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MINIMAL CLOZ-COVERS OF NON-COMPACT SPACES

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ABSTRACT. Observing that for any dense weakly Lindelöf subspace of a space Y, X is $Z^{\#}$-embedded in Y, we show that for any weakly Lindelöf space X, the minimal Cloz-cover ($E_{cc}(X)$, z_X) of X is given by $E_{cc}(X) = \{(\alpha, x) : \alpha$ is a G(X)-ultrafilter on X with $x \in \cap\alpha\}$, $z_X((\alpha, x)) = x$, z_X is $Z^{\#}$-irreducible and $E_{cc}(X)$ is a dense subspace of $E_{cc}(\beta X)$.

1. Introduction

All spaces in this paper are Tychonoff and βX denotes the Stone-Čech compactification of a space X.

In [5], it is shown that the minimal cloz-cover ($E_{cc}(X)$, z_X) of a compact space X is characterized as follows:

$E_{cc}(X)$ is the space $\{(\alpha, x) : \alpha$ is a G(X)-ultrafilter on X with $x \in \cap\alpha\}$

which is a subspace of $\mathcal{L}(G(X)) \times X$ and $z_X((\alpha, x)) = x$,

where $\mathcal{L}(G(X))$ is the ultrafilter-space of G(X). In [9] ([4], resp.), a theory of the minimal basically disconnected cover (ΛX, Λ_X) (quasi-F cover (QF(X), Φ_X), resp.) of a Tychonoff space X is developed and the relation between ΛX and $\Lambda\beta X$ (QF(X) and QF(βX), resp.) is explored. In [6], the minimal basically disconnected (quasi-F, resp.) cover of a locally weakly Lindelöf space X is characterized by the filter space $\Lambda X = \{\alpha : \alpha$ is a fixed $\sigma Z(X)^{\#}$-ultrafilter on X$\}$ (QF(X) $= \{\alpha : \alpha$ is a fixed $Z(X)^{\#}$-ultrafilter on X$\}$, resp.).

In this paper, we show that every (non-compact) weakly Lindelöf space X has the minimal cloz-cover ($E_{cc}(X)$, z_X) and that $E_{cc}(X)$ is characterized by the space $\{(\alpha, x) : \alpha$ is a G(X)-ultrafilter on X with $x \in \cap\alpha\}$ which is a dense subspace of

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$E_{cc}(\beta X)$. Moreover, we find suitable conditions for a space for which the minimal cloz-cover is basically disconnected. For the terminology, we refer to [1] and [7]

2. Covering maps and $Z^{\#}$-irreducible maps

Definition 2.1. Let $f : X \longrightarrow Y$ be a continuous map. Then f is said to be

(a) *perfect* if f is closed and for any $y \in Y$, $f^{-1}(y)$ is a compact subset in X,

(b) *irreducible* if f is onto and for any closed set A in X with $A \neq X$, $f(A) \neq Y$, and

(c) a *covering map* if f is a perfect irreducible map.

Proposition 2.2. *Consider the following commutative diagram:*

$$\begin{array}{ccc} Z & \xrightarrow{f} & X \\ {\scriptstyle j_1}\downarrow & & \downarrow{\scriptstyle j_2} \\ W & \xrightarrow{g} & Y, \end{array}$$

where f, g are continuous maps and j_1, j_2 are dense embeddings. Then we have the following:

(a) if f and g are perfect onto maps, then $g(W - Z) = Y - X$,

(b) if g is a covering map and f is a perfect onto map, then f is a covering map, and

(c) if W, Y are compact spaces and f is a covering map, then g is also a covering map.

proof. (a) It is trivial ([7]).

(b) Take any closed set A in W with $f(A \cap Z) = X$. By (a), $g^{-1}(X) = Z$ and hence $X = f(A \cap Z) = g(A \cap g^{-1}(X)) = g(A) \cap X$. Thus $X \subseteq g(A)$. Since X is dense in Y and $g(A)$ is closed, $g(A) = Y$. Since g is irreducible, $A = W$ and so $A \cap Z = Z$. Thus f is irreducible.

(c) Clearly g is a perfect continuous map. Since $g(W) = g(\mathrm{cl}_W(Z)) = \mathrm{cl}_Y(g(Z)) = \mathrm{cl}_Y(f(Z)) = \mathrm{cl}_Y(X) = Y$, g is onto. Take any closed set A in W with $A \neq W$. Then $A \cap Z \neq Z$ and hence $f(A \cap Z) \neq X$. Since $f(A \cap Z) = g(A) \cap X$ by (a), $g(A) \cap X \neq X$; hence $g(A) \neq Y$. Thus g is irreducible.

Notation 2.3. *For any space X, let*

(a) $C(X) = \{f : f : X \longrightarrow R$ *is a continuous map*$\}$ *and* $C^*(X) = \{f : f : X \longrightarrow R$ *is a bounded continuous map*$\}$, *where R is the space of real numbers endowed with the usual topology,*

(b) for any $f \in C(X)$, $f^{-1}(0) = Z(f)$ *which will be called a zero-set in X, and complements of zero-sets in X will be called cozero-sets in X,*

(c) $Z(X) = \{Z : Z$ *is a zero-set in X*$\}$,

(d) $Z(X)^\# = \{cl_X(int_X(A)) : A \in Z(X)\}$,

(e) $B(X) = \{B : B$ *is a clopen set in X*$\}$, *and*

(f) $R(X) = \{A : A$ *is a regular closed set in X*$\}$,

It is well-known that R(X) is a complete Boolean algebra under the inclusion relation and B(X), $Z(X)^\#$ *are sublattices of R(X) and that for any covering map* $f : X \longrightarrow Y$, *the map* $\phi : R(X) \longrightarrow R(Y)$, *defined by* $\phi(A) = f(A)$, *is a Boolean isomorphism. Moreover for any dense subspace Y of a space X, the* $\psi : R(X) \longrightarrow R(Y)$, *defined by* $\psi(A) = A \cap Y$, *is a Boolean algebra isomorphism ([6]).*

In a lattice, meets and joins will be denoted by $\wedge$ *and* $\vee$, *respectively and for any map* $f : X \longrightarrow Y$ *and* $\mathcal{B} \subseteq 2^X$, *let* $f(\mathcal{B}) = \{f(B) : B \in \mathcal{B}\}$.

Definition 2.4. A covering map $f : X \longrightarrow Y$ is said to be *Z#-irreducible* if $f(Z(X)^\#) = Z(Y)^\#$.

We note that for any covering map $f : X \longrightarrow Y$, $Z(Y)^\# \subseteq f(Z(X)^\#)$ and hence f is Z#-irreducible if and only if $Z(Y)^\# \supseteq f(Z(X)^\#)$.

Proposition 2.5. *Let* $g : Y \longrightarrow W$, $h : W \longrightarrow X$ *be covering maps. Then* $h \circ g$ *is Z#-irreducible if and only if h and g are Z#-irreducible.*

proof. Assume that $h \circ g$ is Z#-irreducible and take any $A \in Z(Y)^\#$, then there is $B \in Z(X)^\#$ with $h \circ g(A) = B$; $h(g(A)) = B = h(cl_W(h^{-1}(int_X(B))))$. Since h is a covering map and $g(A)$, $cl_W(h^{-1}(int_X(B)))$ are regular closed in W, $cl_W(h^{-1}(int_X(B))) = g(A)$. Since $cl_W(h^{-1}(int_X(B))) = cl_W(int_W(h^{-1}(B)))$, $g(A) \in Z(W)^\#$. Thus g is Z#-irreducible.

Take any $A \in Z(W)^\#$. Since g is a covering map, $cl_Y(g^{-1}(int_W(A))) \in Z(Y)^\#$. Since $h \circ g$ is Z#-irreducible, $h \circ g(cl_Y(g^{-1}(int_W(A)))) \in Z(X)^\#$. But $h \circ g(cl_Y(g^{-1}(int_W(A)))) = h(g(cl_Y(g^{-1}(int_W(A))))) = h(A)$. Thus h is Z#-irreducible. The converse is immediate from the definition.

Definition 2.6. Let Y be a space and X a subspace of Y. Then X or $j : X \hookrightarrow Y$ is said to be *$Z^{\#}$-embedded* in Y if for any $A \in Z(X)^{\#}$, there is a $B \in Z(Y)^{\#}$ such that $A = B \cap X$.

Proposition 2.7. *Consider the following commutative diagram :*

$$\begin{array}{ccc} P & \xrightarrow{f} & X \\ {\scriptstyle j_1}\downarrow & & \downarrow{\scriptstyle j_2} \\ Y & \xrightarrow{g} & W, \end{array}$$

where j_1, j_2 are dense embeddings and f, g are covering maps. Then g is $Z^{\#}$-irreducible and j_1 is $Z^{\#}$-embedded if and only if f is $Z^{\#}$-irreducible and j_2 is $Z^{\#}$-embedded.

proof. ($\Longrightarrow$) Take any $A \in Z(P)^{\#}$. Since j_1 is $Z^{\#}$-embedded, there is a $B \in Z(Y)^{\#}$ such that $A = B \cap P$. Note that $f(A) = f(B \cap P) = g(B) \cap X$. Since g is $Z^{\#}$-irreducible, $f(A) \in Z(X)^{\#}$. Thus f is $Z^{\#}$-irreducible. Let $C \in Z(X)^{\#}$. Then $cl_P(f^{-1}(int_X(C))) \in Z(P)^{\#}$. Since j_1 is $Z^{\#}$-embedded, there is a $D \in Z(Y)^{\#}$ such that $D \cap P = cl_P(f^{-1}(int_X(C)))$. Then $C = f(D \cap P) = g(D) \cap X$. Since g is $Z^{\#}$-irreducible, $g(D) \in Z(X)^{\#}$; therefore j_2 is $Z^{\#}$-embedded.

($\Longleftarrow$) Take any $A \in Z(Y)^{\#}$. Then $A \cap P \in Z(P)^{\#}$ for P is dense in Y and $f(A \cap P) = g(A \cap P) = g(A) \cap X$. Since f is $Z^{\#}$-irreducible, $g(A) \cap X \in Z(X)^{\#}$. Since j_2 is $Z^{\#}$-embedded, there is a $B \in Z(W)^{\#}$ with $g(A) \cap X = B \cap X$. Since j_2 is a dense embedding and $g(A)$, B are regular closed, $g(A) = B$. Thus g is $Z^{\#}$-irreducible.

Take any $C \in Z(P)^{\#}$. Since f is $Z^{\#}$-irreducible, $f(C) \in Z(X)^{\#}$. Since j_2 is $Z^{\#}$-embedded, there is a $D \in Z(W)^{\#}$ with $f(C) = D \cap X$. Since g is a covering map, $cl_Y(g^{-1}(int_W(D))) \in Z(Y)^{\#}$. Then $f(cl_Y(g^{-1}(int_W(D))) \cap P) = g(cl_Y(g^{-1}(int_W(D)))) \cap X = D \cap X = f(C)$. Hence $cl_Y(g^{-1}(int_W(D))) \cap P = C$. Thus j_1 is $Z^{\#}$-embedded.

Definition 2.8. A pair (Y, f) is said to be a *cover* of a space X if $f : Y \longrightarrow X$ is a covering map.

Let X, Y be spaces and $f : X \longrightarrow Y$ a continuous map. For any $U \subseteq Y$, let $f_U : f^{-1}(U) \longrightarrow U$ be the restriction and corestriction of f with respect to $f^{-1}(U)$ and U, respectively.

Lemma 2.9. *Let X be a space and (E, f) a cover of βX. Then $(f^{-1}(X), f_X)$ is also a cover of X.*

proof. Clearly, we have a pullback diagram:

$$\begin{array}{ccc} f^{-1}(X) & \xrightarrow{f_X} & X \\ {\scriptstyle j}\downarrow & & \downarrow{\scriptstyle \beta_X} \\ E & \xrightarrow{f} & \beta X. \end{array}$$

Since f is perfect, f_X is also perfect and clearly, f_X is onto. Since X is dense in βX and f is a covering map, f^{-1}(X) is dense in E. Thus j is a dense embedding and hence f_X is a covering map by Proposition 2.2.

3. Minimal Cloz-covers of non-compact spaces

Definition 3.1. Let X be a space.

(a) A cozero-set C in X is said to be a *complemented cozero-set* if there is a cozero-set D in X such that C ∩ D = ∅ and C ∪ D is dense in X. In case, {C, D} is called a *complementary pair of cozero-sets* in X.

(b) G(X) = {cl_X(C) : C is a complemented cozero-set in X}.

For any space X, G(X) = {A ∈ $Z(X)^{\#}$: A′ ∈ $Z(X)^{\#}$}, where A′ denotes the complement of A in R(X), that is, A′ = cl_X(X - A) and G(X) is a subalgebra of R(X) ([5]).

Recall that a subspace Y of a space X is called C*-embedded in X if for any $f \in$ C*(Y), there is a $g \in$ C*(X) with $g|_Y = f$.

Definition 3.2. A space X is said to be

(a) a *cloz-space* if G(X)=B(X), and

(b) a *quasi-F space* if every dense cozero-set in X is C*-embedded, equivalently, for any zero-sets Z_1, Z_2 in X such that $\mathrm{int}_X(Z_1) \cap \mathrm{int}_X(Z_2) = \emptyset$, $\mathrm{cl}_X(\mathrm{int}_X(Z_1)) \cap \mathrm{cl}_X(\mathrm{int}_X(Z_2)) = \emptyset$.

Proposition 3.3. *(a) A space X is a cloz-space if and only if every element of X has a cloz open neighborhood.*

(b) Every dense $Z^{\#}$-embedded subspace of a cloz-space is again a cloz-space.

proof. (a) ($\Longrightarrow$) It is trivial.

($\Longleftarrow$) Let {C, D} be a complemented pair of cozero-sets in X and $x \in cl_X(C)$. Let V be a cloz open neighborhood of x in X. Since $cl_V((C \cup D) \cap V) = cl_X(C \cup D) \cap V = X \cap V = V$, $\{C \cap V, D \cap V\}$ is a complemented pair of cozero-sets in V. Since V is a cloz-space, $cl_V(C \cap V)$ is clopen in V. Moreover, $cl_X(C) \cap V = cl_V(C \cap V) = int_V(cl_V(C \cap V)) = int_X(cl_X(C)) \cap V$. Hence $x \in int_X(cl_X((C)))$. Thus $cl_X(C)$ is again clopen in X and therefore X is a cloz-space.

(b) Let Y be a cloz-space and X a dense $Z^{\#}$-embedded subspace of Y. Let {C, D} be a complemented pair of cozero-sets in X. Since $G(X) \subseteq Z(X)^{\#}$ and X is $Z^{\#}$-embedded in Y, there are $A, B \in Z(Y)^{\#}$ with $cl_X(C) = A \cap X$ and $cl_X(D) = B \cap X$. Note that

$$\begin{aligned}\emptyset &= cl_X(C) \wedge cl_X(D) = (A \cap X) \wedge (B \cap X)\\ &= cl_X(int_X((A \cap X) \cap (B \cap X))) = cl_X(int_X((A \cap B) \cap X))\\ &= cl_X(int_Y(A \cap B) \cap X) = cl_Y(int_Y(A \cap B)) \cap X = (A \wedge B) \cap X.\end{aligned}$$

Since X is dense in Y, $A \wedge B = \emptyset$. Since $X = X \cap Y = (A \cup B) \cap X$, $Y = A \cup B$. Hence $A' = B$. Thus $A \in G(Y)$. Since Y is a cloz-space, A is clopen in Y and hence $cl_X(C)$ is clopen in X.

Definition 3.4. Let C be a full subcategory of the category Tych of Tychonoff spaces and continuous maps and $X \in$ Tych. Then

(a) a pair (Y, f) is called a *C-cover* of X if (Y, f) is a cover of X and $Y \in$ C,

(b) a C-cover (Y, f) is called a *minimal C-cover* of X if for any C-cover (Z, g) of X, there is a covering map $h : Z \to Y$ with $f \circ h = g$.

Lemma 3.5. *([6]) Let C be a full subcategory of Tych such that $Y \in$ C if and only if $\beta Y \in$ C. Suppose that $X \in$ Tych and (E, f) is a minimal C-cover of βX. If $f^{-1}(X) \in$ C, then ($f^{-1}(X)$, f_X) is a minimal C-cover of X.*

Let Cloz (QF, resp.) denote the full subcategory of Tych determined by cloz-spaces (quasi-F spaces, resp).

It is known that every compact space X has the minimal Cloz-cover ($E_{cc}(X)$, z_X) and moreover $E_{cc}(X) = \{(\alpha, x) : \alpha$ is a G(X)-ultrafilter on X with $x \in \cap\alpha\}$ and $z_X((\alpha, x)) = x$ ([5]). It is a natural question whether every space has a Cloz-cover. We will give some partial answers for this problem in this section.

Definition 3.6. A space X is said to be *weakly Lindelöf* if for any open cover $\mathcal{U}$ of X, there is a countable subfamily $\mathcal{V}$ of $\mathcal{U}$ such that $\cup\mathcal{V}$ is dense in X and a space X is called *locally weakly Lindelöf* if every element of X has a weakly Lindelöf neighborhood.

In [3], the minimal QF-cover (QF(X), Φ_X) of a compact space X is constructed as an inverse limit space and in [8], Vermeer constructed the minimal QF-cover (QF(X), Φ_X) of arbitrary space X. In [4] ([6], resp), the minimal QF-cover (QF(X), Φ_X) of a compact (locally weakly Lindelöf, resp.) space X is characterized by QF(X) $= \{\alpha : \alpha$ is a fixed $Z(X)^{\#}$-ultrafilter on X$\}$ and $\Phi_X(\alpha) = \cap\alpha$. Moreover, Φ_X is $Z(X)^{\#}$-irreducible if X is compact ([4]).

For any space X, let (QF(βX), Φ_β) (($E_{cc}(\beta X)$, z_β), resp.) denote the minimal QF(Cloz, resp.)-cover of βX.

Theorem 3.7. *Let X be a space such that $\Phi_\beta^{-1}(X)$ is $Z^{\#}$-embedded in QF(βX). Then ($z_\beta^{-1}(X)$, z_{β_X}) is the minimal Cloz-cover of X, $z_\beta^{-1}(X)$ is dense in $E_{cc}(\beta X)$ and z_{β_X} is $Z^{\#}$-irreducible.*

proof. Clearly, z_β^{-1}(X) is dense in E_{cc}(X). Since every quasi-F space is a cloz-space, there is a covering map g : QF(βX) $\longrightarrow$ $E_{cc}(\beta X)$ with $z_\beta \circ g = \Phi_\beta$. Since the following diagram

$$\begin{array}{ccc} z_\beta^{-1}(X) & \xrightarrow{\ z_{\beta_X}\ } & X \\ {\scriptstyle j_2}\big\downarrow & & \big\downarrow{\scriptstyle \beta} \\ E_{cc}(\beta X) & \xrightarrow{\ z_\beta\ } & \beta X \end{array}$$

is a pullback, there is a unique continuous map $g^0 : \Phi_\beta^{-1}(X) \longrightarrow z_\beta^{-1}(X)$ such that $z_{\beta_X} \circ g^0 = \Phi_{\beta_X}$ and $g \circ j_1 = j_2 \circ g^0$, where $j_1 : \Phi_\beta^{-1}(X) \hookrightarrow$ QF(βX) is the inclusion map. Since j_1, β are $Z^{\#}$-embedded and Φ_β is $Z^{\#}$-irreducible, by Proposition 2.7, Φ_{β_X} is $Z^{\#}$-irreducible. Let $a \in Z_\beta^{-1}(X)$. Then there is $b \in$ QF(βX) with $g(b) = a$. Hence $\Phi_\beta(b) = z_\beta(g(b)) = z_\beta(a) \in$ X and so $b \in \Phi_\beta^{-1}(X)$. Hence $g(b) = g^0(b) = a$. Thus g^0 is onto. Since $z_{\beta_X} \circ g^0 = \Phi_{\beta_X}$ is a covering map, by Proposition 2.2, g^0 is a covering map. Since $z_{\beta_X} \circ g^0 = \Phi_{\beta_X}$ is $Z^{\#}$-irreducible, by Proposition 2.5, z_{β_X} and g^0 are $Z^{\#}$-irreducible. Consider the following commutative diagram:

$$\begin{array}{ccc} \Phi_\beta^{-1}(X) & \xrightarrow{g^0} & z_\beta^{-1}(X) \\ {\scriptstyle j_1}\downarrow & & {\scriptstyle j_2}\downarrow \\ QF(\beta X) & \xrightarrow{g} & E_{cc}(\beta X). \end{array}$$

By Proposition 2.7, j_2 is $Z^\#$-embedded. So, by Proposition 3.3, z_β^{-1}(X) is a cloz-space. Thus, by Lemma 3.5, (z_β^{-1}(X), z_{β_X}) is the minimal Cloz-cover of X.

Recall that a dense weakly Lindelöf subspace of a space is $Z^\#$-embedded and that for any covering map f : X $\longrightarrow$ Y such that Y is weakly Lindelöf, X is weakly Lindelöf ([4]). Using this, we have the following corollary:

Corollary 3.8. *For a weakly Lindelöf space X, (z_β^{-1}(X), z_{β_X}) is the minimal Cloz-cover of X and z_{β_X} is $Z^\#$-irreducible.*

For any weakly Lindelöf space X, (E_{cc}(X), z_X) denotes the minimal Cloz-cover of X.

For any space X, the isomorphism ψ : R(βX) $\longrightarrow$ R(X) (ψ(A) = A $\cap$ X, A $\in$ R(βX)) induces a lattice isomorphism G(βX) $\longrightarrow$ G(X). Thus $(\alpha, x) \in z_\beta^{-1}$(X) if and only if $\alpha_X = \{A \cap X : A \in \alpha\}$ is a G(X)-ultrafilter and $x \in \cap\alpha_X$. Therefore we have the following corollary:

Corollary 3.9. *For any weakly Lindelöf space X, E_{cc}(X) is the space $\{(\alpha, x) : \alpha$ is a G(X)-ultrafilter on X with $x \in \alpha\}$ which is a subspace of $\mathcal{L}$(G(X)) $\times$ X, where $\mathcal{L}$(G(X)) is the ultrafilter space of G(X).*

Definition 3.10. A space X is said to be a *basically disconnected space* if for any zero-set Z in X, int_X(Z) is closed in X.

Recall that a sublattice $\mathcal{A}$ of R(X) is called *σ-complete* if it is closed under countable joins and meets.

Proposition 3.11. *Let X be a weakly Lindelöf space. Then the following are equivalent:*

(1) G(X) = $Z(X)^\#$,
(2) G(X) = $\{cl_X(C)$: C is a cozero-set in X$\}$,
(3) G(E_{cc}(X)) = $Z(E_{cc}(X))^\#$,
(4) E_{cc}(X) is basically disconnected,

(5) $Z(X)^{\#}$ is a σ-complete Boolean subalgebra of $R(X)$,

Proof. (1) $\Rightarrow$ (2) Clearly, $G(X) \subseteq \{cl_X(C) : C$ is a cozero-set in $X\}$. Let C be a cozero-set in X, then $cl_X(int_X(X - C)) \in Z(X)^{\#} = G(X)$ and hence $cl_X(X - cl_X(X - C)) = cl_X(C) \in G(X)$.

(2) $\Rightarrow$ (3) Let $A \in Z(E_{cc}(X))^{\#}$. Since z_X is $Z^{\#}$-irreducible, $z_X(A) \in Z(X)^{\#}$ and hence $z_X(A)' = z_X(A') \in G(X) \subseteq Z(X)^{\#}$. Hence $A' \in Z(E_{cc}(X))^{\#}$ and so $A \in G(E_{cc}(X))$.

(3) $\Rightarrow$ (4) Let Z be a zero-set in $E_{cc}(X)$, then by (3), $cl_{E_{cc}(X)}(int_{E_{cc}(X)}(Z)) \in G(E_{cc}(X))$. Since $E_{cc}(X)$ is a cloz-space, $cl_{E_{cc}(X)}(int_{E_{cc}(X)}(Z))$ is clopen in $E_{cc}(X)$ and hence $int_{E_{cc}(X)}(Z)$ is closed. Thus $E_{cc}(X)$ is basically disconnected.

(4) $\Rightarrow$ (5) Since z_X is $Z^{\#}$-irreducible, $z_X(Z(E_{cc}(X))^{\#}) = Z(X)^{\#}$ and since $E_{cc}(X)$ is basically disconnected, $Z(E_{cc}(X))^{\#}$ is a σ-complete Boolean subalgebra of $R(E_{cc}(X))$ and hence $Z(X)^{\#}$ is a σ-complete Boolean subalgebra of $R(X)$.

(5) $\Rightarrow$ (1) Since $Z(X)^{\#}$ is a subalgebra of $R(X)$, $G(X) = Z(X)^{\#}$.

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