_n = 0$ and $\lim_{n\to\infty}\beta_n = 0$. It follows from Lemma 2.2 that $\lim_{n\to\infty}\alpha_n = 0$. That is, $x_n \to q$ as $n \to \infty$.

From (3.1) and (3.6), we get for all $n \geq 0$,

$$\begin{aligned}(1-d_n)y_n &= p_n - b_nTw_n - c_nu_n \\ &= \Big(1-(1-k)b_n\Big)p_n + b_n\Big((1-k)I - T\Big)p_n \\ &\quad + b_n(Tp_n - Tw_n) - c_nu_n. \qquad (3.13)\end{aligned}$$

It follows from (3.9), (3.11) and (3.13) that

$$\begin{aligned}(1-d_n)\|y_n - q\| &\geq (1-(1-k)b_n)\|p_n - q \\ &\quad + \frac{b_n}{1-(1-k)b_n}[((1-k)I-T)p_n - ((1-k)I-T)q]\| \\ &\quad - b_n\|Tp_n - Tw_n\| - c_n\|u_n - q\| \\ &\geq (1-(1-k)b_n)\|p_n - q\| - b_nt_n - c_n\|u_n - q\| \qquad (3.14)\end{aligned}$$

for all $n \geq 0$. Using (3.1) and (3.14), we immediately conclude that

$$\begin{aligned}\|p_n - q\| &\leq \frac{1-d_n}{1-(1-k)b_n}\|y_n - q\| + \frac{b_n}{1-(1-k)b_n}t_n \\ &\quad + \frac{c_n}{1-(1-k)b_n}\|u_n - q\|\end{aligned}$$

$$\leq (1-kb_n)\|y_n-q\|+k^{-1}b_nt_n+k^{-1}c_n\|u_n-q\| \tag{3.15}$$

for any $n \geq 0$. Thus (3.15) implies that

$$\begin{aligned}\|y_{n+1}-q\| &\leq \|p_n-q\|+\|y_{n+1}-p_n\| \\ &\leq (1-kb_n)\|y_n-q\|+k^{-1}b_nt_n+k^{-1}c_n\|u_n-q\|+\varepsilon_n\end{aligned} \tag{3.16}$$

for all $n \geq 0$.

Suppose that $\sum_{n=0}^{\infty}\varepsilon_n < \infty$. Set $\alpha_n = \|y_n-q\|$, $\omega_n = kb_n$, $\gamma_n = \varepsilon_n$,

$$\beta_n = k^{-2}(t_n + r_n\|u_n-q\|),\ n \geq 0.$$

Using Lemma 2.2, (3.3), (3.4) and (3.16), we conclude that $\alpha_n \to 0$ as $n \to \infty$. Therefore $y_n \to q$ as $n \to \infty$. That is, $\{x_n\}_{n=0}^{\infty}$ is almost T-stable on K.

Suppose that $\lim_{n\to\infty} y_n = q$. It follows from (3.15) that

$$\begin{aligned}\varepsilon_n &\leq \|y_{n+1}-q\|+\|p_n-q\| \\ &\leq \|y_{n+1}-q\|+(1-kb_n)\|y_n-q\|+k^{-1}b_n+k^{-1}c_n\|u_n-q\| \\ &\to 0\end{aligned}$$

as $n \to \infty$. That is, $\varepsilon_n \to 0$ as $n \to \infty$. This completes the proof. □

Using the technique of proof of Theorem 3.1, we have

Theorem 3.2. *Let X, T, K, $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$, $\{x_n\}_{n=0}^{\infty}$, $\{z_n\}_{n=0}^{\infty}$, $\{w_n\}_{n=0}^{\infty}$, $\{y_n\}_{n=0}^{\infty}$, $\{p_n\}_{n=0}^{\infty}$ and $\{\varepsilon_n\}_{n=0}^{\infty}$ be as in Theorem 3.1. Suppose that $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$ and $\{c'_n\}_{n=0}^{\infty}$ are any sequences in* $[0,1]$ *satisfying* (3.1), (3.4) *and*

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} c_n = \lim_{n\to\infty} b'_n = \lim_{n\to\infty} c'_n = 0; \tag{3.17}$$

$$\sum_{n=0}^{\infty} c_n = \infty. \tag{3.18}$$

Then the conclusions of Theorem 3.1 hold.

Theorem 3.3. *Let K be a nonempty convex subset of a uniformly smooth Banach space X and $T: K \to K$ be a strictly hemi-contractive operator with $R(T)$ bounded. Suppose that $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$, $\{x_n\}_{n=0}^{\infty}$, $\{z_n\}_{n=0}^{\infty}$, $\{w_n\}_{n=0}^{\infty}$, $\{y_n\}_{n=0}^{\infty}$, $\{p_n\}_{n=0}^{\infty}$, $\{\varepsilon_n\}_{n=0}^{\infty}$ $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$, $\{c'_n\}_{n=0}^{\infty}$ and $\{r_n\}_{n=0}^{\infty}$ be as in Theorem 3.1. Then there exist nonnegative sequences $\{s_n\}_{n=0}^{\infty}$, $\{t_n\}_{n=0}^{\infty}$ and constant $D > 0$ such that*

(i) *the sequence* $\{x_n\}_{n=0}^{\infty}$ *converges strongly to the unique fixed point* q *of* T *and*

$$\|x_{n+1}-q\|^2 \le (1-kb_n)\|x_n-q\|^2 + Db_n s_n, \quad n \ge 0,$$

(ii) $\|y_{n+1}-q\|^2 \le (1-kb_n)\|y_n-q\|^2 + Db_n t_n + D\varepsilon_n, \quad n \ge 0,$

(iii) $\sum_{n=0}^{\infty}\varepsilon_n < \infty$ *implies that* $\lim_{n\to\infty} y_n = q$, *so that* $\{x_n\}_{n=0}^{\infty}$ *is almost* T*-stable on* K;

(iv) $\lim_{n\to\infty} y_n = q$ *implies that* $\lim_{n\to\infty}\varepsilon_n = 0$.

(v) $\lim_{n\to\infty} s_n = \lim_{n\to\infty} t_n = 0$.

Proof. Let q and M be as in the proof of Theorem 3.1. Then (3.7) holds. Put $f_n = \|j(z_n-q)-j(x_n-q)\|$, $g_n = \|j(x_{n+1}-q)-j(z_n-q)\|$, $h_n = \|j(p_n-q)-j(w_n-q)\|$, $k_n = \|j(w_n-q)-j(y_n-q)\|$ for each $n \ge 0$. Observe that

$$\|(z_n-q)-(x_n-q)\| \le b'_n\|x_n-Tx_n\| + c'_n\|x_n-v_n\| \le 2Md'_n, \tag{3.19}$$

$$\begin{aligned}\|(x_{n+1}-q)-(z_n-q)\| &\le b_n\|x_n-Tz_n\| + c_n\|x_n-u_n\| \\ &\quad + b'_n\|x_n-Tx_n\| + c'_n\|x_n-v_n\| \\ &\le 2M(d_n+d'_n),\end{aligned} \tag{3.20}$$

$$\begin{aligned}\|(p_n-q)-(w_n-q)\| &\le b_n\|y_n-Tw_n\| + c_n\|y_n-u_n\| \\ &\quad + b'_n\|y_n-Ty_n\| + c'_n\|y_n-v_n\| \\ &\le 2M(d_n+d'_n),\end{aligned} \tag{3.21}$$

$$\|(w_n-q)-(y_n-q)\| \le b'_n\|y_n-Ty_n\| + c'_n\|y_n-v_n\| \le 2Md'_n \tag{3.22}$$

for any $n \ge 0$. Using Lemma 2.5, (3.2), (3.3) and (3.19)–(3.22), we infer that

$$\lim_{n\to\infty} f_n = \lim_{n\to\infty} g_n = \lim_{n\to\infty} h_n = \lim_{n\to\infty} k_n = 0. \tag{3.23}$$

In view of (3.1), (3.5), (3.7) and Lemma 2.3 and Lemma 2.6, we have

$$\begin{aligned}\|z_n-q\|^2 &= \|(1-d'_n)(x_n-q) + b'_n(Tx_n-q) + c'_n(v_n-q)\|^2 \\ &\le (1-d'_n)^2\|x_n-q\|^2 + 2b'_n\operatorname{Re}\langle Tx_n-q,\ j(z_n-q)\rangle \\ &\qquad + 2c'_n\operatorname{Re}\langle v_n-q,\ j(z_n-q)\rangle \\ &\le (1-d'_n)^2\|x_n-q\|^2 + 2b'_n\operatorname{Re}\langle Tx_n-q,\ j(x_n-q)\rangle \\ &\quad + 2b'_n\operatorname{Re}\langle Tx_n-q,\ j(z_n-q)-j(x_n-q)\rangle \\ &\qquad + 2c'_n\|v_n-q\|\|z_n-q\| \\ &\le [(1-d'_n)^2 + 2b'_n(1-k)]\|x_n-q\|^2 \\ &\quad + 2b'_n\|Tx_n-q\|\|j(z_n-q)-j(x_n-q)\| + 2M^2c'_n\end{aligned}$$

$$\leq [(1-d_n')^2 + 2b_n'(1-k)]\|x_n - q\|^2 + 2Mb_n'f_n + 2M^2c_n' \tag{3.24}$$

for any $n \geq 0$. Using (3.1), (3.5), (3.7), (3.24) and Lemma 2.3 and Lemma 2.6, we conclude that

$$\begin{aligned}
\|x_{n+1} - q\|^2 &= \|(1-d_n)(x_n - q) + b_n(Tz_n - q) + c_n(u_n - q)\|^2 \\
&\leq (1-d_n)^2\|x_n - q\|^2 + 2b_n \operatorname{Re}\langle Tz_n - q,\ j(x_{n+1} - q)\rangle \\
&\qquad + 2c_n \operatorname{Re}\langle u_n - q,\ j(x_{n+1} - q)\rangle \\
&\leq (1-d_n)^2\|x_n - q\|^2 + 2b_n \operatorname{Re}\langle Tz_n - q,\ j(z_n - q)\rangle \\
&\quad + 2b_n \operatorname{Re}\langle Tz_n - q,\ j(x_{n+1} - q) - j(z_n - q)\rangle + 2M^2c_n \\
&\leq (1-d_n)^2\|x_n - q\|^2 + 2(1-k)b_n\|z_n - q\|^2 \\
&\qquad + 2Mb_ng_n + 2M^2c_n \\
&\leq \{(1-d_n)^2 + 2(1-k)b_n[(1-d_n')^2 + 2b_n'(1-k)]\}\|x_n - q\|^2 \\
&\quad + 2(1-k)b_n(2Mb_n'f_n + 2M^2c_n') + 2Mb_ng_n + 2M^2c_n \\
&\leq (1-kb_n)\|x_n - q\|^2 + Db_ns_n
\end{aligned} \tag{3.25}$$

for any $n \geq 0$, where $s_n = b_n'f_n + c_n' + g_n + r_n$, $D = 6M^2$. Let $\alpha_n = \|x_n - q\|^2$, $\omega_n = kb_n$, $\beta_n = k^{-1}Ds_n$ and $\gamma_n = 0$ for each $n \geq 0$. Thus Lemma 2.2, (3.2)–(3.4), (3.23) and (3.25) yield that $\alpha_n \to 0$ as $n \to \infty$. That is, $\lim_{n\to\infty} x_n = q$. Similarly, we have

$$\begin{aligned}
\|w_n - q\|^2 &= \|(1-d_n')(y_n - q) + b_n'(Ty_n - q) + c_n'(v_n - q)\|^2 \\
&\leq (1-d_n')^2\|y_n - q\|^2 + 2b_n' \operatorname{Re}\langle Ty_n - q,\ j(w_n - q)\rangle \\
&\qquad + 2c_n' \operatorname{Re}\langle v_n - q,\ j(w_n - q)\rangle \\
&\leq (1-d_n')^2\|y_n - q\|^2 + 2b_n' \operatorname{Re}\langle Ty_n - q,\ j(y_n - q)\rangle \\
&\quad + 2b_n' \operatorname{Re}\langle Ty_n - q,\ j(w_n - q) - j(y_n - q)\rangle \\
&\qquad + 2c_n'\|v_n - q\|\|w_n - q\| \\
&\leq [(1-d_n')^2 + 2b_n'(1-k)]\|y_n - q\|^2 \\
&\qquad + 2b_n'\|Ty_n - q\|\|j(w_n - q) - j(y_n - q)\| + 2M^2c_n' \\
&\leq [(1-d_n')^2 + 2b_n'(1-k)]\|y_n - q\|^2 + 2Mb_n'h_n + 2M^2c_n'
\end{aligned} \tag{3.26}$$

for all $n \geq 0$. Using (3.1), (3.5), (3.7), (3.26) and Lemma 2.3 and Lemma 2.6, we infer that

$$\|p_n - q\|^2 = \|(1-d_n)(y_n - q) + b_n(Tw_n - q) + c_n(u_n - q)\|^2$$

$$
\begin{aligned}
&\le (1-d_n)^2\|y_n-q\|^2 + 2b_n \operatorname{Re}\langle Tw_n - q,\ j(p_n-q)\rangle \\
&\qquad\qquad\qquad\qquad + 2c_n \operatorname{Re}\langle u_n - q,\ j(p_n-q)\rangle \\
&\le (1-d_n)^2\|y_n-q\|^2 + 2b_n \operatorname{Re}\langle Tw_n - q,\ j(w_n-q)\rangle \\
&\qquad + 2b_n \operatorname{Re}\langle Tw_n - q,\ j(p_n-q) - j(w_n-q)\rangle + 2M^2c_n \\
&\le (1-d_n)^2\|y_n-q\|^2 + 2(1-k)b_n\|w_n-q\|^2 \\
&\qquad\qquad\qquad\qquad + 2Mb_nk_n + 2M^2c_n \\
&\le \{(1-d_n)^2 + 2(1-k)b_n[(1-d'_n)^2 + 2b'_n(1-k)]\}\|y_n-q\|^2 \\
&\qquad + 2(1-k)b_n(2Mb'_nh_n + 2M^2c'_n) + 2Mb_nk_n + 2M^2c_n \\
&\le (1-kb_n)\|y_n-q\|^2 + Db_nt_n
\end{aligned}
\tag{3.27}
$$

for any $n \ge 0$, where $t_n = b'_nh_n + c'_n + k_n + r_n$, It follows from (3.7), (3.27) that

$$
\begin{aligned}
\|y_{n+1}-q\|^2 &\le (\|y_{n+1}-p_n\| + \|p_n-q\|)^2 \\
&\le \|p_n-q\|^2 + \|y_{n+1}-p_n\|(\|y_{n+1}-p_n\| + 2\|p_n-q\|) \\
&\le (1-kb_n)\|y_n-q\|^2 + Db_nt_n + \varepsilon_n(2M+2M) \\
&\le (1-kb_n)\|y_n-q\|^2 + Db_nt_n + D\varepsilon_n
\end{aligned}
\tag{3.28}
$$

for all $n \ge 0$.

Suppose that $\sum_{n=0}^{\infty}\varepsilon_n < \infty$. Let $\alpha_n = \|y_n-q\|^2$, $\omega_n = kb_n$, $\beta_n = k^{-1}Dt_n$ and $\gamma_n = D\varepsilon_n$ for each $n \ge 0$. Thus Lemma 2.2, (3.2)–(3.4), (3.23) and (3.28) yield that $\alpha_n \to 0$ as $n \to \infty$. That is, $\lim_{n=\infty} y_n = q$.

Conversely, suppose that $\lim_{n=\infty} y_n = q$. By virtue of (3.27) and (3.4), we obtain that

$$
\begin{aligned}
\varepsilon_n &\le \|y_{n+1}-q\| + \|p_n-q\| \\
&\le \|y_{n+1}-q\| + [(1-kb_n)\|y_n-q\|^2 + Db_nt_n]^{\frac{1}{2}} \\
&\to 0
\end{aligned}
$$

as $n \to \infty$. This implies that $\lim_{n=\infty}\varepsilon_n = 0$. This completes the proof. □

Similarly, we have

Theorem 3.4. *Let X, T, K, $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$, $\{x_n\}_{n=0}^{\infty}$, $\{z_n\}_{n=0}^{\infty}$, $\{w_n\}_{n=0}^{\infty}$, $\{y_n\}_{n=0}^{\infty}$, $\{p_n\}_{n=0}^{\infty}$, $\{\varepsilon_n\}_{n=0}^{\infty}$ be as in Theorem* 3.3. *Suppose that, $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$ and $\{c'_n\}_{n=0}^{\infty}$ are any sequences in* $[0,1]$ *satisfying* (3.1), (3.4), (3.17) *and* (3.18). *Then the conclusion of theorem 3.3 hold.*

Remark 3.1. The convergence results in Theorem 3.1 and Theorem 3.2 extend, improve and unify Theorems 3.4 and 4.2 of Chang [1], Theorems 3.4 and 4.2 of Chang, Cho, Lee & Kang [2], Theorem of Chidume [3], Theorem 2 of Chidume [4], Theorem 4 of Chidume [5], Theorem 4 of Chidume [6], Theorem 1 of Chidume [7], Theorem 2 of Chidume & Osilike [9], Theorem 4 of Chidume & Osilike [10], Theorem 1 of Chidume & Osilike [11], Theorem 1 of Chidume & Osilike [12], Theorem 1 of Liu [19], the Theorem of Schu [25] and Theorem 4.2 of Tan & Xu [26] in the following ways:

(i) Theorem 3.1 and Theorem 3.2 hold in arbitrary Banach spaces whereas the results in Chang [1], Chang, Cho, Lee & Kang [2], Chidume [3, 4, 5, 6, 7], Chidume & Osilike [9, 10, 11, 12], Schu [25] and Tan & Xu [26] are fulfilled in the restricted L_p (or l_p) spaces, p-uniformly smooth Banach spaces, real uniformly smooth Banach spaces, real smooth Banach spaces, real Banach spaces, respectively;

(ii) The boundedness of $R(T)$ in Theorem 3.1 and Theorem 3.2 is weaker than the boundedness of the subsets K in Chang [1], Chang, Cho, Lee & Kang [2], Chidume [3, 4, 5, 6, 7], Chidume & Osilike [9, 10, 11, 12], Liu [19], Schu [25] and Tan & Xu [26];

(iii) The Mann iteration methods in Chang [1], Chang, Cho, Lee & Kang [2], Chidume [3], Chidume & Osilike [10], Liu [19], and Schu [25] and the Ishikawa iteration methods in Chang [1], Chang, Cho, Lee & Kang [2], Chidume [4, 5, 6], Chidume & Osilike [9, 11, 12], and Tan & Xu [26] are replaced by the more general Ishikawa iteration method with errors;

(iv) The uniformly continuous strongly pseudocontractive operators in Chang [1], Chang, Cho, Lee & Kang [2], Chidume [6, 7], Chidume & Osilike [12] and Schu [25], the Lipschitz strongly pseudocontractive operators in Chidume [3, 4, 5], Chidume & Osilike [10, 11], Liu [19], and Tan & Xu [26] and the Lipschitz strictly hemi-contractive operators in Chidume & Osilike [9] are replaced by the uniformly continuous strictly hemi-contractive operators;

(v) The iteration parameters α_n, β_n in Chidume [4] and Chidume & Osilike [9] deal with the geometry of the underlying Banach space X. The iteration parameters $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$, $\{c'_n\}_{n=0}^{\infty}$ and $\{r_n\}_{n=0}^{\infty}$ in Theorem 3.1 and Theorem 3.2 are not dependent on the geometric structure of X;

(vi) The conditions $0 \le \alpha_n \le \beta_n < 1$ in Chidume [4] and Chidume [6] are omitted. The following example reveals that Theorem 3.1 generalizes indeed the corresponding results in Chang [1], Chang, Cho, Lee & Kang [2], Chidume [3, 4, 5, 6, 7], Chidume & Osilike [9, 10, 11, 12], Liu [19], Schu [25] and Tan & Xu [26].

Example 3.1. Let R denote the reals with the usual norm, $K = [0, \infty)$ and define $T : K \to K$ by $Tx = \left(\sin \frac{x}{3}\right)^2$ for all $x \in X$. Set

$$t = \frac{3}{2},\ a_n = 1 - \frac{1}{2\sqrt{1+n}} - \frac{1}{2(1+n)},\ b_n = \frac{1}{2\sqrt{1+n}},$$
$$c_n = \frac{1}{2(1+n)},\ a'_n = 1 - \frac{2}{2+n},\ b'_n = c'_n = \frac{1}{2+n}$$

for all $n \ge 0$. It is easy to verify that

$$|Tx - Ty| \le 2\left|\sin\frac{x}{3} - \sin\frac{y}{3}\right| \le 4\left|\sin\frac{x-y}{6}\right| \le \frac{2}{3}|x-y|, \quad x, y \in K. \tag{3.29}$$

That is, T is both Lipschitz and uniformly continuous in K. Thus (3.29) yields that

$$\begin{aligned} |(1+r)(x-y) - rt(Tx - Ty)| &\ge (1+r)|x-y| - rt|Tx - Ty| \\ &= |x-y| + r(|x-y| - t|Tx - Ty|) \\ &\ge |x-y| \end{aligned}$$

for any $x, y \in K$ and $r > 0$. Hence T is strongly pseudocontractive. Clearly $F(T) = \{0\}$. Thus Lemma 2.4 ensures that T is strictly hemi-contractive. Since K is unbounded and $\sum_{n=0}^{\infty} c_n = \infty$, the results in Chang [1], Chang, Cho, Lee & Kang [2], Chidume [3, 4, 5, 6, 7], Chidume & Osilike [9, 10, 11, 12], Liu [19], Schu [25] and Tan & Xu [26] are not applicable. Let

$$r_n = \frac{1}{\sqrt{1+n}}$$

for each $n \ge 0$. Then the conditions of Theorem 3.1 are satisfied.

Remark 3.2. Theorem 3.3 and Theorem 3.4 generalize Theorems 3.2 and 4.1 of Chang [1], Theorems 3.3 and 4.1 of Chang, Cho, Lee & Kang [2], Theorem of Chidume [3], Theorem of Chidume [8], Theorem 2 of Chidume & Osilike [9], Theorem 4 of Chidume & Osilike [10], Theorem 4.2 of Tan & Xu [26], Theorem 3.3 of Xu [28], Theorem 2 of Zhou [29] and Theorem 2.1 of Zhou & Jia [30] to the more general class of uniformly smooth Banach spaces and the Ishikawa iteration method with errors.

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