J. Korea Soc. Math. Ed. Series B: The Pure and Applied Mathematics
4, No. 1 (1997) pp. 7–18

A STUDY OF SOME TESTS OF TREND IN CONTINGENCY TABLES

EUN SOOK JEE

ABSTRACT. Consider an $r \times c$ contingency table under the full multinomial model in which each classification is ordered. The problem is to test the null hypothesis of independence. A number of tests have been proposed for this problem. In this article we show that all of these tests can be improved on in some sense for most cases. In fact the preceding tests sometimes are inadmissible in a strict sense. Furthermore, we show by example that in some cases improved tests can yield substantially improved power functions.

Consider an $r \times c$ contingency table under the full multinomial model where each classification is ordered. Let $X = (X_{ij})$ be the $r \times c$ matrix of cell frequencies; let $p = (p_{ij})$ be the matrix of cell probabilities; let r_i be the ith row total of cell frequencies; let $r' = (r_1, \cdots, r_{r-1})$ be the $1 \times (r-1)$ vector consisting of the first $(r-1)$ row totals; let c_j be the jth column total of cell frequencies; and let $c' = (c_1, \cdots, c_{c-1})$ be the $1 \times (c-1)$ vector consisting of the first $(c-1)$ column totals. Let $m = (r, c)$ and $n = \sum\sum X_{ij}$. Under the full multinomial model, $X \sim M(n, rc, p)$. The problem is to test independence against the alternative that all local log odds ratios are nonnegative with at least one local log odds ratio positive. We express the testing problem as testing the null hypothesis $H : p_{ij} = p_{i\cdot}p_{\cdot j}$ for $i = 1, 2, \cdots, r; j = 1, 2, \cdots c$, where $p_{i\cdot} = \sum_{j=1}^{c} p_{ij}$ and $p_{\cdot j} = \sum_{i=1}^{r} p_{ij}$, against $K : \log\{p_{ij}p_{(i+1)(j+1)}/p_{i(j+1)}p_{(i+1)j}\} \geq 0$, $i = 1, 2, \cdots, r-1$; $j = 1, 2, \cdots, c-1$, with strict inequality for at least one pair (i, j). Such an alternative makes sense when the categories of the contingency table are ordered.

Models for local odds ratios in tables with ordered categories were discussed in Agresti (1984;see especially chaps. 5, 9, and 10). In that book, tests based

Received by the editors March 4, 1997 and, in revised form, May 23, 1997.
1991 *Mathematics Subject classification.* 62H17.
Key words and phrases. Contingency table, Goodman-Kruskal gamma test, Inadmissible test, p value, Unbiased test.
This study was supported by Research Funds from Kwangwoon University, 1996.

on $C - D$, the number of concordant pairs minus the number of discordant pairs, were discussed. The gamma test, originally suggested by Goodman and Kruskal (1954), was also discussed and illustrated in Agresti (1984). The gamma test was recommended by Subramanyam and Rao (1988). Hirotsu (1982) studied a class of tests that form a subset of an essentially complete class of tests for this problem. Cohen and Sackrowitz (1991) formed a different subset of an essentially complete class and found the class of exact, unbiased, and admissible tests. Patefield (1982) did a numerical study for the case $r = 3,\ c = 2$ and compared the powers of exact tests based on the likelihood ratio statistic, two linear statistics, and the Goodman-Kruskal gamma test. Robertson, Wright, and Dykstra (1988, p.262) listed this model and discussed the likelihood ratio test. Agresti, Mehta, and Patel (1990) (hereinafter referred to as AMP) offered an algorithm that enables exact tests via linear statistics.

In this article we attempt to address the question: Which tests can be recommended? We demonstrate not only that the Goodman-Kruskal gamma test is (usually) inadmissible but also that a better test with substantially improved power can be found. For $r = 2,\ c \geq 3$(or $r \geq 3,\ c = 2$) the test based on $(C - D)$ is a linear test and can be admissible and unbiased. However, tests based on $C - D$ often are inadmissible for $r \geq 3,\ c \geq 3$. AMP (1990) suggested some linear statistics that can be used to test H versus K. Should an exact test using such statistics be desired, then randomization often could be necessary and the resulting test would be inadmissible. In some such cases substantial improvement is possible.

If a test of exact size is desired, it would require auxiliary randomization as is the usual case in discrete models.. Hence if the tests based on some statistic S were to be exact in size, randomization usually would be required on some tables of observed frequencies for which, say $S = C_\alpha$. We suggest a test based on a statistic T_p in this paper. However, instead of randomizing on some tables where $T_p = C_\alpha$, we use a different statistic to discriminate among those tables to achieve an exact test. Such a procedure would be preferable, because the class of tables which require randomization would be considerably smaller.

In most practical situations statisticians prefer not to use auxiliary randomizations in carrying out a test. One popular procedure that avoids randomization is to examine the value of the test procedure. As defined by Lehmann (1986, p. 70), the p value is a concept used with "a nested family of rejection regions corresponding to different significance levels" and is "the smallest significance level at which the hypothesis would be rejected for the given observation." Thus for the test based on T_p, one would compute

$$P_H(T_p \geq t_0), \tag{1}$$

the probability under the independence model that T_p is at least t_0, where t_0 is the observed value of T_p. To convert this to a decision of reject or accept, one rejects if (1)$\leq \alpha$, where is a preassigned number. Our proposal is to follow up the T_p test when $T_p = c_\alpha$ with a procedure that orders sample points according to their probabilities. Such a method also generates nested rejection regions and can be done using p values. The advantage of our procedure is that our p values will be greater (and in many cases will be smaller) than those of the T_p test procedure. Our list of available p values will be greater. Another way of seeing the advantage is as follows: The p value approach is conservative in the sense that the probability of the type I error is always less than or equal to a preassigned value. Our finer grid of p values enables us to reject more and still maintain the conservative nature of our test regarding the type I error. Thus the probability of our type I error is below the preassigned value and our power is never lower (and usually is higher for most alternative points).

The preceding arguments demonstrate the desirable features of our test procedure in cases of exact testing and in cases where auxiliarly randomization is to be avoided. Some comments regarding computational feasibility are made at the end of Section 1.

In Section 1 we state the theorem of Cohen and Sackrowitz (1991), which gives the class of unbiased (hence exact), admissible tests. We also define all the tests to be studied. In Section 2 we show by example that the exact gamma test is (usually) inadmissible; for $r \geq 3$, $c \geq 3$, exact tests based on $C-D$ are (usually) inadmissible. We show numerically the improvements that can be made. We show numerically the improvements that can be made. We also evaluate the test we recommend, providing some numerical support in power calculations.

We remark that although the article's language is somewhat decision theoretic, its nature is virtually entirely applications oriented. In fact, the article represents an instance where decision theory ideas offer useful and practical findings.

1. UNBIASED, ADMISSIBLE TESTS

Let $Y_{ij} = \sum_{l=1}^{j}\sum_{k=1}^{i} X_{kl}$, $Y_i = (Y_{i1}, \cdots, Y_{i(c-1)})$, $i = 1, 2, \cdots, r-1$, and $Y = (Y_1, \cdots, Y_{r-1})$. Let $\varphi(x)$ be an exact test of size α. Note that (m, Y) is a one-one linear transformation from the space of X. Express $\varphi(x)$ in terms of (m, Y) by $\varphi_m(y)$; that is, the conditional test for each fixed $m = (r, c)$ expressed as a function of y is $\varphi_m(y)$. We know that because $\varphi(x)$ is of size α, then $\varphi_m(y)$ is of size α for all m (see Cohen and Sackrowitz 1991).

Suppose for each m, $\varphi_m(y)$ is monotonically nondecreasing in y in the sense that $\varphi_m(y)$ is nondecreasing in any component variable when all others are fixed. Let $A_{\varphi_m} = \{y : \varphi_m(y) < 1\}$ for each fixed m. A_{ϕ_m} is the set of points for which the conditional test does not reject with probability 1; that is, except for possible randomization, the acceptance region of the test.) Then, from Cohen and Sackrowitz(1991) we have the following theorem.

Theorem 1.1. *For each fixed m, if $\varphi_m(y)$ is monotone nondecreasing in y, then the test $\varphi_m(y)$ is conditionally unbiased and the test $\varphi(x)$ is unconditionally unbiased. Furthermore, the original test $\varphi(x)$ is admissible if and only if for each fixed m, A_{φ_m} is convex and $\varphi_m(y)$ is zero at nonextreme points of A_{φ_m}.*

Thus an exact test is unbiased and admissible if and only if conditionally, given m, the acceptance regions are monotone (in the sense that the corresponding $\varphi_m(y)$ is monotone) and convex with randomization possible only at extreme points.

Note that a point $b \in A$ is called an extreme point if b is not an interior point of any line segment in A.

At this point we will define the tests we wish to study. The first test statistic is

$$C - D = \sum_{i<k}\sum_{j<l} x_{ij}x_{kl} - \sum_{i<k}\sum_{j>l} x_{ij}x_{kl}. \tag{2}$$

(See Agresti 1984, chap.9.) In(1) $C = \sum_{i<k}\sum_{j<l} x_{ij}x_{kl}$. The Goodman-Kruskal statistic is

$$\Gamma = (C - D)/(C + D). \tag{3}$$

The family of linear statistics studied by AMP(1990) is

$$T = \sum_{i=1}^{r}\sum_{j=1}^{c} u_i v_j x_{ij}, \tag{4}$$

where the u_i's and v_j's are monotone nonincreasing (or nondecreasing). In particular if $u_i = (r - i)$ and $v_j = (c - j)$, then (4) becomes

$$T_p = \sum_{j=1}^{c} \sum_{i=1}^{r} y_{ij}, \tag{5}$$

which is a version of Pearson's correlation coefficient. For the test that we recommend, we will need to order conditional probabilities under the independence model, with $m = (r, c)$, of those tables for which $T_p = t_0$. Suppose we let $\mathcal{Z}' = \{Z : Z\ is\ r \times c,\ Z1 = r,\ Z'1 = c\}$, where 1 is a vector of ones of an appropriate dimension. Then the conditional probabilities of interest can be expressed as

$$P(Z) = \{\sum_{j=1}^{c} c_j!\}\{\prod_{i=1}^{r} r_i!\}/n! \prod_{i=1}^{r} \prod_{j=1}^{c} z_{ij}! \tag{6}$$

for those tables $Z \in \mathcal{Z}'$ such that $T_p = t_0$.

The exact test of size α that we recommend is as follows: Let C_α be a constant(depending on m) such that $P\{T_p \geq C_\alpha\} \geq \alpha$ and $P\{T_p > C_\alpha\} = \lambda < \alpha$. Our test rejects if $T_p > C_\alpha$.

If $T_p = C_\alpha$, consider all tables $x|m$, $T_p = C_\alpha$. Order the tables according to their probabilities. (Ordering outcomes in a sample space according to their probabilities was first done by Freeman and Halton (1951)). Reject for those tables whose probabilities are smallest and are summed to $(\alpha - \lambda)$. Randomization may be required. Such randomization can occur only at extreme points of the convex acceptance section (given m, $T_p = C_\alpha$), as we see from Cohen(1987). The overall resulting test, which we call T_p plus order or simple $\varphi_{T_{po}}$, satisfies the properties of Theorem 1.1. It is a more desirable test than T_p, which randomizes on this entire set $\{x : m\ fixed,\ T_p = C_\alpha\}$. Usually such a set contains nonextreme points, which implies the inadmissibility of the exact test based on T_p in such cases.

If testing is to be carried out using p values, the p value of the T_p test is

$$P_H(T_p \geq t_0), \tag{7}$$

where t_0 is the observed value of T_p. The p value for the test $\varphi_{T_{po}}$ is obtained as follows: Let X be the observed table with marginal totals $m = (r, c)$. Let $B = \{Z : Z \in \mathcal{Z}',\ T_p = t_0,\ p(Z) \leq p(X)\}$. Then the p value of $\varphi_{T_{po}}$ is $P_H(T_p > t_0) + \sum_{Z \in B} P_H(Z)$.

Carrying out tests based on p values avoids auxiliary randomization. In terms of the inadmissibility results previously stated for exact tests, the T_p test no longer would be inadmissible when performed without randomization. It would be too conservative, however. The Γ test and tests based on $C - D$ still usually would be inadmissible, even when performed without auxiliary randomization.

We now make some comments regarding computation of the p values for the $\varphi_{T_{po}}$ procedure. The article by AMP (1990) contained an efficient algorithm for computing p values for the T_p procedure. With minor modifications this algorithm can be used for the $\varphi_{T_{po}}$ procedure.

The goal of computational methods is to obtain p values by generating as few tables as possible. The AMP algorithm used notions of "shortest path" (SP) and "longest path" (LP) to attain considerable early stopping of table generation for determining p values for the test based on T_p above. The AMP algorithm could be modified as follows: During table generation, the algorithm could check the two conditions

$$\sum(r-i)(c-j)x_{ij} + SP > t_0 \tag{8}$$

and

$$\sum(r-i)(c-j)x_{ij} + LP < t_0, \tag{9}$$

where t_0 is the observed value of T_p and LP and SP are determined by Theorems 1 and 2 of AMP. If (8) holds, then every possible table completion will fall in the rejection region, whereas if (9) holds, then every possible table completion will fall in the acceptance region.

To this point the only departure from the AMP algorithm is that the inequality in (8) is strict, so that tables will tend to go farther toward completion (i.e., in AMP table generation also will stop if the left side of (8) is equal t_0). A second modification of the AMP algorithm is for tables such that $T_p = t_0$, we still would have to compare the probability of the generated table to that of the observed table to decide whether or not to include the table in the rejection region.

2. EVALUATION OF TESTS

In this section we discuss the following two examples:

(1) The exact test based on Γ does not have convex acceptance sections and thus is inadmissible by virtue of Theorem 1.1. We illustrate a better test and show that the amount of improvement in power is considerable. For this same example we find the conditional power of the tests T_p and $\varphi_{T_{po}}$. We also compare the conditional values for these two tests. We will see that the p value for the test based on $\varphi_{T_{po}}$ seems more sensible. Although we show the preceding results for a single 3×2 table, it is expected that such results will carry to general tables in many cases.

We remark that in tables of order $r \times 2$ or $2 \times c$, the test based on $C - D$ is linear in the y'_{ij}s with nonnegative coefficients. Therefore, this test clearly satisfies the conditions of Theorem 1.1 and is admissible provided randomization is done only at extreme points of a convex acceptance region.

(2) For tables where $r \geq 3$ and $c \geq 3$, we show by example in a 3×3 table that the test based on $C - D$ does not have convex acceptance sections and thus is inadmissible.

Additional remarks will be made in discussing each example.

Example A. We study a 3×2 table (Table 1) whose marginal totals are drawn from the example in Table 1 of Patefield (1982).

The .01 size conditional test based on Γ is

$$\begin{aligned} \varphi_\Gamma(y_{11}, y_{21}) &= 1 \text{ if } (y_{11}, y_{21}) = (5,15), (5,14), (4,15), (4,14) \\ &= .35572 \text{ if } (y_{11}, y_{21}) = (5,13) \\ &= 0 \text{ otherwise.} \end{aligned} \tag{10}$$

The .01 size conditional test, which is uniformly better than φ_Γ, is

$$\begin{aligned} \varphi^*(y_{11}, y_{21}) &= 1 \text{ if } (y_{11}, y_{21}) = (5,15), (5,14), (4,15), (3,15), \\ &= 8/11 \text{ if } (y_{11}, y_{21}) = (4,14) \\ &= .59382 \text{ if } (y_{11}, y_{21}) = (5,13) \\ &= 0 \text{ otherwise.} \end{aligned} \tag{11}$$

Table 1. Examination Results Compared With Tutor's Prediction

	Treatment	Control	
B: Better than predicted			5
S: Same as predicted			19
W: Worse than predicted			8
	15	17	32

For this same example we study the size .01 tests based on T_p called φ_{T_p}, a better test $\varphi^*_{T_p}$, and the test $\varphi_{T_{po}}$. All the tests are determined as follows:

$$\begin{aligned}\varphi_{T_p} &= 1 \text{ if } (y_{11}, y_{21}) = (5,15),(5,14),(4,15)\\ &= .70431 \text{ if } (y_{11}, y_{21}) = (5,13),(4,14),(3,15)\\ &= 0 \text{ otherwise,}\end{aligned} \tag{12}$$

$$\begin{aligned}\varphi^*_{T_p} &= 1 \text{ if } (y_{11}, y_{21}) = (5,15),(5,14),(4,15),(3,15)\\ &= .62367 \text{ if } (y_{11}, y_{21}) = (4,14)\\ &= .77472 \text{ if } (y_{11}, y_{21}) = (5,13)\\ &= 0 \text{ otherwise,}\end{aligned} \tag{13}$$

and

$$\begin{aligned}\varphi_{T_{po}} &= 1 \text{ if } (y_{11}, y_{21}) = (5,15),(5,14),(4,15),(3,15),(5,13)\\ &= .49463 \text{ if } (y_{11}, y_{21}) = (4,14)\\ &= 0 \text{ otherwise.}\end{aligned} \tag{14}$$

Table 2. Power Functions of Five Tests for Example A

		Power				
Odds ratios		φ_Γ	φ^*	φ_{T_p}	$\varphi^*_{T_p}$	$\varphi_{T_{p0}}$
1.00	1.00	.0100	.0100	.0100	.0100	.0100
1.00	1.78	.0381	.0392	.0377	.0381	.0367
1.00	4.00	.1274	.1496	.1374	.1440	.1370
1.00	16.00	.2885	.4588	.3998	.4502	.4394
1.00	inf	.3597	.7451	.6311	.7451	.7451
1.78	1.00	.0264	.0271	.0280	.0282	.0296
1.78	1.78	.0880	.0880	.0880	.0880	.0880
1.78	4.00	.2545	.2663	.2556	.2591	.2502
1.78	16.00	.5022	.6329	.5819	.6205	.6050
1.78	inf	.5879	.8953	.8048	.8953	.8953
4.00	1.00	.0665	.0737	.0789	.0811	.0902
4.00	1.78	.1861	.1928	.2000	.2020	.2135
4.00	4.00	.4492	.4492	.4492	.4492	.4492
4.00	16.00	.7603	.8061	.7809	.7945	.7800
4.00	inf	.8366	.9799	.9381	.9799	.9799
16.00	1.00	.1389	.1723	.1895	.1994	.2332
16.00	1.78	.3198	.3646	.3896	.4029	.4505
16.00	4.00	.6355	.6633	.6832	.6914	.7264
16.00	16.00	.9297	.9297	.9297	.9297	.9297
16.00	inf	.9828	.9994	.9945	.9994	.9994
inf	1.00	.1878	.2475	.2752	.2929	.3494
inf	1.78	.3907	.4741	.5129	.5375	.6165
inf	4.00	.7023	.7706	.8022	.8224	.8870
inf	16.00	.9590	.9724	.9786	.9825	.9951
inf	inf	1.0000	1.0000	1.0000	1.0000	1.0000

Here the test in (12) is the exact test of size .01 based on the AMP (1990) statistic. It randomizes with the same probability on all points for which $T_p = 94$; That is, $(y_{11}, y_{21}) = (5, 13)$, $(4, 14)$, and (3, 15). Note that the point (4, 14) is not an extreme point of a convex acceptance region. The test in (13) is constructed to beat the test in (12). The point (3, 15) is taken out of the acceptance region, and hence the points (4, 14) and (5, 13) become extreme points on which randomization is permitted. The test $\varphi_{T_{p0}}$ in (14) randomizes only on an extreme point (4, 14) of its convex acceptance region.

Table 3. Conditional p Values for Tests φ_{T_p} and $\varphi_{T_{p0}}$

Table		p value	
x_{11}	x_{21}	φ_{T_p}	$\varphi_{T_{p0}}$
0	15	.3092	.1461
1	13	.3092	.1492
1	14	.1461	.0525
2	11	.3092	.2178
2	12	.1461	.0630
2	13	.0524	.0138
3	9	.3092	.3092
3	10	.1461	.1461
3	11	.0524	.0295
4	7	.3092	.1804
4	8	.1461	.1004
4	9	.0524	.0524
4	10	.0133	.0133
4	11	.0022	.0009
5	5	.3092	.1473
5	6	.1461	.0559
5	7	.0524	.0188
5	8	.0133	.0068
5	9	.0022	.0022
5	10	.0002	.0002

In Table 2 we note that both $\varphi^*_{T_p}$ and $\varphi_{T_{p0}}$ are more desirable than φ_{T_p}. (They also are preferable to φ_{Γ} as well.)

In the above comparison of the five tests, it really only makes sense that each test have the same size. Otherwise, the comparisons of power would not be valid.

Another item of some interest to be noted from the table is that when the two odds ratios are equal, all tests have the same power. This can be demonstrated mathematically using the formula

$$p(X = x|m) = \frac{p_1^{x_{11}} p_2^{x_{11}+x_{21}} \prod_{i=1,\ j=1}^{3,\ 2} x_{ij}!}{\sum\limits_{\{z \in Z\}} p_1^{z_{11}} p_2^{z_{21}} \prod_{i=1,\ j=1}^{3,\ 2} z_{ij}!}$$

where $p_1 = p_{11}p_{22}/p_{12}p_{21}$ and $p_2 = p_{21}p_{32}/p_{22}p_{31}$.

It turns out that the sample points where randomization is required by the various tests all have the same conditional probability when the odd ratios are equal.

In terms of conditional p values, we note the discrepancy between the tests p_{T_p} and $p_{T_{p0}}$ in Table 3. Table 3 is based on the marginal totals of Table 1, and it lists several possible observable tables for the given marginals. This is accomplished by noting the coordinates.

For those observable tables whose p values are less than or equal to .3092 the table contrasts the p value for the two tests. We note many tables with large discrepancies and note that the p value for the $\varphi_{T_{p0}}$ test is much more refined. We also note in passing that, for Patefield's example, the observed table was $(x_{11}, x_{21}) = (5, 8)$ and the contrast in p values was .0133 versus .0068.

Remark 2.1. Although our results regarding inadmissibility and p values are noted for this example, it is clear that many examples could be found.

Remark 2.2. The discrepancy between φ_{T_p} and $\varphi_{T_{po}}$ will be less marked if the total sample size n is very large. For large n, Simon (1978) studied the "efficacy" (some measure related to asymptotic local efficiency) of the various possible indices that can be used as test statistics. Many of the indices have the same efficacy.

Table 4. Frequency of Visits by Length of Stay for 132 Long-Term Schizophrenic Patients

Frequency of visiting	Length of hospital stay: At least 2 years but less than 10 years	At least 10 years but less than 20 years	At least 20 years	Total
Goes home, or visited regularly				62
Visited less than once a month. Does not go home.				27
Never visited and never goes home.				43
Totals	58	45	29	132

Example B. We consider a 3×3 table (Table 4) whose marginal totals are drawn from the example in Table $4-1$ of Fienberg (1980). Label a possible observed table with the above marginal totals as $(x_{11},\ x_{12},\ x_{21},\ x_{22})$.

Now consider the following three observable tables: $T_1 = (54, 8, 4, 5)$, $T_2 = (56, 6, 2, 5)$, and $T_3 = (58, 4, 0, 5)$. The $C - D$ values for these three tables are 3,415, 3,421, and 3,419. Because T_2 is a convex combination of T_1 and T_3, we note that the acceptance region of the $C - D$ test is not convex for any critical value between 3,419 and 3,421. Hence the example shows that $C - D$ generally cannot be admissible if $r \geq 3$ and $c \geq 3$.

Remark 2.3. One might question whether the test based on Γ still would be inadmissible if the alternative space were enlarged. The alternative we consider, called likelihood ratio dependence is implied by what is called quadrant dependence. (See, for example, Ledwina 1984). The fact that the acceptance sections for φ_Γ are not convex implies that it also would be inadmissible for a quadrant dependence alternative. This follows because φ_Γ would be inadmissible for a totally unrestricted alternative.

Remark 2.4. Consider the statistic T in equation (4), of which T_p is a special case. The statistic T is a function of $\{y_{ij}\}$ and as such can be checked as to whether or not it satisfies the properties of Theorem 1.1. Special cases of T for different u_i's and v_j's may of particular interest. In any case such statistics can be studied, just as T_p was as to its properties by the methods in this article.

REFERENCES

1. Agresti, A., *Analysis of Ordinal Categorical Data*, New York: John Wiley. (1984).
2. Agresti, A., Mehta, C. R., and Patel, N. R., *Exact Inference for Contingency Tables With Ordered Categories*, Journal of the American Statistical Association **85** (1990), 453-458.
3. Cohen, A., *Optimality Properties of Some Tests for Homogeneity in Contingency Tables*, Statistics **4** (1987), 591-597.
4. Cohen, A., and Sackrowitz, H. B., *Tests for Independence in Contingency Tables With Ordered Categories*, Journal of Multivariate Analysis **36** (1991), 56-67.
5. Fienberg, S. E., *The Analysis not Cross-Classified Data (2nd ed.)*, Cambridge, MA; MIT Press (1980).
6. Freeman, G. H., and Halton, J. H., *Note on an Exact Treatment of Contingency, Goodness of Fit and Other Problems of Significance*, Biometrika **38** (1951), 141-149.
7. Goodman, L. A., and Kruskal, W. H., *Measures of Associations for Cross Classifications*, Journal of the American Statistical Association **49** (1954), 732-764..
8. Hirotsu, C., *Use of Cumulant Efficient Scores for Testing Ordered Alternatives in Discrete Models*, Biometrika **69** (1982), 567-577.
9. Ledwina, T., *A Note on the Admissibility of Some Tests of Independence Against 'Positive Dependence' in $R \times C$ Contingency Table*, Mathematische Operations forschung und Statistik, Series Statistics **15** (1984), 565-570.
10. Lehmann, E. L., *Testing Statistical Hypotheses (2nd ed.)*, New York: John Wiley (1986).
11. Patefield. N. M., *Exact Tests for Trends for Ordered Contingency Tables.*, Applied Statistics **31** (1982), 32-43.
12. Simon, G. A., *Efficacies of Measures of Association for Ordinal Contingency Tables*, Journal of the American Statistical Association **73** (1978), 545-551.
13. Subramanyam, K., and Rao, M. B., *Analysis of Odds Ratios in $2 \times n$ Ordinal Contingency Tables*, Journal of Multivariate Analysis **27** (1988), 478-493.

DEPARTMENT OF MATHEMATICS, KWANGWOON UNIVERSITY, NOWON-GU, SEOUL 139-701, KOREA