J. Korea Soc. Math. Ed. Ser. B: Pure Appl. Math.
5(1998), no. 1, pp. 23–32

A RESULT OF DUALITY FOR BISHOP'S PROPERTY (β)

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ABSTRACT. In the present paper the author studies the decomposition property (δ) of the bounded linear operators.

1. Introduction

Bishop [2] introduced the notion of an operator with a duality theory and gave a certain sufficient condition for an operator to have a duality theory. In the spectral decomposition theory of bounded linear operators the decomposition property (δ) is an elementary and important property. All spectral, decomposable, and hyponormal operators have this property, but there are some ordinary operators which do not have this property, for example, the right shift operator on a Hilbert space $\ell_2(\mathbb{N})$. In 1984, Putinar [15] constructed a functional model for hyponormal operators that showed them to be subscalar, and hence to possess a certain property introduced by Bishop.

Given a complex Banach space X, let $\mathcal{L}(X)$ denotes the Banach algebra of all continuous linear operators on X, and T^* denote the dual operator of T.

Definition 1.1. *An operator $T \in \mathcal{L}(X)$ is said to have Bishop's property (β) if for every open subset U of the complex plane $\mathbb{C}$ and for every sequence of analytic functions $f_n : U \to X$ such that $(T - \lambda I)f_n(\lambda)$ converges uniformly to zero on each compact subset of U, it follows that $f_n(\lambda) \to 0$ as $n \to \infty$, uniformly on each compact subset of U.*

Received by the editors Nov. 3, 1997 and, in revised form April 8, 1998.
1991 *Mathematics Subject Classifications.* Primary 47A11, Secondary 47B40.
Key words and phrases. Bishop's property, decomposition property, decomposable operator, single-valued extension property.

Example 1.2. *The left shift operator L on the Hilbert space $\ell_2(\mathbb{N})$ is an example of an operator without Bishop's property (β).*

Proof. Let $U := \mathbb{C} \setminus (\{0\} \cup \{\lambda : |\lambda| = 1\})$, and let $\{e_i\}_{i=1}^{\infty}$ be the usual orthonormal basis for $\ell_2(\mathbb{N})$. We define $f_n : U \longrightarrow \ell_2(\mathbb{N})$ by

$$(*) \qquad f_n(\lambda) := \begin{cases} \sum_{i=0}^{\infty} n\lambda^{i-1} e_{i+1} & \text{for } 0 < |\lambda| < 1 \\ 0 & \text{for } 1 < |\lambda|. \end{cases}$$

The infinite series $(*)$ converges on U and then f_n is analytic on U. Then clearly $(L-\lambda I)f_n(\lambda) \equiv 0$ and so $(L-\lambda I)f_n(\lambda)$ converges uniformly to zero on each compact subset of U. But

$$\|f_n(\lambda)\| = \frac{n}{|\lambda|}(1 - |\lambda|^2)^{-\frac{1}{2}}$$

on $0 < |\lambda| < 1$. This completes the proof.

Putinar must be given credit as one of the first to recognize the importance of Bishop's property (β) in localizing the analytic functional calculus of an operator and the corresponding decomposition of its spectrum. Clearly, Bishop's property (β) implies that T has the *single-valued extension property* (SVEP) which means that for every open subset U of $\mathbb{C}$, the only analytic solution $f : U \to X$ of the equation $(T - \lambda I)f(\lambda) = 0$ for all $\lambda \in U$ is the constant $f \equiv 0$.

Definition 1.3. *An operator $T \in \mathcal{L}(X)$ is said to have decomposition property (δ) if given an arbitrary open covering $\{U, V\}$ of $\mathbb{C}$, every $x \in X$ has a decomposition $x = u + v$ where $u, v \in X$ satisfy $u = (T - \lambda I)f(\lambda)$ on $\mathbb{C} \setminus \overline{U}$ and $v = (T - \lambda I)g(\lambda)$ on $\mathbb{C} \setminus \overline{V}$ for some pair of X-valued analytic functions f and g on $\mathbb{C} \setminus \overline{U}$ and $\mathbb{C} \setminus \overline{V}$, respectively.*

Definition 1.4. *An operator $T \in \mathcal{L}(X)$ is called decomposable if for every open covering $\{U, V\}$ of the complex plane $\mathbb{C}$, there exists a pair of T-invariant closed linear subspaces Y and Z of X such that $\sigma(T|Y) \subseteq U$, $\sigma(T|Z) \subseteq V$ and $X = Y + Z$, where σ denotes the spectrum.*

In [5], Foias showed that every decomposable operator (and therefore spectral operators in the sense of Dunford, all generalized scalar operators in the sense of Colojoara and Foias, compact operators, and unitary, normal, and self-adjoint operators on a Hilbert space) has condition (δ). It follows easily from Proposition

1.3.8 of [5] that an operator $T \in \mathcal{L}(X)$ is decomposable if and only if it has both properties (β) and (δ).

Theorem 1.5 (Lange's Theorem [13]). *A bounded linear operator T on a reflexive Banach space is decomposable if and only if both T and its adjoint T^* have condition (β). Moreover, T is decomposable if and only if T^* is.*

Albrecht and Eschmeier [1] have recently shown that the property (β) characterizes, up to similarity, the restrictions of decomposable operators onto closed invariant subspaces.

Theorem 1.6 (Albrecht and Eschmeier [1]). *The properties (β) and (δ) are dual to each other. i.e., an operator $T \in \mathcal{L}(X)$ satisfies Bishop's property (β) if and only if its adjoint operator T^* satisfies decomposition property (δ), and the corresponding statement remains valid if both properties are interchanged.*

Miller and Miller [14] showed that an operator T has property (δ) if and only if its adjoint T^* has Bishop's property (β). Within this class of operators, it is shown that quasisimilarity preserves essential spectra.

2. Decomposition property (δ)

The next Theorem is due to Finch [9], its corollary provides a useful test for operators that fail to have condition (δ).

Theorem 2.1. *Let T be a bounded linear operator on a Banach space X. If the range of T is all of X, but T is not one-one, then T does not have the SVEP.*

Corollary 2.2. *Let T be a bounded linear operator on a Hilbert space X. If T is bounded from below, but T is not dense in X, then T does not have condition (δ).*

Proof. Since $0 \in \sigma_{com}(T)\backslash\sigma_{ap}(T) = \sigma_p(T^*)\backslash\sigma_{sur}(T^*)$, we have T^* does not have the SVEP. Thus T^* does not have condition (β), and hence T does not have condition (δ).

It is clear that the right shift operator R on $\ell_2(\mathbb{N})$ is an example of an operator without decomposition property (δ). In general, if T is an isometry but is not onto then T does not have condition (δ).

We shall need the following known criterion [1].

Theorem 2.3. *An operator $T \in \mathcal{L}(X)$ has decomposition property (δ) if and only if there exist a decomposable operator $R \in \mathcal{L}(Z)$ on some Banach space Z and a continuous linear surjection $Q \in \mathcal{L}(Z, X)$ such that $TQ = QR$.*

Example 2.4. *The left shift operator on $\ell_2(\mathbb{N})$ has property (δ).*

Proof. Since the right shift operator R on $\ell_2(\mathbb{N})$ is subnormal as the restriction of the bilateral right shift on $\ell_2(\mathbb{Z})$. Clearly, R has property (β). Since L is the adjoint of R, it follows from theorem 1.6 that L has property (δ).

The left shift operator L on $\ell_2(\mathbb{N})$ is an example of a bounded linear operator that has condition (δ), but whose adjoint does not. This shows that condition (δ) is not preserved under the adjoint operation. Also, it is clear that the identity operator I on $\ell_2(\mathbb{N})$ has condition (δ). If T does not have condition (δ), then I commutes with T, but $T = IT = TI$ does not have condition (δ). Unlike the compact operators, the operators that have condition (δ) do not form an ideal in the algebra of operators on a Banach space. The natural related operator in the context of the spectral theory is the restriction operator. We give an example of an operator T and a $T-$invariant subspace Y such that T has condition (δ), but $T|Y$ does not.

Example 2.5. *Let T be the right bilateral shift operator on $\ell_2(\mathbb{Z})$, and let $Y := span\{e_i : i = 1, 2, \cdots, \}^-$, where $\{e_i : i \in \mathbb{Z}\}$ is the usual orthonormal basis for $\ell_2(\mathbb{Z})$. Now T is unitary and so certainly has condition (δ), but $T|Y$ is isomorphic to the right shift on $\ell_2(\mathbb{N})$, and hence does not have condition (δ), by Example 1.2.*

The next theorem is immediate.

Theorem 2.6. *Let T be a bounded linear operator on a Banach space X. Then T has decomposition property (δ) if and only if so does the quotient operator T^Y induced by X/Y for every $T-$invariant closed linear subspace Y of X.*

Proof. Assume that T has property (δ). Let Y be a closed T-invariant subspace of X and let $P : X \longrightarrow X/Y$ be the projection operator. Then there exists a decomposable operator $R \in \mathcal{L}(Z)$ on some Banach space Z and a continuous linear surjection $Q \in \mathcal{L}(Z, X)$ such that $TQ = QR$. Clearly, $T^Y(PR) = (PR)Q$ and hence T^Y has property (δ). This completes the proof.

Theorem 2.7. *If $T \in \mathcal{L}(X)$ and if A is a linear isomorphism between the Banach spaces X and Y, then T has property (δ) if and only if ATA^{-1} does.*

Proof. Assume that T has property (δ). Then there exist a decomposable operator $R \in \mathcal{L}(Z)$ on some Banach space Z and a continuous linear surjection $Q \in \mathcal{L}(Z, X)$ such that $TQ = QR$. Thus AQ is a continuous linear surjection such that $(AQ)R = (ATA^{-1})AQ$. Hence ATA^{-1} has property (δ). Conversely, Assume that ATA^{-1} has property (δ). Then there exist a decomposable operator $R \in \mathcal{L}(Z)$ on some Banach space Z and a continuous linear surjection $Q \in \mathcal{L}(Z, Y)$ such that $(ATA^{-1})Q = QR$. Thus $A^{-1}Q$ is a continuous linear surjection such that $(A^{-1}Q)R = (TA^{-1})Q$. Hence T has property (δ).

Corollary 2.8. *Suppose that $\phi : \mathcal{L}(X) \longrightarrow \mathcal{L}(Y)$ is a spectrum-preserving surjective linear mapping. If $T \in \mathcal{L}(X)$ has property (δ), then either $\phi(T) \in \mathcal{L}(Y)$ has property (δ) or $\phi(T) \in \mathcal{L}(Y)$ has Bishop's property (β).*

Proof. The statement follows from Theorem 2.7 and Theorem 2 of [16].

Corollary 2.9. *Let X be an infinite-dimensional complex Banach space. Assume that $\phi : \mathcal{L}(X) \longrightarrow \mathcal{L}(X)$ is a point spectrum preserving linear mapping. Then $T \in \mathcal{L}(X)$ is decomposable if and only if $\phi(T)$ does.*

Proof. The statement follows from Theorem 2.7 and Theorem 3 of [11].

Corollary 2.10. *Let H be an infinite-dimensional complex Hilbert space. Assume that $\phi : \mathcal{L}(H) \longrightarrow \mathcal{L}(H)$ is a surjectivity spectrum preserving surjective linear mapping. Then $T \in \mathcal{L}(H)$ is decomposable if and only if $\phi(T)$ does.*

Proof. The statement follows from Theorem 2.7 and Theorem 4 of [11].

The next example show that the property (δ) fails to preserved under the sum of operators.

Example 2.11. *Let T be a right shift operator on $\ell_2(\mathbb{N})$. Then $\frac{1}{2}(T + T^*)$ and $\frac{1}{2}(T - T^*)$ are normal, and so they have decomposition property (δ). But $T = \frac{1}{2}(T + T^*) + \frac{1}{2}(T - T^*)$ does not have decomposition property (δ).*

The property (δ) is preserved under the direct sum of operators.

Theorem 2.12. *A bounded linear operator $T_1 \oplus T_2$ on the Banach space $X_1 \oplus X_2$ has property (δ) if and only if T_i has property (δ), ($i = 1, 2$).*

Proof. Suppose that $T_1 \oplus T_2 \in \mathcal{L}(X_1 \oplus X_2)$ has property (δ). Then by Theorem 2.3, there exist a decomposable operator $R \in \mathcal{L}(Z)$ on some Banach space Z and a continuous linear surjection $Q \in \mathcal{L}(Z, X_1 \oplus X_2)$ such that $QR = (T_1 \oplus T_2)Q$. Let $P_i : X_1 \oplus X_2 \longrightarrow X_i, (i = 1,2)$ be the projection operator. Then $Q_i := P_i \circ Q : Z \longrightarrow X_1$ is a continuous linear surjection, and $Q_iR = T_iQ_i$. Hence $T_i \in \mathcal{L}(X_i)$ has property (δ). Conversely, assume that T_i has property (δ), $(i = 1,2)$. Then by Theorem 2.3, there exist a decomposable operator $R_i \in \mathcal{L}(Z_i)$ on some Banach space Z_i and a continuous linear surjection $Q_i \in \mathcal{L}(Z_i, X_i)$ such that $Q_iR_i = T_iQ_i$, $(i = 1,2)$. It follows from (Theorem 2.1 [5]) that $R_1 \oplus R_2 \in \mathcal{L}(Z_1 \oplus Z_2)$ is decomposable. Also, $Q_1 \oplus Q_2 \in \mathcal{L}(Z_1 \oplus Z_2, X_1 \oplus X_2)$ is a continuous linear surjection, and $(Q_1 \oplus Q_2)(R_1 \oplus R_2) = (T_1 \oplus T_2)(Q_1 \oplus Q_2)$. Hence $T_1 \oplus T_2$ has property (δ).

Corollary 2.13. *A bounded linear operator $T_1 \oplus T_2$ on the Banach space $X_1 \oplus X_2$ has Bishop's property (β) if and only if T_i does $(i = 1,2)$.*

In some cases we can conclude that T has the decomposition property (δ) if we know that $T|Y$ does for certain $T-$invariant subspaces Y. Let σ be a spectral set of $\sigma(T)$, and let f be a scalar-valued function, analytic on a neighborhood U of $\sigma(T)$, such that $f(\lambda) \equiv 1$ on σ and $f(\lambda) \equiv 0$ on $\sigma(T) \setminus \sigma$. Define

$$\mathcal{E}(\sigma) := \frac{1}{2\pi i} \int_\Gamma f(\lambda)(\lambda I - T)^{-1} d\lambda,$$

where Γ is a finite union of rectifiable Jordan curves such that $\sigma(T)$ lies inside Γ and $\Gamma \subset U$. It is well known [6] that each $\mathcal{E}(\sigma)$ is a projection such that $\mathcal{E}(\sigma)X$ reduces T.

Theorem 2.14. *Let $T \in \mathcal{L}(X)$. Suppose that $\sigma(T)$ is disconnected with spectral sets $\sigma_1, \sigma_2, \cdots \sigma_n$ such that $\sigma(T) = \cup\{\sigma_i : i = 1,2,\cdots n\}$. Let $Y_i = \mathcal{E}(\sigma_i)X$, $(i = 1,2,\cdots n)$. Then T has property (δ) if and only if each restriction $T|Y_i$ does $(i = 1,2,\cdots n)$.*

Proof. It is clear that $X = \bigoplus_i X_i$ and $T = \bigoplus_i (T|Y_i)$. It follows from theorem 2. 12 that T has property (δ) if and only if $T|Y_i$ $(i = 1,2,\cdots n)$ has property (δ).

Theorem 2.15. *Let T be a bounded linear operator on a Banach space X. If the spectrum of T is totally disconnected, then T has condition (δ).*

Proof. Suppose that if U_1 and U_2 are open sets that cover $\sigma(T)$. Each point of $\sigma(T)$ has a clopen neighborhood in either U_1 or U_2. Because $\sigma(T)$ is compact, a finite number of these clopen neighborhoods cover $\sigma(T)$. Let $N_1, N_2, \cdots, N_k$ be those clopen neighborhoods that are contained in U_1. For each point λ of $\sigma(T)$ such that λ is in $U_2 \setminus (\bigcup_{i=1}^k N_i)$, we may choose a clopen neighborhood $D_\lambda \subseteq U_2$ such that $D_\lambda \cap N_j = \phi$, $j = 1, 2, \cdots, k$. Then clearly

$$(\bigcup_{i=1}^{k} N_i) \bigcup \{D_\lambda : \lambda \in \sigma(T) \bigcap (U_2 \setminus (\bigcup_{i=1}^{k} N_i))\}$$

is an open cover of $\sigma(T)$ and so there exists a finite subcover. Let σ be all members of the subcover from the set $\{N_1, N_2, \cdots, N_k\}$, and let τ be the remaining neighborhoods in the subcover. Since σ and τ are clopen and disjoint, there exist scalar-valued analytic functions f and g such that

$$f \equiv \begin{cases} 1 & \text{on } \sigma \\ 0 & \text{on } \tau \end{cases} \qquad \text{and} \qquad g \equiv \begin{cases} 1 & \text{on } \tau \\ 0 & \text{on } \sigma \end{cases}$$

Define

$$\mathcal{E}(\sigma) := \frac{1}{2\pi i} \int_{\Gamma_1} f(\lambda)(\lambda I - T)^{-1} d\lambda, \text{ and } \mathcal{E}(\tau) := \frac{1}{2\pi i} \int_{\Gamma_2} g(\lambda)(\lambda I - T)^{-1} d\lambda,$$

where Γ_i is a finite union of rectifiable Jordan curves such that $\sigma(T)$ lies inside Γ_i and $\Gamma_i \subseteq U_i$. Let $Y := \mathcal{E}(\sigma)$ and $Z := \mathcal{E}(\tau)$. It follows from ([6], VII 3.20) that both Y and Z are T-invariant subspaces such that $X = Y + Z$, $\sigma(T|Y) \subseteq \sigma \subseteq U_1$ and $\sigma(T|Z) \subseteq \tau \subseteq U_2$. Hence $x \in X$ has a decomposition $x = u_1 + u_2$ where $u_i \in X$ satisfy $u_i = (T - \lambda)h_i(\lambda)$ on $\mathbb{C} \setminus \overline{U}_i$ for some pair of X-valued analytic functions h_i on $\mathbb{C} \setminus \overline{U}_i$, and so T has condition (δ). This completes the proof.

The next example show that the decomposition property (δ) fails to preserved under compact perturbations.

Example 2.16. *This is an example of an operator that has decomposition property (δ), and a compact perturbation of it that does not decomposition property (δ).*

Proof. In [10], Herrero gave an example of a compact operator $K \in \mathcal{L}(\ell_2(\mathbb{Z}))$ such that $T + K$ has the following properties, where T is the right bilateral shift on $\ell_2(\mathbb{Z})$

$$\sigma(T + K) = \{\lambda \in \mathbb{C} : |\lambda| = 1\},$$

and if $Y \neq \{0\}$ is an $(T + K)-$invariant subspace of $\ell_2(\mathbb{Z})$, then either

$$(*) \quad \sigma((T + K)|Y) = \sigma(T + K) \quad \text{or} \quad \sigma((T + K)|Y) = \{\lambda \in \mathbb{C} : |\lambda| \leq 1\}.$$

By Schauder's theorem, $T^* + K^*$ is also compact perturbation of a unitary operator. Since T and T^* are unitary, they have the decomposition property (δ). Suppose that both $T + K$ and $T^* + K^*$ have the decomposition property (δ). Then $T + K$ is decomposable. In order to show that this is impossible. Let

$$U := \{\lambda \in \mathbb{C} : Re\lambda < \frac{1}{2}\}, \quad V := \{\lambda \in \mathbb{C} : Re\lambda > -\frac{1}{2}\}.$$

Since $\{U, V\}$ is an open cover of $\sigma(T + K)$, there exist $T + K$-invariant subspace Y_1 and Y_2 such that

$$\ell_2(\mathbb{Z}) = Y_1 + Y_2, \quad \sigma((T + K)|Y_1) \subseteq U \quad \text{and} \quad \sigma((T + K)|Y_2) \subseteq V.$$

Case 1. Suppose that either Y_1 or Y_2 is trivial, say $Y_1 = \{0\}$. Then $Y_2 = \ell_2(\mathbb{Z})$ and $\sigma(T + K|Y_2) = \sigma(T + K) \not\subseteq V$. This contradicts that $\sigma((T + K)|Y_2) \subseteq V$.
Case 2. Suppose that neither Y_1 nor Y_2 is trivial. Then $\sigma((T + K)|Y_1) \subseteq U$. This contradicts condition $(*)$.

In all, at least one of $T + K$ and $T^* + K^*$ does not have the decomposition property (δ). This completes the proof.

Under stronger hypotheses we can conclude that condition (δ) is preserved under certain types of perturbations. In [13], it is shown that if T is decomposable and S is an operator that commutes with T such that S has totally disconnected spectrum, then $T + S$ is decomposable. From this, the next theorem is immediate.

Theorem 2.17. *Let T be a decomposable operator on a Banach space X, and let S be a bounded linear operator on X that commutes with T. If*
(1) S is a quasinilpotent operator, or
(2) S is a compact operator, or
(3) S has discrete spectrum,
then $T + S$ is decomposable. In particular $T + S$ has condition (δ).

Corollary 2.18. *Let T be an arbitrary bounded linear operator on a Banach space X, let Q be a quasinilpotent operator on X that commutes with T, and let K be a compact operator on X. Then QT, KT, and TK all have condition (δ).*

It is well known [5] that a convergent sequence of decomposable operators on a reflexive Banach space converge to a decomposable operator. Also Vasilescu [17] has shown that the uniform limit of commuting operators with the single valued extension property also has the single valued extension property, but we have been unable to extend the result to the decomposition property (δ) even under hypotheses as strong as Vasilescu. We construct a sequence of operators with decomposition property (δ) that converges in the strong operator topology to an operator without property (δ).

Example 2. 19. *In this example we construct a sequence of operators with condition* (δ) *that converges in the strong operator topology to an operator without condition* (δ). *Define* $T_n : \ell_2(\mathbb{N}) \longrightarrow \ell_2(\mathbb{N})$ $(n = 1, 2, \cdots)$ *by*

$$T_n e_k := \begin{cases} e_{k+1} & \text{for } 1 \leq k \leq n-1 \\ 0 & \text{for } k = n \\ e_k & \text{for } n+1 \leq k, \end{cases}$$

where $\{e_k : k \in \mathbb{N}\}$ *is the usual orthonormal basis, and extend* T_n *linearly and continuously to all of* $\ell_2(\mathbb{N})$. *Clearly, the* T_n *are bounded linear operators. Let* A *be the right shift on* $\mathbb{C}^n$. *We may define the operator*

$$A \oplus I : \mathbb{C}^n \oplus \ell_2(\mathbb{N}) \longrightarrow \mathbb{C}^n \oplus \ell_2(\mathbb{N}),$$

where I *is the identity operator on* $\ell_2(\mathbb{N})$. *Then* $\sigma(A \oplus I) = \sigma(A) \cup \sigma(I) = \{0, 1\}$ *and so* $A \oplus I$ *has condition* (δ) *by Theorem 2.15. Since* T_n *is isomorphic to* $A \oplus I$, *it follows from Theorem 2.7 that each* T_n *has condition* (δ). *Let* R *be the right shift on* $\ell_2(\mathbb{N})$. *Since for any* $x = (x_k)$ *in* $\ell_2(\mathbb{N})$, *we have*

$$\|(R - T_n)x\|^2 \leq 2 \sum_{k=n+1}^{\infty} |x_k|^2.$$

This implies that T_n *converge to* R *in the strong operator topology. By Corollary 2.2,* R *does not have condition* (δ). *This completes the example.*

It is known [5] that if $T \in \mathcal{L}(X)$ has the SVEP, and f is an analytic scalar-valued function on some neighborhood of $\sigma(T)$, then $f(T)$ also has the SVEP. Moreover, it is known [7] that if $T \in \mathcal{L}(X)$ is decomposable, and f is an analytic scalar-valued function on some neighborhood of $\sigma(T)$, then $f(T)$ is decomposable.

Theorem 2.20. *Let* T *be a bounded linear operator on a Banach space* X. *Let* f *be a scalar-valued analytic function defined on a neighborhood of* $\sigma(T)$. *Then if* T *and* T^* *have condition* (δ), *so do* $f(T)$ *and* $f(T^*)$.

Proof. Since T and T^* have condition (δ), they are decomposable. Thus $f(T)$ and $f(T^*)$ are decomposable and therefore have condition (δ).

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