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CERTAIN INTEGRAL REPRESENTATIONS OF EULER TYPE FOR THE EXTON FUNCTION X_2

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ABSTRACT. Exton [Hypergeometric functions of three variables, *J. Indian Acad. Math.* **4** (1982), 113–119] introduced 20 distinct triple hypergeometric functions whose names are X_i ($i = 1, \ldots, 20$) to investigate their twenty Laplace integral representations whose kernels include the confluent hypergeometric functions ${}_0F_1$, ${}_1F_1$, a Humbert function Ψ_2, a Humbert function Φ_2. The object of this paper is to present 16 (presumably new) integral representations of Euler type for the Exton hypergeometric function X_2 among his twenty X_i ($i = 1, \ldots, 20$), whose kernels include the Exton function X_2 itself, the Appell function F_4, and the Lauricella function F_C.

1. INTRODUCTION

Exton [5] introduced 20 distinct triple hypergeometric functions whose names are X_i ($i = 1, \ldots, 20$) to investigate their twenty Laplace integral representations which include the confluent hypergeometric functions ${}_0F_1$, ${}_1F_1$, a Humbert function Ψ_2, a Humbert function Φ_2 in their kernels. The Exton functions X_i have been studied a lot until today, for example, see [2, 3, 6, 7, 8, 9, 10, 11]. Choi et al. [2] gave 13 integral representations of Euler type which contain the Exton function X_1 itself, the Exton function X_2, the Appell function F_4, and the Srivastava function $F^{(3)}$ in their kernels. In the sequel of the work by Choi et al. [2], here, we choose to investigate the Exton function X_2 to present (presumably new) 16 integral representations of Euler type which contain the Exton function X_2 itself, the Appell function F_4, and the Lauricella function F_C in their kernels.

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Exton [5] defined the function X_2 by the following triple series

$$X_2\left(a_1, a_2; c_1, c_2, c_3; x, y, z\right) = \sum_{m,n,p=0}^{\infty} \frac{\left(a_1\right)_{2m+2n+p}\left(a_2\right)_p}{\left(c_1\right)_m\left(c_2\right)_n\left(c_3\right)_p\, m!\, n!\, p!} x^m y^n z^p, \tag{1.1}$$

where $(\lambda)_m$ denotes the Pochhammer symbol defined by

$$(\lambda)_m := \frac{\Gamma(\lambda+m)}{\Gamma(\lambda)} \quad \left(\lambda \in \mathbb{C} \setminus \mathbb{Z}_0^-;\ m \in \mathbb{N}_0\right),$$

$\mathbb{C}$, $\mathbb{Z}_0^-$, and $\mathbb{N}_0$ being the set of complex numbers, the set of nonpositive integers, and the set of nonnegative integers, respectively. The precise three-dimensional region of convergence of (1.1) is given by Srivastava and Karlsson in [11, p. 106, 58a].

It may be recalled the Laplace integral representation of (1.1) (see [5]) in passing that

$$\begin{aligned} &X_2\left(a_1, a_2; c_1, c_2, c_3; x, y, z\right) \\ &= \frac{1}{\Gamma\left(a_1\right)} \int_0^{\infty} e^{-s} s^{a_1-1}{}_0F_1\left(-; c_1; x s^2\right){}_0F_1\left(-; c_1; y s^2\right){}_1F_1\left(a_2; c_3; z s\right) ds, \end{aligned} \tag{1.2}$$

provided $\Re\left(a_1\right) > 0$.

2. INTEGRAL REPRESENTATIONS OF EULER TYPE FOR X_2

Each of the following integral representations for X_2 holds true.

(2.1)
$$\begin{aligned} &X_2\left(a_1, a_2; c_1, c_2, c_3; x, y, z\right) \\ &= \frac{\Gamma\left(c_1\right)}{\Gamma\left(c_1-d\right)\Gamma(d)} \int_0^1 \xi^{c_1-d-1}(1-\xi)^{d-1} X_2\left(a_1, a_2; d, c_2, c_3; x(1-\xi), y, z\right) d\xi \\ &\qquad \left(\Re\left(c_1\right) > \Re(d) > 0\right); \end{aligned}$$

(2.2)
$$\begin{aligned} &X_2\left(a_1, a_2; c_1, c_2, c_3; x, y, z\right) \\ &= \frac{\Gamma\left(c_3\right)}{\Gamma\left(c_3-d\right)\Gamma(d)} \int_0^1 \xi^{c_3-d-1}(1-\xi)^{d-1} X_2\left(a_1, a_2; c_1, c_2, c_3; x, y, z(1-\xi)\right) d\xi \\ &\qquad \left(\Re\left(c_3\right\} > \Re(d) > 0\right); \end{aligned}$$

(2.3)
$$\begin{aligned} &X_2\left(a_1, a_2; c_1, c_2, c_3; x, y, z\right) \\ &= \frac{\Gamma(a)}{\Gamma\left(a_1\right)\Gamma\left(a-a_1\right)} \int_0^1 \xi^{a_1-1}(1-\xi)^{a-a_1-1} X_2\left(a, a_2; c_1, c_2, c_3; x\xi^2, y\xi^2, z\xi\right) d\xi \\ &\qquad \left(\Re(a) > \Re\left(a_1\right) > 0\right); \end{aligned}$$

$$
\begin{aligned}
X_2(a_1,a_2;c_1,c_2,c_3;x,y,z) &= \frac{\Gamma(c_3)}{\Gamma(a_2)\Gamma(c_3-a_2)}\int_0^1 \xi^{a_2-1}(1-\xi)^{c_3-a_2-1} \\
&\cdot (1-z\xi)^{-a_1}\, F_4\left(\frac{a_1}{2},\frac{1}{2}+\frac{a_1}{2};c_1,c_2;\frac{4x}{(1-z\xi)^2},\frac{4y}{(1-z\xi)^2}\right)d\xi \\
&(\Re(c_3)>\Re(a_2)>0),
\end{aligned} \tag{2.4}
$$

where F_4 denotes (throughout this paper) an Appell function (see [11, p. 23]) defined by

$$
F_4(a,b;c_1,c_2;x,y)=\sum_{m,n=0}^{\infty}\frac{(a)_{m+n}(b)_{m+n}}{(c_1)_m(c_2)_n\, m!n!}\,x^m\,y^n \quad \left(\sqrt{|x|}+\sqrt{|y|}<1\right); \tag{2.5}
$$

$$
\begin{aligned}
X_2(a_1,a_2;c_1,c_2,c_3;x,y,z) &= \frac{\Gamma(c_3)(1+\lambda)^{a_2}}{\Gamma(a_2)\Gamma(c_3-a_2)} \\
&\cdot\int_0^1 \xi^{a_2-1}(1-\xi)^{c_3-a_2-1}(1+\lambda\xi)^{a_1-c_3}\left[1+\lambda\xi-(1+\lambda)z\xi\right]^{-a_1} \\
&\cdot F_4\left(\frac{a_1}{2},\frac{1}{2}+\frac{a_1}{2};c_1,c_2;\frac{4x(1+\lambda\xi)^2}{[1+\lambda\xi-(1+\lambda)z\xi]^2},\frac{4y(1+\lambda\xi)^2}{[1+\lambda\xi-(1+\lambda)z\xi]^2}\right)d\xi \\
&(\Re(c_3)>\Re(a_2)>0;\ \lambda>-1);
\end{aligned} \tag{2.6}
$$

(2.7)

$$
\begin{aligned}
X_2(a_1,a_2;c_1,c_2,c_3;x,y,z) &= \frac{\Gamma(c_3)(\beta-\gamma)^{a_2}(\alpha-\gamma)^{c_3-a_2}}{\Gamma(a_2)\Gamma(c_3-a_2)(\beta-\alpha)^{c_3-a_1-1}}\int_\alpha^\beta (\beta-\xi)^{c_3-a_2-1} \\
&\cdot(\xi-\alpha)^{a_2-1}(\xi-\gamma)^{a_1-c_3}\left[(\beta-\alpha)(\xi-\gamma)-(\beta-\gamma)z(\xi-\alpha)\right]^{-a_1} \\
&\cdot F_4\left(\frac{a_1}{2},\frac{1}{2}+\frac{a_1}{2};c_1,c_2;\sigma x,\sigma y\right)d\xi,\quad \sigma=\frac{4(\beta-\alpha)^2(\xi-\gamma)^2x}{[(\beta-\alpha)(\xi-\gamma)-(\beta-\gamma)(\xi-\alpha)z]^2} \\
&(\Re(c_3)>\Re(a_2)>0;\ \gamma<\alpha<\beta);
\end{aligned}
$$

(2.8)

$$
\begin{aligned}
X_2(a_1,a_2;c_1,c_2,c_3;x,y,z) &= \frac{\Gamma(c_3)(\gamma-\beta)^{a_2}(\gamma-\alpha)^{c_3-a_2}}{\Gamma(a_2)\Gamma(c_3-a_2)(\beta-\alpha)^{c_3-a_1-1}}\int_\alpha^\beta (\beta-\xi)^{c_3-a_2-1} \\
&\cdot(\xi-\alpha)^{a_2-1}(\gamma-\xi)^{a_1-c_3}\left[(\beta-\alpha)(\gamma-\xi)-(\gamma-\beta)(\xi-\alpha)z\right]^{-a_1} \\
&\cdot F_4\left(\frac{a_1}{2},\frac{1}{2}+\frac{a_1}{2};c_1,c_2;\sigma x,\sigma y\right)d\xi,\quad \sigma=\frac{4(\beta-\alpha)^2(\gamma-\xi)^2}{[(\beta-\alpha)(\gamma-\xi)-(\gamma-\beta)(\xi-\alpha)z]^2} \\
&(\Re(c_3)>\Re(a_2)>0;\ \alpha<\beta<\gamma);
\end{aligned}
$$

(2.9)
$$X_2(a_1,a_2;c_1,c_2,c_3;x,y,z) = \frac{2\Gamma(c_3)}{\Gamma(a_2)\Gamma(c_3-a_2)} \int_0^{\frac{\pi}{2}} \left(\sin^2\xi\right)^{a_2-\frac{1}{2}} \cdot \left(\cos^2\xi\right)^{c_3-a_2-\frac{1}{2}} \left(1-z\sin^2\xi\right)^{-a_1} F_4\left(\frac{a_1}{2},\frac{1}{2}+\frac{a_1}{2};c_1,c_2;\sigma x,\sigma y\right) d\xi$$
$$\left(\Re(c_3) > \Re(a_2) > 0\right),$$

where

$$\sigma = \frac{4}{\left(1-z\sin^2\xi\right)^2};$$

(2.10)
$$X_2(a_1,a_2;c_1,c_2,c_3;x,y,z) = \frac{2\Gamma(c_3)(1+\lambda)^{a_2}}{\Gamma(a_2)\Gamma(c_3-a_2)} \int_0^{\frac{\pi}{2}} \frac{(\sin\xi)^{2a_2-1}(\cos\xi)^{2c_3-2a_2-1}}{\left(1+\lambda\sin^2\xi\right)^{c_3-a_1}} \cdot \left[\left(1+\lambda\sin^2\xi\right)-(1+\lambda)z\sin^2\xi\right]^{-a_1} F_4\left(\frac{a_1}{2},\frac{1}{2}+\frac{a_1}{2};c_1,c_2;\sigma x,\sigma y\right) d\xi$$
$$\left(\Re(c_3) > \Re(a_2) > 0;\ \ \lambda > -1\right),$$

where

$$\sigma = \frac{4\left(1+\lambda\sin^2\xi\right)^2}{\left[\left(1+\lambda\sin^2\xi\right)-(1+\lambda)z\sin^2\xi\right]^2};$$

(2.11)
$$X_2(a_1,a_2;c_1,c_2,c_3;x,y,z) = \frac{2\Gamma(c_3)\lambda^{a_2}}{\Gamma(a_2)\Gamma(c_3-a_2)} \cdot \int_0^{\frac{\pi}{2}} \frac{(\sin\xi)^{2a_2-1}(\cos\xi)^{2c_3-2a_2-1}}{\left(\cos^2\xi+\lambda\sin^2\xi\right)^{c_3-a_1}} F_4\left(\frac{a_1}{2},\frac{1}{2}+\frac{a_1}{2};c_1,c_2;\sigma x,\sigma y\right) d\xi$$
$$\left(\Re(c_3) > \Re(a_2) > 0;\ \ \lambda > 0\right),$$

where

$$\sigma = \frac{4}{\left[\cos^2\xi+\lambda\sin^2\xi-z\lambda\sin^2\xi\right]^2};$$

(2.12)
$$X_2(a_1,a_2;c_1,c_2,c_3;x,y,z) = \frac{\Gamma(a_2+a_1)}{\Gamma(a_1)\Gamma(a_2)} \int_0^1 \xi^{a_1-1}(1-\xi)^{a_2-1} \cdot F_C\left(\frac{a_1+a_2}{2},\frac{1}{2}+\frac{a_1+a_2}{2};c_1,c_2,c_3;4x\xi^2,4y\xi^2,4z\xi(1-\xi)\right) d\xi$$
$$\left(\Re(a_1) > 0;\ \ \Re(a_2) > 0\right),$$

where F_C denotes (throughout this paper) a Lauricella function (see [11, p. 33]) defined by

(2.13)
$$F_C(a,b;c_1,c_2,c_3;x,y,z)=\sum_{m,n,\ell=0}^{\infty}\frac{(a)_{m+n+\ell}\,(b)_{m+n+\ell}}{(c_1)_m\,(c_2)_n\,(c_2)_\ell\;m!\,n!\,\ell!}x^m\,y^n\,z^\ell$$
$$\left(\sqrt{|x|}+\sqrt{|y|}+\sqrt{|z|}<1\right);$$

(2.14)
$$X_2(a_1,a_2;c_1,c_2,c_3;x,y,z)=\frac{\Gamma(a_2+a_1)}{\Gamma(a_1)\Gamma(a_2)}\frac{(\beta-\gamma)^{a_1}(\alpha-\gamma)^{a_2}}{(\beta-\alpha)^{a_1-1}}\int_\alpha^\beta(\beta-\xi)^{a_2-1}$$
$$\cdot(\xi-\alpha)^{a_1-1}(\xi-\gamma)^{-a_1-a_2}F_C\left(\frac{a_1+a_2}{2},\frac{1}{2}+\frac{a_1+a_2}{2};c_1,c_2,c_3;\sigma_1x,\sigma_1y,\sigma_2z\right)d\xi$$
$$(\Re(a_1)>0;\ \Re(a_2)>0;\ \gamma<\alpha<\beta),$$

where

$$\sigma_1=\frac{4(\beta-\gamma)^2(\xi-\alpha)^2}{(\beta-\alpha)^2(\xi-\gamma)^2}\quad\text{and}\quad\sigma_2=\frac{4(\alpha-\gamma)(\beta-\gamma)(\xi-\alpha)(\beta-\xi)}{(\beta-\alpha)^2(\xi-\gamma)^2};$$

(2.15)
$$X_2(a_1,a_2;c_1,c_2,c_3;x,y,z)=\frac{\Gamma(a_2+a_1)}{\Gamma(a_1)\Gamma(a_2)}\frac{(\gamma-\beta)^{a_1+p}(\gamma-\alpha)^{a_2}}{(\beta-\alpha)^{a_1+a_2-1}}\int_\alpha^\beta(\beta-\xi)^{a_2-1}$$
$$\cdot(\xi-\alpha)^{a_1-1}(\gamma-\xi)^{-a_1-a_2}F_C\left(\frac{a_1+a_2}{2},\frac{1}{2}+\frac{a_1+a_2}{2};c_1,c_2,c_3;\sigma_1x,\sigma_1y,\sigma_2z\right)d\xi$$
$$(\Re(a_1)>0;\ \Re(a_2)>0;\ \alpha<\beta<\gamma),$$

where

$$\sigma_1=\frac{4(\gamma-\beta)^2(\xi-\alpha)^2}{(\beta-\alpha)^2(\gamma-\xi)^2}\quad\text{and}\quad\sigma_2=\frac{(\gamma-\beta)(\gamma-\alpha)(\beta-\xi)(\xi-\alpha)}{(\beta-\alpha)^2(\gamma-\xi)^2};$$

(2.16)
$$X_2(a_1,a_2;c_1,c_2,c_3;x,y,z)=\frac{2\Gamma(a_2+a_1)}{\Gamma(a_1)\Gamma(a_2)}\int_0^{\frac{\pi}{2}}\left(\sin^2\xi\right)^{a_1-\frac{1}{2}}\left(\cos^2\xi\right)^{a_2-\frac{1}{2}}$$
$$\cdot F_C\left(\frac{a_1+a_2}{2},\frac{1}{2}+\frac{a_1+a_2}{2};c_1,c_2,c_3;4x\sin^4\xi,4y\sin^4\xi,z\sin^2 2\xi\right)d\xi$$
$$(\Re(a_1)>0;\ \Re(a_2)>0);$$

(2.17)

$$X_2(a_1, a_2; c_1, c_2, c_3; x, y, z) = \frac{2\Gamma(a_2 + a_1)(1+\lambda)^{a_1}}{\Gamma(a_1)\Gamma(a_2)} \int_0^{\frac{\pi}{2}} \frac{\left(\sin^2\xi\right)^{a_1-\frac{1}{2}}\left(\cos^2\xi\right)^{a_2-\frac{1}{2}}}{\left(1+\lambda\sin^2\xi\right)^{a_1+a_2}}$$
$$\cdot F_C\left(\frac{a_1+a_2}{2}, \frac{1}{2}+\frac{a_1+a_2}{2}; c_1, c_2, c_3; \sigma_1 x, \sigma_1 y, \sigma_2 z\right) d\xi$$
$$(\Re(a_1) > 0;\ \Re(\ a_2) > 0;\ \lambda > -1),$$

where

$$\sigma_1 = \frac{4(1+\lambda)^2\sin^4\xi}{\left(1+\lambda\sin^2\xi\right)^2} \quad \text{and} \quad \sigma_2 = \frac{(1+\lambda)\sin^2 2\xi}{\left(1+\lambda\sin^2\xi\right)^2};$$

(2.18)

$$X_2(a_1, a_2; c_1, c_2, c_3; x, y, z) = \frac{2\Gamma(a_2 + a_1)\lambda^{a_1}}{\Gamma(a_1)\Gamma(a_2)} \int_0^{\frac{\pi}{2}} \frac{\left(\sin^2\xi\right)^{a_1-\frac{1}{2}}\left(\cos^2\xi\right)^{a_2-\frac{1}{2}}}{\left(\cos^2\xi+\lambda\sin^2\xi\right)^{a_1+a_2}}$$
$$\cdot F_C\left(\frac{a_1+a_2}{2}, \frac{1}{2}+\frac{a_1+a_2}{2}; c_1, c_2, c_3; \sigma_1 x, \sigma_1 y, \sigma_2 z\right) d\xi$$
$$(\Re(a_1) > 0;\ \Re(\ a_2) > 0;\ \lambda > 0),$$

where

$$\sigma_1 = \frac{4\lambda^2\sin^4\xi}{\left(\cos^2\xi+\lambda\sin^2\xi\right)^2} \quad \text{and} \quad \sigma_2 = \frac{\lambda\sin^2 2\xi}{\left(\cos^2\xi+\lambda\sin^2\xi\right)^2}.$$

3. Proof of Results

It is noted that each of the integral representations in Section 2 can be proved mainly by expressing the series definition of the involved special function in each integrand and changing the order of the integral sign and the summation, and finally using the following well-known relationship between the Beta function $B(\alpha, \beta)$ and the Gamma function Γ:

$$B(\alpha,\ \beta) = \begin{cases} \displaystyle\int_0^1 t^{\alpha-1}(1-t)^{\beta-1}\,dt & (\Re(\alpha) > 0;\ \Re(\beta) > 0), \\ \displaystyle\frac{\Gamma(\alpha)\,\Gamma(\beta)}{\Gamma(\alpha+\beta)} & (\Re(\alpha) < 0;\ \Re(\beta) < 0;\ \alpha,\ \beta \in \mathbb{C}\setminus\mathbb{Z}_0^-). \end{cases} \tag{3.1}$$

As an illustration, we try to prove only (2.6). By applying the Appell function F_4 to the right-hand side of (2.6), we have

$$\tag{3.2} \begin{aligned} &\frac{\Gamma(c_2)}{\Gamma(a_2)\Gamma(c_2-a_2)}\int_0^1 \xi^{a_2-1}(1-\xi)^{c_2-a_2-1}(1-z\xi)^{-a_1} \\ &\cdot\left[\sum_{m,n=0}^{\infty}\frac{4^{m+n}\left(\frac{a_1}{2}\right)_{m+n}\left(\frac{1}{2}+\frac{a_1}{2}\right)_{m+n}}{(c_1)_m(c_2-a_2)_n m!n!}\left(\frac{x}{(1-z\xi)^2}\right)^m\left[\frac{y(1-\xi)}{(1-z\xi)^2}\right]^n\right]d\xi. \end{aligned}$$

By using the following identity

$$\tag{3.3} (\lambda)_{2n} = 2^{2n}\left(\frac{1}{2}\lambda\right)_n\left(\frac{1}{2}\lambda+\frac{1}{2}\right)_n \qquad (n \in \mathbb{N}_0)$$

in the integrand of (3.2), we obtain

$$\tag{3.4} \begin{aligned} &\frac{\Gamma(c_2)}{\Gamma(a_2)\Gamma(c_2-a_2)}\int_0^1 \xi^{a_2-1}(1-\xi)^{c_2-a_2-1} \\ &\cdot\left[\sum_{m,n=0}^{\infty}\frac{(a_1)_{2m+2n}}{(c_1)_m(c_2-a_2)_n m!n!}x^m[y(1-\xi)]^n(1-z\xi)^{-a_1-2m-2n}\right]d\xi. \end{aligned}$$

Taking into account the binomial series

$$(1-z\xi)^{-a_1-2m-2n} = \sum_{p=0}^{\infty}\frac{(a_1+2m+2n)_p}{p!}(z\xi)^p$$

in the integrand of (3.4), and changing the order of the integral sign and the summations in the resulting expression, we find

$$\tag{3.5} \frac{\Gamma(c_2)}{\Gamma(a_2)\Gamma(c_2-a_2)}\sum_{m,n,p=0}^{\infty}\frac{(a_1)_{2m+2n+p}}{(c_1)_m(c_2-a_2)_n m!n!p!}x^m y^n z^p B(a_2+p, c_2-a_2+n)\,d\xi$$

which, upon applying (3.1) to the integral, is seen to yield the X_2 function in (2.6).

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