

RESULTS ON AN INTUITIONISTIC FUZZY TOPOLOGICAL SPACE

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ABSTRACT. In this paper, we introduce the concepts of r -gp-open map, weakly r -gp-open map, intuitionistic fuzzy r -compactness, nearly intuitionistic fuzzy r -compactness and almost intuitionistic fuzzy r -compactness defined by intuitionistic gradations of openness, and obtain some characterizations.

1. INTRODUCTION

In [8], Hazra, Samanta and Chattopadhyay introduced the concept of fuzzy topology redefined by a gradation of openness and investigated some fundamental properties, which is an extended concept of fuzzy topological spaces [2] in Chang's sense. Atanassov [1] introduced the concept of intuitionistic fuzzy set which is a generalization of fuzzy set in Zadeh's sense [12]. Çoker [4] introduced Chang's type intuitionistic fuzzy topological spaces, which it is an extended concept of fuzzy topological spaces redefined by a gradation of openness. In [10], Mondal and Samanta introduced and investigated the concept of intuitionistic gradation of openness which is a generalization of the concept of gradation of openness defined by Chattopadhyay et. al. In [9], we introduced the concepts of r -closure and r -interior defined by intuitionistic gradation of openness, which are the extended concepts of fuzzy closure and fuzzy interior of a fuzzy set [5, 6, 7].

In this paper, we introduce the concepts of r -gp-open map, weakly r -gp-open map, intuitionistic fuzzy r -compactness, nearly intuitionistic fuzzy r -compactness and almost intuitionistic fuzzy r -compactness defined by intuitionistic gradations

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of openness, and obtain some characterizations in terms of r-closure and r-interior operators defined by intuitionistic gradation of openness

2. PRELIMINARIES

Let X be a set and $I = [0, 1]$ be the unit interval of the real line. I^X will denote the set of all fuzzy sets of X . 0_X and 1_X will denote the characteristic functions of ϕ and X , respectively.

Definition 1 ([3, 11]). Let X be a non-empty set and $\tau : I^X \rightarrow I$ be a mapping satisfying the following conditions:

- (1) $\tau(0_X) = \tau(1_X) = 1$;
- (2) $\forall A, B \in I^X, \tau(A \cap B) \geq \tau(A) \wedge \tau(B)$;
- (3) For every subfamily $\{A_i : i \in J\} \subseteq I^X, \tau(\cup_{i \in J} A_i) \geq \wedge_{i \in J} \tau(A_i)$.

Then the mapping $\tau : I^X \rightarrow I$ is called a *fuzzy topology* (or *gradation of openness* [3]) on X . We call the ordered pair (X, τ) a *fuzzy topological space*. The value $\tau(A)$ is called the *degree of openness* of A .

Definition 2 ([1]). An intuitionistic fuzzy set A in a set X is an *object* having the form $A = \{(x, \mu_A(x), \gamma_A(x)) : x \in X\}$ where the functions $\mu_A : X \rightarrow I$ and $\gamma_A : X \rightarrow I$ denote the degree of membership and the degree of nonmembership of each element $x \in X$ to the set A , respectively, and $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$ for each $x \in X$.

Definition 3 ([10]). An intuitionistic gradation of openness (briefly *IGO*) of fuzzy subsets of a set X is an ordered pair (τ, τ^*) of functions $\tau, \tau^* : I^X \rightarrow I$ such that

- (IGO1) $\tau(A) + \tau^*(A) \leq 1$, for all $A \in I^X$;
- (IGO2) $\tau(0_X) = \tau(1_X) = 1, \tau^*(0_X) = \tau^*(1_X) = 0$;
- (IGO3) $\forall A, B \in I^X, \tau(A \cap B) \geq \tau(A) \wedge \tau(B)$ and $\tau^*(A \cap B) \leq \tau^*(A) \vee \tau^*(B)$;
- (IGO4) For every subfamily $\{A_i : i \in J\} \subseteq I^X, \tau(\cup_{i \in J} A_i) \geq \wedge_{i \in J} \tau(A_i)$ and $\tau^*(\cup_{i \in J} A_i) \leq \vee_{i \in J} \tau^*(A_i)$.

Then the triplet (X, τ, τ^*) is called an intuitionistic fuzzy topological space (briefly *IFTS*) on X . τ and τ^* may be interpreted as gradation of openness and gradation of nonopenness, respectively.

Definition 4 ([10]). Let X be a nonempty set and $\mathcal{F}, \mathcal{F}^* : I^X \rightarrow I$ be two functions satisfying

- (IGC1) $\mathcal{F}(A) + \mathcal{F}^*(A) \leq 1$, for all $A \in I^X$;
 (IGC2) $\mathcal{F}(0_X) = \mathcal{F}(1_X) = 1, \mathcal{F}^*(0_X) = \mathcal{F}^*(1_X) = 0$;
 (IGC3) $\forall A, B \in I^X, \mathcal{F}(A \cup B) \geq \mathcal{F}(A) \wedge \mathcal{F}(B)$ and $\mathcal{F}^*(A \cup B) \leq \mathcal{F}^*(A) \vee \mathcal{F}^*(B)$;
 (IGC4) for every subfamily $\{A_i : i \in J\} \subseteq I^X, \mathcal{F}(\bigcap_{i \in J} A_i) \geq \bigwedge_{i \in J} \mathcal{F}(A_i)$ and $\mathcal{F}^*(\bigcap_{i \in J} A_i) \leq \bigvee_{i \in J} \mathcal{F}^*(A_i)$.

Then the ordered pair $(\mathcal{F}, \mathcal{F}^*)$ is called an intuitionistic gradation of closedness [10] (briefly *IGC*) on X . \mathcal{F} and \mathcal{F}^* may be interpreted as gradation of closedness and gradation of nonclosedness, respectively.

Theorem 5 ([10]). *Let X be a nonempty set. If (τ, τ^*) is an IGO on X , then the pair $(\mathcal{F}, \mathcal{F}^*)$, defined by $\mathcal{F}_\tau(A) = \tau(A^c)$, $\mathcal{F}^*_{\tau^*}(A) = \tau^*(A^c)$ where A^c denotes the complement of A , is an IGC on X . And if $(\mathcal{F}, \mathcal{F}^*)$ is an IGC on X , then the pair $(\tau_{\mathcal{F}}, \tau^*_{\mathcal{F}^*})$, defined by $\tau_{\mathcal{F}}(A) = \mathcal{F}(A^c)$, $\tau^*_{\mathcal{F}^*}(A) = \mathcal{F}^*(A^c)$ is an IGO on X .*

Definition 6 ([10]). Let (X, τ, τ^*) and (Y, σ, σ^*) be two IFTSs. A mapping $f : X \rightarrow Y$ is called a *gp-map* if $\tau(f^{-1}(A)) \geq \sigma(A)$ and $\tau^*(f^{-1}(A)) \leq \sigma^*(A)$ for every $A \in I^Y$.

Definition 7 ([9]). Let (X, τ, τ^*) be an IFTS, $A \in I^X$ and $r \in [0, 1)$. Then the r -closure (resp., r -interior) of A , denoted by $cl_r A$ (resp., $i_r A$), is defined by $cl_r A = \bigcap \{K \in I^X : \mathcal{F}_\tau(K) > 0 \text{ and } \mathcal{F}^*_{\tau^*}(K) \leq r, A \subseteq K\}$ (resp., $i_r A = \bigcup \{K \in I^X : \tau(K) > 0 \text{ and } \tau^*(K) \leq r, K \subseteq A\}$).

Theorem 8 ([9]). *Let (X, τ, τ^*) be an IFTS and $A, B \in I^X, r \in [0, 1)$. Then*

- (1) $cl_r(0_X) = 0_X$,
- (2) $A \subseteq cl_r A$,
- (3) $cl_r A = cl_r(cl_r A)$,
- (4) $cl_r A \cup cl_r B \subseteq cl_r(A \cup B)$.

Definition 9 ([9]). Let (X, τ, τ^*) and (Y, σ, σ^*) be two IFTSs, and $r \in [0, 1)$. A mapping $f : X \rightarrow Y$ is a *r -gp-map* iff $\sigma(A) \leq \tau(f^{-1}(A))$ and $\tau^*(f^{-1}(A)) \leq \sigma^*(A)$, for each a fuzzy set A in Y such that $\sigma(A) > 0$ and $\sigma^*(A) \leq r$. A mapping $f : X \rightarrow Y$ is a *weakly r -gp-map* iff $\tau(f^{-1}(A)) > 0$ and $\tau^*(f^{-1}(A)) \leq r$, for each fuzzy set $A \in I^Y$ such that $\sigma(A) > 0$ and $\sigma^*(A) \leq r$.

Theorem 10 ([9]). *Let (X, τ, τ^*) and (Y, σ, σ^*) be two IFTSs, $r \in [0, 1)$. If a mapping $f : X \rightarrow Y$ is a weakly r -gp-map, then we have*

- (1) $f(cl_r A) \subseteq cl_r f(A)$ for every $A \in I^X$,

- (2) $cl_r(f^{-1}(A)) \subseteq f^{-1}(cl_r A)$ for every $A \in I^Y$,
 (3) $f^{-1}(i_r A) \subseteq i_r(f^{-1}(A))$ for every $A \in I^Y$,

3. MAIN RESULTS

We introduce the concepts of r -gp-open map, weakly r -gp-open map and several types compactness in intuitionistic topological spaces and investigate some properties of them.

Definition 11. Let (X, τ, τ^*) and (Y, σ, σ^*) be two IFTSs, $r \in [0, 1]$. A mapping $f : X \rightarrow Y$ is called

- (1) a r -gp-open map if $\tau(A) \leq \sigma(f(A))$ and $\sigma^*(f(A)) \leq \tau^*(A)$, for every $A \in I^X$ such that $\tau(A) > 0$ and $\tau^*(A) \leq r$;
 (2) a r -gp-closed map if $\mathcal{F}_\tau(A) \leq \mathcal{F}_\sigma(f(A))$ and $\mathcal{F}^*_{\sigma^*}(f(A)) \leq \mathcal{F}^*_{\tau^*}(A)$, for every $A \in I^X$ such that $\mathcal{F}_\tau(A) > 0$ and $\mathcal{F}^*_{\tau^*}(A) \leq r$.

Definition 12. Let (X, τ, τ^*) and (Y, σ, σ^*) be two IFTSs, $r \in [0, 1]$. A mapping $f : X \rightarrow Y$ is called

- (1) a weakly r -gp-open map if $\sigma(f(A)) > 0$ and $\sigma^*(f(A)) \leq r$, for every $A \in I^X$ such that $\tau(A) > 0$ and $\tau^*(A) \leq r$;
 (2) a weakly r -gp-closed map if $\mathcal{F}_\sigma(f(A)) > 0$ and $\mathcal{F}^*_{\sigma^*}(f(A)) \leq r$, for every $A \in I^X$ such that $\mathcal{F}_\tau(A) > 0$ and $\mathcal{F}^*_{\tau^*}(A) \leq r$.

Every r -gp-open (resp., r -gp-closed) maps are weakly r -gp-open (resp., r -gp-closed) maps but the converse may not be true.

Example 13. Let $X = I$ and let N denote the set of all natural numbers. For each $n \in N$, we consider $\mu_n \in I^X$ such that $\mu_n(x) = \frac{1}{n}x$ for $x \in X$.

Define $\tau, \tau^* : I^X \rightarrow I$ by

$$\begin{aligned} \tau(0_X) &= \tau(1_X) = 1, \tau^*(0_X) = \tau^*(1_X) = 0; \\ \tau(\mu_n) &= \frac{n}{n+2}, \tau^*(\mu_n) = \frac{2}{n+2} \text{ for each } n \in N; \\ \tau(\mu) &= 0, \tau^*(\mu) = 1 \text{ for all other fuzzy set } \mu \in I^X. \end{aligned}$$

And define $\sigma, \sigma^* : I^X \rightarrow I$ by

$$\begin{aligned} \sigma(0_X) &= \sigma(1_X) = 1, \sigma^*(0_X) = \sigma^*(1_X) = 0; \\ \sigma(\mu_n) &= \frac{1}{n+1}, \sigma^*(\mu_n) = \frac{1}{n+1} \text{ for each } n \text{ in } N; \end{aligned}$$

$$\sigma(\mu) = 0, \sigma^*(\mu_n) = 1 \text{ for all other fuzzy set } \mu \in I^X.$$

Then the pairs (τ, τ^*) and (σ, σ^*) are two intuitionistic gradations of openness on X .

Let $r = \frac{1}{2}$ and $f : (X, \tau, \tau^*) \rightarrow (X, \sigma, \sigma^*)$ be the identity mapping. Then f is a weakly r -gp-open map but not a r -gp-open map. In the same way, we can show that a weakly r -gp-closed map may not be a r -gp-closed map.

Theorem 14. *Let (X, τ, τ^*) and (Y, σ, σ^*) be two IFTSs, and $r \in [0, 1)$. If $f : X \rightarrow Y$ is a weakly r -gp-open map, then $f(i_r A) \subseteq i_r(f(A))$ for every $A \in I^X$.*

Proof. For $A \in I^X$, we have that

$$\begin{aligned} f(i_r A) &= f[\cup\{U \in I^X : \tau(U) > 0 \text{ and } \tau^*(U) \leq r, U \subseteq A\}] \\ &\subseteq [\cup\{f(U) \in I^Y : \tau(U) > 0 \text{ and } \tau^*(U) \leq r, f(U) \subseteq f(A)\}] \\ &\subseteq \cup\{f(U) \in I^Y : \sigma(f(A)) > 0 \text{ and } \sigma^*(U) \leq \tau^*(U) \leq r, f(U) \subseteq f(A)\} \\ &\subseteq \cup\{K \in I^Y : \sigma(K) > 0 \text{ and } \sigma^*(K) \leq r, K \subseteq f(A)\} \\ &= i_r(f(A)). \end{aligned}$$

Thus the proof is obtained. \square

Corollary 1. *Let (X, τ, τ^*) and (Y, σ, σ^*) be two IFTSs, and $r \in [0, 1)$. If $f : X \rightarrow Y$ is a r -gp-open map then $f(i_r A) \subseteq i_r(f(A))$ for every $A \in I^X$.*

Theorem 15. *Let (X, τ, τ^*) and (Y, σ, σ^*) be two IFTSs, and $r \in [0, 1)$. If $f : X \rightarrow Y$ is an injective weakly r -gp-closed map, then $cl_r(f(A)) \subseteq f(cl_r A)$ for every $A \in I^X$.*

Proof. Let $A \in I^X$; then since f is an injective weakly r -gp-closed map, we have

$$\begin{aligned} f(cl_r A) &= f[\cap\{U \in I^X : \mathcal{F}_\tau(U) > 0 \text{ and } \mathcal{F}^*_\tau(U) \leq r, A \subset U\}] \\ &= \cap\{f(U) \in I^X : \mathcal{F}_\tau(U) > 0 \text{ and } \mathcal{F}^*_\tau(U) \leq r, f(A) \subseteq f(U)\} \\ &\supseteq \cap\{f(U) \in I^X : \mathcal{F}_\sigma(f(U)) > 0 \text{ and } \mathcal{F}^*_{\sigma^*}(f(U)) \leq r, f(A) \subseteq f(U)\} \\ &\supseteq cl_r f(A). \end{aligned}$$

Thus it follows $cl_r(f(A)) \subseteq f(cl_r A)$ for every $A \in I^X$. \square

Since every r -gp-close map is a weakly r -gp-closed map, we get the following theorem.

Theorem 16. *Let (X, τ, τ^*) and (Y, σ, σ^*) be two IFTSs, and $r \in [0, 1)$. If $f : X \rightarrow Y$ is an injective r -gp-closed map, then $cl_r(f(A)) \subseteq f(cl_r A)$ for every $A \in I^X$.*

Definition 17. Let (X, τ, τ^*) be an IFTS, and $r \in [0, 1)$. A family $\{A_i \in I^X : \tau(A_i) > 0 \text{ and } \tau^*(A_i) \leq r, i \in J\}$ is called an *intuitionistic fuzzy r -cover* if $\cup_{i \in J} A_i = 1_X$.

Definition 18. For $r \in [0, 1)$, an IFTS (X, τ, τ^*) is said to be *intuitionistic fuzzy r -compact* if for every intuitionistic fuzzy r -cover $\{A_i \in I^X : \tau(A_i) > 0 \text{ and } \tau^*(A_i) \leq r, i \in J\}$ of X , there exists a finite subset J_o of J such that $\cup_{i \in J_o} A_i = 1_X$.

Theorem 19. Let (X, τ, τ^*) and (Y, σ, σ^*) be two IFTSs, and $r \in [0, 1)$ and let $f : X \rightarrow Y$ be a surjective weakly r -gp-map. If (X, τ, τ^*) is intuitionistic fuzzy r -compact, then so is (Y, σ, σ^*) .

Proof. Obvious. □

Definition 20. For $r \in [0, 1)$, an IFTS (X, τ, τ^*) is called *nearly intuitionistic fuzzy r -compact* if for every intuitionistic fuzzy r -cover $\{A_i \in I^X : \tau(A_i) > 0 \text{ and } \tau^*(A_i) \leq r, i \in J\}$ of X , there exists a finite subset J_o of J such that $\cup_{i \in J_o} i_r(cl_r(A_i)) = 1_X$.

Theorem 21. For $r \in [0, 1)$, an intuitionistic fuzzy r -compact IFTS (X, τ, τ^*) is nearly intuitionistic fuzzy r -compact.

Proof. Let $\{A_i \in I^X : \tau(A_i) > 0 \text{ and } \tau^*(A_i) \leq r, i \in J\}$ be an intuitionistic fuzzy r -cover of X ; then there exists a finite subset J_o of J such that $\cup_{i \in J_o} A_i = 1_X$. Since $\tau(A_i) > 0$ for all $i \in J$, by Theorem 2.8 we have $A_i = i_r(A_i) \subseteq i_r(cl_r(A_i))$. Thus $1_X = \cup_{i \in J_o} A_i \subseteq \cup_{i \in J_o} i_r(cl_r(A_i))$. Hence (X, τ, τ^*) is nearly intuitionistic fuzzy r -compact. □

Remark 22. In Theorem 3.12, the converse of implication may not be true. For if (X, τ, τ^*) is an IFTS and $\tau^*(\mu) = 0$ for all $\mu \in I^X$, then the (X, τ, τ^*) is a fuzzy topological space in Sostak's sense. Since a nearly fuzzy compact space is not fuzzy compact, so we can say a nearly intuitionistic fuzzy r -compact IFTS is not always intuitionistic fuzzy r -compact.

Definition 23. For $r \in [0, 1)$, an IFTS (X, τ, τ^*) is said to be *almost intuitionistic fuzzy r -compact* if for every intuitionistic fuzzy r -cover $\{A_i \in I^X : \tau(A_i) > 0 \text{ and } \tau^*(A_i) \leq r, i \in J\}$ of X , there exists a finite subset J_o of J such that $\cup_{i \in J_o} cl_r(A_i) = 1_X$.

Theorem 24. For $r \in [0, 1)$, a nearly intuitionistic fuzzy r -compact IFTS (X, τ, τ^*) is almost intuitionistic fuzzy r -compact.

Proof. Let $\{A_i \in I^X : \tau(A_i) > 0 \text{ and } \tau^*(A_i) \leq r, i \in J\}$ be an intuitionistic fuzzy

r -cover X ; then there exists a finite subset J_o of J such that $\cup_{i \in J_o} i_r(cl_r(A_i)) = 1_X$. Since $i_r(cl_r(A_i)) \subseteq cl_r(A_i)$ for each $i \in J$, $1_X = \cup_{i \in J_o} i_r(cl_r(A_i)) \subseteq \cup_{i \in J_o} cl_r(A_i)$. So $\cup_{i \in J_o} cl_r(A_i) = 1_X$. Hence (X, τ, τ^*) is almost intuitionistic fuzzy r -compact. \square

As Remark 3.13, we can say that an almost intuitionistic fuzzy r -compact IFTS is not always a nearly intuitionistic fuzzy r -compact IFTS.

Theorem 25. *Let (X, τ, τ^*) and (Y, σ, σ^*) be two IFTSs, $r \in [0, 1)$ and $f : X \rightarrow Y$ a surjective, weakly r -gp-map. If X is almost intuitionistic fuzzy r -compact, then so is Y .*

Proof. It is obvious. \square

Corollary 2. *Let (X, τ, τ^*) and (Y, σ, σ^*) be two IFTSs, and $r \in [0, 1)$ and let $f : X \rightarrow Y$ be a surjective, weakly r -gp-map. If X is nearly intuitionistic fuzzy r -compact, then Y is almost intuitionistic fuzzy r -compact.*

Theorem 26. *Let (X, τ, τ^*) and (Y, σ, σ^*) be two IFTSs, $r \in [0, 1)$ and $f : X \rightarrow Y$ a surjective, weakly r -gp-map and r -gp-open map. If X is nearly intuitionistic fuzzy r -compact, then so is Y .*

Proof. Let $\{A_i \in I^Y : \sigma(A_i) > 0 \text{ and } \sigma^*(A_i) \leq r, i \in J\}$ be an intuitionistic fuzzy r -cover of Y . Then $1_X = f^{-1}(1_Y) = \cup_{i \in J} f^{-1}(A_i)$. Since f is a weakly r -gp-map, we have an intuitionistic fuzzy r -cover $\{f^{-1}(A_i) \in I^X : \tau(f^{-1}(A_i)) > 0 \text{ and } \tau^*(f^{-1}(A_i)) \leq r, i \in J\}$ of X . And since X is nearly intuitionistic fuzzy r -compact, there exists a finite subset J_o of J such that $\cup_{i \in J_o} i_r(cl_r(f^{-1}(A_i))) = 1_X$. Thus by hypothesis, we have

$$\begin{aligned} 1_Y &= \cup_{i \in J_o} f(i_r(cl_r(f^{-1}(A_i)))) \\ &\subseteq \cup_{i \in J_o} i_r(f cl_r(f^{-1}(A_i))) \\ &\subseteq \cup_{i \in J_o} i_r(f(f^{-1}(cl_r(A_i)))) \\ &= \cup_{i \in J_o} i_r(cl_r(A_i)). \end{aligned}$$

Thus (Y, σ, σ^*) is nearly intuitionistic fuzzy r -compact. \square

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