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REMARKS ON WEAKLY $s\gamma$-CONTINUOUS FUNCTIONS

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ABSTRACT. We introduce the concepts of strongly $s\gamma$-closed graph, $s\gamma$-compactness and $s\gamma$-T_2 space and study the relationships between such concepts and weakly $s\gamma$-continuous functions.

1. INTRODUCTION

Let X be a nonempty set and $P(X)$ the power set of X. A subclass $\mathcal{S} \subseteq P(X)$ is called a *supratopology* on X [3] if $X \in \int$ and $\mathcal{S}$ is closed under arbitrary union. $(X, \mathcal{S})$ is called a supratopological space. The members of $\mathcal{S}$ are called *supraopen* sets. The complement of supraopen sets are called *supraclosed* sets.

Let $(X, \mathcal{S})$ be a supratopological space and let $S(x) = \{A \in \tau : x \in A\}$ for each $x \in X$. Then we call $\mathbf{S}_x = \{A \subseteq X :$ there exists $\mu \subseteq S(x)$ such that μ is finite and $\cap\mu \subseteq A\}$ the *supra-neighborhood filter* at x [4]. A filter $\mathbf{F}$ on X *supra-converges* [4] to x if $\mathbf{F}$ is finer than the supra-neighborhood filter $\mathbf{S}_x$. A subset U of X is called an *$s\gamma$-open* set [4] in X if whenever a filter $\mathbf{F}$ on X supra-converges to x and $x \in U$, $U \in \mathbf{F}$. The class of all $s\gamma$-open sets in X will be denoted by $s\gamma(X)$. In particular, The class of all $s\gamma$-open sets induced by the supratopology $\mathcal{S}$ will be denoted by $s\gamma_{\mathcal{S}}(X)$.

For $A \subseteq X$, the *$s\gamma$-interior* of A in X, denoted by $s_\gamma I(A)$, is the union of all $s\gamma$-open sets contained in A.

$s_\gamma C(A) = \{x \in X : A \cap U \neq \emptyset$ for all $U \in \mathbf{S}_x\}$. We call $s_\gamma C(A)$ the *$s\gamma$-closure* of A.

Theorem 1.1 ([4]). *Let (X, τ) be a supratopological space and $A \subseteq X$.*

(1) *$s_\gamma I(A) \subseteq A$ and $A \subseteq s_\gamma C(A)$;*

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(2) *A is $s\gamma$-open set if and only if* $A = s_\gamma I(A)$;
(3) *A is $s\gamma$-closed if and only if* $A = s_\gamma C(A)$;
(4) $s_\gamma I(A) = X - s_\gamma C(X - A)$ *and* $s_\gamma C(A) = X - s_\gamma I(X - A)$.

Let (X, τ) be a topological space. Then τ^* is called an *associated supratopology* with τ if $\tau \subseteq \tau^*$.

Definition 1.2. Let (X, τ) and (Y, μ) be topological spaces and τ^* an associated supratopology with τ. A function $f : X \to Y$ is called

(1) *$s\gamma$-continuous* [4] if the inverse image of each open set of Y is an $s\gamma$-open set in X;

(2) *weakly continuous* [2] if for each x and each open set V of $f(x)$, there exists an open set U in X such that $f(U) \subset cl(V)$;

(3) *weakly $s\gamma$-continuous* [5] if for $x \in X$ and each open subset V containing $f(x)$, there is an $s\gamma$-open subset U containing x such that $f(U) \subseteq cl(V)$.

2. Main Results

Definition 2.1. Let X be a supratopological space. Then X is said to be $s\gamma$-T_2 if for every two distinct points x and y in X, there exist two disjoint $s\gamma$-open sets U and V such that $x \in U$ and $y \in V$.

Let X be a topological space. Then X is said to be *Urysohn* [1] if for every two distinct points x and y in X, there exist two open sets U and V such that $cl(U) \cap cl(V) = \emptyset$.

Theorem 2.2. *Let (X, τ) and (Y, μ) be topological spaces and τ^* be an associated supratopology with τ. If $f : (X, \tau) \to (Y, \mu)$ is a weakly $s\gamma$-continuous injection and Y is Urysohn, then X is $s\gamma$-T_2.*

Proof. Let x_1 and x_2 be two distinct elements in X, then $f(x_1) \neq f(x_2)$. There exist two open sets U and V in Y containing $f(x_1)$, $f(x_2)$, respectively, such that $cl(U) \cap cl(V) = \emptyset$. Since f is weakly $s\gamma$-continuous, there exist $s\gamma$-open sets U_1, V_2 containing x_1, x_2, respectively, such that $f(U_1) \subseteq cl(U)$, $f(V_2) \subseteq cl(V)$. It follows $U_1 \cap V_2 = \emptyset$. Hence X is $s\gamma$-T_2. □

Remark 2.3. Let (X, τ) and (Y, μ) be topological spaces and τ^* be an associated supratopology with τ. If $\tau = \tau^*$, then obviously $s\gamma(X) = \tau$.

Corollary 2.4. Let (X,τ) and (Y,μ) be topological spaces. If $f:(X,\tau)\to(Y,\mu)$ is a weakly continuous injection and Y is Urysohn, then X is T_2.

Proof. It follows from Theorem 2.2 and Remark 2.3. □

Definition 2.5. Let (X,τ) and (Y,μ) be topological spaces and τ^* be an associated supratopology with τ. Let $f:(X,\tau)\to(Y,\mu)$ be a function. We call f has a *strongly $s\gamma$-closed graph* if for each $(x,y)\notin G(f)$, there exist an $s\gamma$-open set U and an open set V containing x and y, respectively, such that $(U\times cl(V))\cap G(f)=\emptyset$.

Lemma 2.6. Let $f:(X,\tau)\to(Y,\mu)$ be a function on topological spaces (X,τ), (Y,μ) and τ^* an associated supratopology with τ. Then f has a strongly $s\gamma$-closed graph if for each $(x,y)\notin G(f)$, there exist an $s\gamma$-open set U containing and an open set V containing x and y, respectively, such that $f(U)\cap cl(V)=\emptyset$.

Proof. Obvious. □

Theorem 2.7. *Let (X,τ) and (Y,μ) be topological spaces and τ^* be an associated supratopology with τ. If $f:(X,\tau)\to(Y,\mu)$ is weakly $s\gamma$-continuous and Y is Urysohn, then f has a strongly $s\gamma$-closed graph.*

Proof. Let $(x,z)\notin G(f)$. Then $z\neq f(x)$ and since Y is Urysohn, there exist two open sets U and V containing z and $f(x)$, respectively, such that $cl(U)\cap cl(V)=\emptyset$. Since f is weakly $s\gamma$-continuous, there exists an $s\gamma$-open set H containing x such that $f(H)\subseteq cl(V)$. It implies $f(H)\cap cl(U)=\emptyset$. Hence f has a strongly $s\gamma$-closed graph. □

Let (X,τ) and (Y,μ) be topological spaces. We call $f:X\to Y$ has a *strongly closed graph* if for each $(x,y)\notin G(f)$, there exist open sets U and V containing x and y, respectively, such that $(U\times cl(V))\cap G(f)=\emptyset$.

Corollary 2.8. Let (X,τ) and (Y,μ) be topological spaces. If $f:(X,\tau)\to(Y,\mu)$ is weakly continuous and Y is Urysohn, then f has a strongly closed graph.

Proof. It follows from Remark 2.3 and Theorem 2.7. □

Theorem 2.9. *Let (X,τ) and (Y,μ) be topological spaces and τ^* be an associated supratopology with τ. If $f:(X,\tau)\to(Y,\mu)$ is a weakly $s\gamma$-continuous injection with a strongly $s\gamma$-closed graph, then X is $s\gamma$-T_2.*

Proof. Let x_1 and x_2 be two distinct elements in X, then $f(x_1) \neq f(x_2)$. This implies that $(x_1, f(x_2)) \in (X \times Y) - G(f)$. Since f has a strongly $s\gamma$-closed graph, there exist an $s\gamma$-open set U and an open set V containing x_1 and $f(x_2)$, respectively, such that $f(U) \cap cl(V) = \emptyset$. Since f is weakly $s\gamma$-continuous, there exist an $s\gamma$-open set W containing x_2 such that $f(W) \subseteq cl(V)$. It implies $f(W) \cap f(U) = \emptyset$. Therefore $W \cap U = \emptyset$ and so X is an $s\gamma$-T_2 space. □

Corollary 2.10. Let (X, τ) and (Y, μ) be topological spaces. If $f : (X, \tau) \to (Y, \mu)$ is a weakly continuous injection with a strongly closed graph, then X is T_2.

Definition 2.11. A subset A of a supratopological space (X, τ) is called *$s\gamma$-compact* relative to A if every collection $\{U_i : i \in J\}$ of $s\gamma$-open subsets of X such that $A \subseteq \cup\{U_i : i \in J\}$, there exists a finite subset J_0 of J such that $A \subseteq \cup\{U_i : i \in J_0\}$.

A subset A of a topological space X is said to be *quasi H-closed* relative to A [6] if every collection $\{U_i : i \in J\}$ of open subsets of X such that $A \subseteq \cup\{U_i : i \in J\}$, there exists a finite subset J_0 of J such that $A \subseteq \cup\{cl(U_i) : i \in J_0\}$.

Theorem 2.12. *Let (X, τ) and (Y, μ) be topological spaces and τ^* be an associated supratopology with τ. If $f : (X, \tau) \to (Y, \mu)$ is weakly $s\gamma$-continuous and A is an $s\gamma$-compact subset of X, then $f(A)$ is quasi H-closed relative to (Y, μ).*

Proof. Let $\{V_i : i \in J\}$ be a cover of $f(A)$ by open subsets of Y. For each $x \in A$, there exists $i \in J$ such that $f(x) = y \in V_i(x)$. Since f is weakly $s\gamma$-continuous, there exists an $s\gamma$-open set $U_i(x)$ containing x such that $f(U_i(x)) \subseteq cl(V_i(x))$. The family $\{U(x) : x \in A\}$ is a cover of A by $s\gamma$-open sets in X. Since A is $s\gamma$-compact, there is a finite subcover $\{U_1(x), U_2(x), \cdots, U_n(x) : x \in A, j = 1, 2, \cdots, n\}$ such that $A \subseteq \cup U_j(x)$. Then

$$f(A) \subseteq f(\cup U_j(x)) = \cup f(U_j(x)) \subseteq \cup cl(V_j(x)),$$

$1 \leq j \leq n$.

Thus $f(A)$ is quasi H-closed relative to (Y, μ). □

Corollary 2.13. Let (X, τ) and (Y, μ) be topological spaces. If $f : (X, \tau) \to (Y, \mu)$ is weakly continuous and A is a compact subset of X, then $f(A)$ is quasi H-closed relative to (Y, μ).

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