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BCK-ALGEBRAS WITH SUPREMUM

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ABSTRACT. The notion of a BCK-algebra with supremum (briefly, sBCK-algebra) is introduced, and several examples are given. Related properties are investigated. We show that every sBCK-algebra with an additional condition has the condition (S). The notion of a dry ideal of an sBCK-algebra is introduced. Conditions for an sBCK-algerba to be an spBCK-algebra are provided. We show that every sBCK-algebra satisfying additional condition is a semi-Brouwerian algebra.

1. INTRODUCTION

BCK-algebras entered into mathematics in 1966 through the work of Imai and Iséki [3], and have been applied to many branches of mathematics, such as group theory, functional analysis, probability theory and topology. Such algebras generalize Boolean rings as well as Boolean D-posets ($= MV$-algebras). There is a deep relation between BCK-algebras and posets. A way to make a new BCK-algebra from old is established by Abujabal [1]. Jun et al. [6] gave a method to make a BCK-algebra from a poset and an upper set. Hao [2] gave a method for constructing a proper BCC-algebra by the extension of a BCK-algebra with a small atom. Iséki [4] gave a method to make a BCI-algebra by using a group and a BCK-algebra. Jun et al. [7] gave a method to make a BCK-algebra by using a poset. We show that if a poset has the least element, then the induced BCK-algebra is bounded. In this paper, we introduce the notion of a (positive implicative) BCK-algebra with supremum, and give several examples. We investigate related properties, and show that every sBCK-algebra with an additional condition has the condition (S). We also introduce the notion of a dry ideal of an sBCK-algebra. We show that every sBCK-algebra satisfying additional condition is a semi-Brouwerian algebra, and provide conditions for an sBCK-algerba to be an spBCK-algebra.

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2. PRELIMINARIES

We first display basic concepts on BCK-algebras. Let $K(\tau)$ be the class of all algebras of type τ. A *BCK-algebra* is a system $(X, *, 0) \in K(\tau)$, where $\tau = (2, 0)$, such that

(a1) $(\forall x, y, z \in X)\ (((x * y) * (x * z)) * (z * y) = 0)$,
(a2) $(\forall x, y \in X)\ ((x * (x * y)) * y = 0)$,
(a3) $(\forall x \in X)\ (x * x = 0, 0 * x = 0)$,
(a4) $(\forall x, y \in X)\ (x * y = 0, y * x = 0 \Rightarrow x = y)$.

We can define a partial ordering $\leq$ by $x \leq y$ if and only if $x * y = 0$. A BCK-algebra $(X, *, 0)$ is said to be *bounded* if there exists the bound 1 such that $x \leq 1$ for all $x \in X$. A BCK-algebra $(X, *, 0)$ is said to be *positive implicative BCK-algebra* (briefly, *pBCK-algebra*) if it satisfies:

$$(\forall x, y, z \in X)\,((x * y) * z = (x * z) * (y * z)), \tag{2.1}$$

or equivalently it satisfies:

$$(\forall x, y \in X)\,(x * y = (x * y) * y). \tag{2.2}$$

Note that every pBCK-algebra is a BCK-algebra but the converse is not true. Denote by $\mathbb{BCK}$ (resp. b$\mathbb{BCK}$ and p$\mathbb{BCK}$) the set of all BCK-algebras (resp. bounded BCK-algebras and positive implicative BCK-algebras).

Let $(X, *, 0)$ be a BCK-algebra, For any $a, b \in X$, consider a set

$$X(a, b) := \{x \in X \mid x * a \leq b\} \tag{2.3}$$

A BCK-algebra $(X, *, 0)$ is said to *have the condition* (S) if it satisfies:

$$(\forall a, b \in X)\,(X(a, b) \text{ has a greatest element}). \tag{2.4}$$

In any BCK-algebra $(X, *, 0)$, the following hold:

(b1) $(\forall x \in X)\ (x * 0 = x)$,
(b2) $(\forall x, y, z \in X)\ ((x * y) * z = (x * z) * y)$,
(b3) $(\forall x, y, z \in X)\ ((x * z) * (y * z) \leq x * y)$,
(b4) $(\forall x, y, z \in X)\ (x \leq y \Rightarrow x * z \leq y * z, z * y \leq z * x)$.

Definition 2.1. A subset A of a BCK-algebra $(X, *, 0)$ is called an *ideal* of X if it satisfies:

(c1) $0 \in A$,
(c2) $(\forall x \in A)\ (\forall y \in X)\ (y * x \in A \Rightarrow y \in A)$.

Note that every ideal A of a BCK-algebra $(X, *, 0)$ satisfies:

$$(\forall x \in A)\,(\forall y \in X)\,(y \leq x \;\Rightarrow\; y \in A). \tag{2.5}$$

3. BCK-algebras with Supremum

Example 3.1 ([8]). Let $(X, \leq)$ be a poset with the least element 0. The operation $*$ on X is defined by the following prescription

$$x * y := \begin{cases} 0 & \text{if } x \leq y, \\ x & \text{otherwise.} \end{cases} \tag{3.1}$$

Then $(X, *, 0)$ is a BCK-algebra.

For any elements x and y of a BCK-algebra $(X, *, 0)$, let $x \vee y$ denote the supremum of $\{x, y\}$ if it exists. Consider the following identity:

$$(x \vee y) * z = (x * z) \vee (y * z) \tag{3.2}$$

where $x, y, z \in X$.

Example 3.2. Let $X = \{0, a, b, 1\}$ be a set with the following Cayley table:

$*$	0	a	b	1
0	0	0	0	0
a	a	0	0	0
b	b	a	0	0
1	1	a	a	0

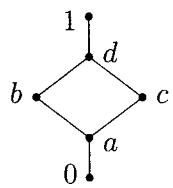
Then $(X, *, 0) \in \mathrm{b}\mathbb{BCK}$ in which there exists $x \vee y$ for all $x, y \in X$, and X satisfies the identity (3.2).

We have the following question.

(Q1) In a BCK-algebra $(X, *, 0)$, if every pair of elements of X has supremum, then does the identity (3.2) hold?

Herein, we give answer to this question negatively.

Example 3.3. Using Example 3.1, we make a BCK-algebra. Let $X = \{0, a, b, c, d, 1\}$ be a poset with the following Hasse diagram:



Using (3.1), we have the following Cayley table:

$*$	0	a	b	c	d	1
0	0	0	0	0	0	0
a	a	0	0	0	0	0
b	b	b	0	b	0	0
c	c	c	c	0	0	0
d	d	d	d	d	0	0
1	1	1	1	1	1	0

Then $(X, *, 0) \in$ b$\mathbb{BCK}$. Note that every pair of elements of X has supremum. But it does not verify the identity (3.2) since

$$(b \vee c) * b = d * b = d \neq c = 0 \vee c = (b * b) \vee (c * b).$$

Definition 3.4. A *BCK-algebra* (resp. *pBCK-algebra*) *with supremum* (briefly, *sBCK-algebra* (resp. *spBCK-algebra*)) is a system $X := (X, *, \vee, 0) \in K(\tau)$, where $\tau = (2, 2, 0)$, such that

(a5) $(X, *, 0) \in \mathbb{BCK}$ (resp. $(X, *, 0) \in$ p$\mathbb{BCK}$).
(a6) $(\forall x \in X)\ (x \vee x = x)$,
(a7) $(\forall x, y, z \in X)\ ((x \vee y) \vee z = x \vee (y \vee z))$,
(a8) $(\forall x, y \in X)\ ((x * y) \vee y = x \vee y)$,
(a9) $(\forall x, y, z \in X)\ (((x * z) \vee (y * z)) * ((y \vee x) * z) = 0)$.

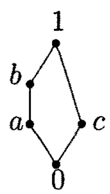
Obviously, every spBCK-algebra is an sBCK-algebra, but the converse is not true as seen in the following example.

Example 3.5. Let X be a set with a special element 0 such that $(X, \vee)$ is a join($\vee$)-semilattice. Let $*$ be a binary operation on X defined as (3.1). Then $X := (X, *, \vee, 0)$ is an sBCK-algebra. Here, if $(X, *, 0)$ is not positive implicative, then $X := (X, *, \vee, 0)$ is not an spBCK-algebra.

Example 3.6. Let $(X, *, 0)$ be the BCK-algebra which is described in Example 3.3. Then $X := (X, *, \vee, 0)$ is an spBCK-algebra, where $\vee$ is meant to be the supremum.

Example 3.7. Let $X = \{0, a, b, c, 1\}$ be a bounded BCK-algebra with the following Cayley table and Hasse diagram:

$*$	0	a	b	c	1
0	0	0	0	0	0
a	a	0	0	a	0
b	b	b	0	b	0
c	c	c	c	0	0
1	1	1	1	1	0



Then $X := (X, *, \vee, 0)$ is an spBCK-algebra, where $x \vee y$ is the supremum of $\{x, y\}$ for all $x, y \in X$.

Example 3.8. (1) Let $(X, *, 0)$ be any BCK-algebra. Define a binary operation $\vee$ on X by $x \vee y = 0$ for all $x, y \in X$. Then (a7), (a8) and (a9) hold, but (a6) does not hold. Hence $X := (X, *, \vee, 0)$ is not an sBCK-algebra.

(2) Let $(X, *, 0)$ be a BCK-algebra which is not bounded. We define a binary operation $\vee$ on X by

$$x \vee y := \begin{cases} \sup\{x, y\} & \text{if it exists,} \\ 0 & \text{otherwise.} \end{cases} \tag{3.3}$$

Obviously, (a6) is valid. If we take $x1(\neq 0), y1(\neq 0) \in X$ such that there is no $\sup\{x1, y1\}$, then

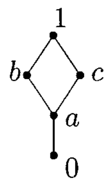
$$(x1 \vee y1) \vee y1 = 0 \vee y1 = y1 \neq 0 = x1 \vee y1 = x1 \vee (y1 \vee y1),$$

i.e., (a7) is not valid. Hence $X := (X, *, \vee, 0)$ is not an sBCK-algebra.

(3) In Example 3.2, $X := (X, *, \vee, 0)$ satisfies (a6), (a7) and (a9), but it does not satisfy the identity (a8) since $(b * a) \vee a = a \vee a = a \neq b = b \vee a$. Hence $X := (X, *, \vee, 0)$ is not an sBCK-algebra.

(4) Let $X = \{0, a, b, c, 1\}$ be a bounded BCK-algebra with the following Cayley table and Hasse diagram:

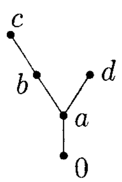
$*$	0	a	b	c	1
0	0	0	0	0	0
a	a	0	0	0	0
b	b	a	0	a	0
c	c	a	a	0	0
1	1	b	a	b	0



Then $(X, \leq)$ is a join($\vee$)-semilattice, and so (a6) and (a7) are valid. On the other hand, (a9) is also valid. But (a8) is not valid since $(c * a) \vee a \neq c \vee a$. Hence $X := (X, *, \vee, 0)$ is not an sBCK-algebra.

(5) Let $X = \{0, a, b, c, d\}$ be a BCK-algebra with the following Cayley table and Hasse diagram:

$*$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	0	0	0
b	b	a	0	0	a
c	c	b	a	0	b
d	d	a	a	a	0



We define a binary operation $\vee$ on X by

$$x \vee y := \begin{cases} \sup\{x, y\} & \text{if it exists,} \\ 0 & \text{otherwise.} \end{cases} \tag{3.4}$$

Then (a9) does not hold since

$$((c * d) \vee (d * d)) * ((d \vee c) * d) = (b \vee 0) * (0 * d) = b * 0 = b \neq 0.$$

Hence $X := (X, *, \vee, 0)$ is not an sBCK-algebra.

Theorem 3.9. *Let $X := (X, *, \vee, 0)$ be an sBCK-algebra. Then $(X, \vee)$ is a join semilattice.*

Proof. Straightforward. □

Note that if X is a bounded BCK-algebra, then there is always $\sup\{x, y\}$ for all $x, y \in X$. Hence we can guess that every bounded BCK-algebra is an sBCK-algebra. But this is not valid as seen in the Example 3.8(4). Now consider an sBCK-algebra $X := (X, *, \vee, 0)$ in Example 3.5. If $X := (X, *, 0)$ is a non-bounded BCK-algebras, this shows that an sBCK-algebra is not equivalent to a bounded BCK-algebra in general.

Proposition 3.10. *In every sBCK-algebra $X := (X, *, \vee, 0)$, we have the following assertions:*

(i) $(\forall x \in X)\ (0 \vee x = x)$.
(ii) $(\forall x, y \in X)\ (x \vee y = y \vee x)$.
(iii) $(\forall x, y \in X)\ (x * y = 0 \Leftrightarrow x \vee y = y)$.
(iv) $(\forall x, y \in X)\ (x \vee y = x \vee (y * x))$.

Proof. (i) For every $x \in X$, we have

$$0 \vee x = (x * x) \vee x = x \vee x = x$$

by (a3), (a8) and (a6).

(ii) Let $x, y \in X$. Then

$$y \vee x = (y * 0) \vee (x * 0) \leq (x \vee y) * 0 = x \vee y$$

by (b1) and (a9), and so (ii) is valid.

(iii) Let $x, y \in X$. If $x * y = 0$, then

$$x \vee y = (x * y) \vee y = 0 \vee y = y.$$

by (a8) and (i). Conversely, assume that $x \vee y = y$. Then

$$\begin{aligned} 0 &= ((y * y) \vee (x * y)) * ((x \vee y) * y) \\ &= (0 \vee (x * y)) * (y * y) \\ &= (x * y) * 0 \\ &= x * y \end{aligned}$$

by (a9), (a3), (i) and (b1).

(iv) For any $x, y \in X$, we have

$$x \vee (y * x) = (y * x) \vee x = y \vee x = x \vee y$$

by (ii) and (a8). □

We have the following question.

(Q2) In an sBCK-algebra $X := (X, *, \vee, 0)$, does the following inequality hold ?

$$(\forall x, y \in X)\,((x \vee y) * x \leq y). \tag{3.5}$$

Herein, we give answer to this question negatively.

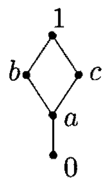
Example 3.11. (1) Consider the sBCK-algebra $X := (X, *, \vee, 0)$ which is described in Example 3.6. Since $(b \vee c) * b = d * b = d \not\leq c$, X does not satisfy the inequality (3.5).

(2) Let $X := (X, *, \vee, 0)$ be the sBCK-algebra which is given in Example 3.7. Then the inequality (3.5) does not hold in X since $(a \vee c) * a = 1 * a = 1 \not\leq c$.

We provide an sBCK-algebras that satisfy the inequality (3.5).

Example 3.12. Let $X = \{0, a, b, c, 1\}$ be a BCK-algebra with the following Cayley table and Hasse diagram:

$*$	0	a	b	c	1
0	0	0	0	0	0
a	a	0	0	0	0
b	b	b	0	b	0
c	c	c	c	0	0
1	1	1	c	b	0



Then $X := (X, *, \vee, 0)$ is an sBCK-algebra, where $x \vee y$ is the supremum of $\{x, y\}$ for all $x, y \in X$. We can verify that X satisfies the inequality (3.5).

Theorem 3.13. *Let $X := (X, *, \vee, 0)$ be an sBCK-algebra. If X satisfies the inequality* (3.5)*, then X has the condition* (S).

Proof. Assume that X satisfies the inequality (3.5). Then $x \vee y \in X(x,y)$ for all $x, y \in X$. Given $x, y \in X$, let $z \in X(x,y)$. Then $z * x \leq y$. It follows from (a8) and Proposition 3.10(ii) that

$$z \leq z \vee x = (z * x) \vee x \leq y \vee x = x \vee y$$

so that $x \vee y$ is a greatest element of $X(x,y)$. Therefore X has the condition (S). □

Proposition 3.14. *Let $X := (X, *, \vee, 0)$ be an sBCK-algebra that satisfies the inequality* (3.5). *For every $a, b \in X$, let*

$$\mathcal{D} := \{x \in X \mid a \leq b \vee x\}. \tag{3.6}$$

Then $a * b$ is the least element in $\mathcal{D}$.

Proof. Since $a * b \leq a * b$, we have $a \leq b \vee (a * b)$. Hence $a * b \in \mathcal{D}$. Let $x \in \mathcal{D}$. Then $a \leq b \vee x$, and so $a * b \leq (b \vee x) * b \leq x$ by (b4) and (3.5). Therefore $a * b$ is the least element in $\mathcal{D}$. □

Theorem 3.15. *Let $X := (X, *, \vee, 0)$ be an sBCK-algebra that satisfies the inequality* (3.5). *Then*

$$(\forall a, b \in X)\,(X(a,b) = \{x \in X \mid x \leq a \vee b\}). \tag{3.7}$$

Proof. It is easy to verify that $X(a,b) \subseteq \{x \in X \mid x \leq a \vee b\}$. Let $u \in X$ be such that $u \leq a \vee b$. Then $u * a \leq (a \vee b) * a$ by (b4). It follows that

$$\begin{aligned}
(u * a) * b &= (((u * a) * b) * 0) * 0 \\
&= (((u * a) * b) * ((u * a) * ((a \vee b) * a))) * (((a \vee b) * a) * b) \\
&= (((u * a) * b) * (((a \vee b) * a) * b)) * ((u * a) * ((a \vee b) * a)) \\
&\leq ((u * a) * ((a \vee b) * a)) * ((u * a) * ((a \vee b) * a)) = 0
\end{aligned}$$

so that $(u * a) * b = 0$, that is, $u \in X(a,b)$. This completes the proof. □

Proposition 3.16. *Let $X := (X, *, \vee, 0)$ be an sBCK-algebra that satisfies the inequality* (3.5). *Then any ideal A of X satisfies:*

$$(\forall x, y \in A)\,(\exists z \in A)\,(x \leq z,\ y \leq z). \tag{3.8}$$

Proof. Let $x, y \in A$. Then $(x \vee y) * x \leq y$, which implies from (2.5) and (c2) that $x \vee y \in A$. Moreover, $x \leq x \vee y$ and $y \leq x \vee y$. This completes the proof. □

Theorem 3.17. *Let $X := (X, *, \vee, 0)$ be an sBCK-algebra that satisfies the inequal-*

ity (3.5). *If A is an ideal of $(X,*,0)$ in which there exists a maximal element w, then $A = \{x \in X \mid x \leq w\}$. Moreover, if A is finite, then $A = \{x \in X \mid x \leq a\}$ for some $a \in X$.*

Proof. For any $x \in A$, we have $(x \vee w) * w \in A$ by (3.5) and (2.5). It follows from (c2) that $x \vee w \in A$. Since $w \leq x \vee w$ and w is a maximal element in A, we get $w = x \vee w$. Hence $x \leq x \vee w = w$, and so $A \subseteq \{x \in X \mid x \leq w\}$. Obviously, $\{x \in X \mid x \leq w\} \subseteq A$. Now assume that A is finite. Then A has a maximal element, say a. Hence $A = \{x \in X \mid x \leq a\}$ by the above discussion. □

Definition 3.18. Let $X := (X,*,\vee,0)$ be an sBCK-algebra. An ideal A of X is said to be *dry* if it satisfies:

$$(\forall x, y \in X)\,(x \in A \Rightarrow (x \vee y) * y \in A). \tag{3.9}$$

Example 3.19. Consider the sBCK-algebra $X = \{0, a, b, c, 1\}$ which is described in Example 3.12. Then we can verify that $A = \{0, a, b\}$ is a dry ideal of X.

Example 3.20. Let $X := (X,*,\vee,0)$ be the sBCK-algebra which is given in Example 3.6. Consider subsets $A = \{0, a, b\}$, $B = \{0, a, c\}$ and $C = \{0, a, b, c\}$ of X. Then A, B and C are all ideals of X, but they are not dry since $(b \vee c) * c = d * c = d \notin \{0, a, b\} \subseteq \{0, a, b, c\}$ and $(c \vee b) * b = d * b = d \notin \{0, a, c\} \subseteq \{0, a, b, c\}$. On the other hand, we know that $D = \{0, a, b, c, d\}$ is a dry ideal of X.

Theorem 3.21. *In an sBCK-algebra $X := (X,*,\vee,0)$ that satisfies the inequality* (3.5)*, every ideal is dry.*

Proof. Let A be an ideal of X, $x \in A$ and $y \in X$. Then $(x \vee y) * y \leq x$ by (3.5). It follows from (2.5) that $(x \vee y) * y \in A$. Hence A is a dry ideal of X. □

Theorem 3.22. *Let $X := (X,*,\vee,0)$ be an sBCK-algebra that satisfies the inequality* (3.5)*. Then a nonempty subset A of X is an ideal of X if and only if it satisfies* (2.5) *and*

$$(\forall x, y \in X)\,(x, y \in A \Rightarrow x \vee y \in A). \tag{3.10}$$

Proof. Assume that A is an ideal of X and let $x, y \in A$. Obviously (2.5) is valid. Using (3.5) we have $(x \vee y) * x \leq y$. It follows from (2.5) and (c2) that $x \vee y \in A$. Conversely, let A be a nonempty subset of X that satisfies (2.5) and (3.10). Then there exists x in A. Since $0 \leq x$, we get $0 \in A$ by (2.5). Let $x, y \in X$ be such that $x * y \in A$ and $y \in A$. By Proposition 3.14, we have $x \leq y \vee (x * y)$. On the other hand, $y \vee (x * y) \in A$ by (3.10). It follows from (2.5) that $x \in A$. Hence A is an ideal of X. □

The notion of semi-Brouwerian algebras was introduced by P. V. R. Murty [9] in 1974.

Definition 3.23. A *semi-Brouwerian algebra* is a system $(X, *, \circ, 0) \in K(\tau)$, where $\tau = (2, 2, 0)$, such that

(d1) $(\forall x \in X)\ (x \circ x = x,\ x * x = 0)$,
(d2) $(\forall x, y \in X)\ (x \circ y = y \circ x)$,
(d3) $(\forall x, y \in X)\ ((x * y) \circ y = x \circ y)$,
(d4) $(\forall x, y, z \in X)\ ((x * y) * z = x * (y \circ z))$.

Theorem 3.24. *Every sBCK-algebra satisfying the inequality* (3.5) *is a semi-Brouwerian algebra.*

Proof. Let $X := (X, *, \vee, 0)$ be an sBCK-algebra that satisfies the inequality (3.5). It is sufficient to show that X satisfies (d4). Let $x, y, z \in X$. Then

$$(x * ((x * y) * z)) * z = (x * z) * ((x * y) * z) \leq x * (x * y) \leq y,$$

and so $x * ((x * y) * z) \leq y \vee z$. It follows that $x * (y \vee z) \leq (x * y) * z$. On the other hand,

$$(x * y) * (x * (y \vee z)) \leq (y \vee z) * y \leq z,$$

and thus $(x*y)*z \leq x*(y\vee z)$. Therefore (d4) is valid, and the proof is complete. □

Theorem 3.25. *Every sBCK-algebra satisfying the inequality* (3.5) *is an spBCK-algebra.*

Proof. Let $X := (X, *, \vee, 0)$ be an sBCK-algebra that satisfies the inequality (3.5). It is sufficient to show that X satisfies the identity (2.2). Let $x, y \in X$. Then

$$(x * y) * y = x * (y \vee y) = x * y$$

by (d4) and (a6). Hence $X := (X, *, \vee, 0)$ is an spBCK-algebra. □

REFERENCES

1. H.A.S. Abujabal: A relative one-point union of BCK-algebras. *Math. Japonica* **45** (1997), no. 1, 103-111.
2. J. Hao: Ideal theory of BCC-algebras. *Scientiae Mathematicae* **1** (1998), no. 3, 373-381.

3. Y. Imai & K. Iséki: On axiom systems of propositional calculi. *Proc. Jpn. Acad.* **42** (1966), 19-21.
4. K. Iséki: Some examples of BCI-algebras. *Math. Seminar Notes* **8** (1980), 237-240.
5. K. Iséki & S. Tanaka: An introduction to the theory of BCK-algebras. *Math. Japonica* **23** (1978), no. 1, 1-26.
6. Y.B. Jun, J.Y. Kim & H.S. Kim: On Q-upper algebras. *Order* **22** (2005), 191-200.
7. Y.B. Jun, K.J. Lee & C.H. Park: A method to make BCK-algebras. *Commun. Korean Math. Soc.* **22**(2007),no. 4,503-508.
8. J. Meng & Y.B. Jun: *BCK-algebras.* Kyungmoon Sa Co. Korea, 1994.
9. P.V.R. Murty: Semi-Brouwerian algebras. *J. Austral Math. Soc.* **18** (1974), 293-302.
10. J. Neggers & H.S. Kim: *Basic Posets,* World Scientific Publishing Co. 1998.

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