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THE CURVATURE OF HALF LIGHTLIKE SUBMANIFOLDS OF A SEMI-RIEMANNIAN MANIFOLD OF QUASI-CONSTANT CURVATURE

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Abstract. We study half lightlike submanifolds M of semi-Riemannian manifolds $\widetilde{M}$ of quasi-constant curvatures. The main result is a characterization theorem for screen homothetic Einstein half lightlike submanifolds of a Lorentzian manifold of quasi-constant curvature subject to the conditions; (1) the curvature vector field of $\widetilde{M}$ is tangent to M, and (2) the co-screen distribution is a conformal Killing one.

1. Introduction

Chen and Yano [1] introduced the notion of a Riemannian manifold of quasi-constant curvature as a Riemannian manifold $(\widetilde{M}, \widetilde{g})$ equipped with the curvature tensor $\widetilde{R}$ satisfying the following condition:

$$\begin{aligned}\widetilde{g}(\widetilde{R}(X,Y)Z,W) &= \alpha\{\widetilde{g}(Y,Z)\widetilde{g}(X,W) - \widetilde{g}(X,Z)\widetilde{g}(Y,W)\} \\ &+ \beta\{\widetilde{g}(X,W)\theta(Y)\theta(Z) - \widetilde{g}(X,Z)\theta(Y)\theta(W) \\ &+ \widetilde{g}(Y,Z)\theta(X)\theta(W) - \widetilde{g}(Y,W)\theta(X)\theta(Z)\},\end{aligned} \tag{1.1}$$

where α, β are scalar functions and θ is a 1-form defined by

$$\theta(X) = \widetilde{g}(X, \zeta), \tag{1.2}$$

and ζ is a unit vector field on $\widetilde{M}$, which called the *curvature vector field* of $\widetilde{M}$. It is well known that if the curvature tensor $\widetilde{R}$ is of the form (1.1), then $\widetilde{M}$ is conformally flat. If $\beta = 0$, then $\widetilde{M}$ is a space of constant curvature α.

Recently Jin [7] and Jin and Lee [8] studied lightlike submanifolds M in a semi-Riemannian manifold $\widetilde{M}$ of quasi-constant curvature subject to the conditions; (1)

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the curvature vector field ζ of $\widetilde{M}$ is tangent to M and (2) the screen distribution is totally geodesic in M. They proved two characterization theorems for such lightlike submanifolds (see [7, 8]).

The classification of Einstein half lightlike submanifolds M was studied by Jin [5]. Its main result focused on the geometry of Einstein half lightlike submanifolds M of a Lorentz space form $\widetilde{M}(c)$ of constant curvature c, whose co-screen distribution is a Killing one and whose shape operator is conformal to the shape operator of its screen distribution by some non-vanishing smooth function φ. The reason for this geometric restrictions on M was due to the fact that such a class admits an integrable screen distribution and a symmetric induced Ricci tensor. After that, Jin [6] generalized the main result of [5] for Einstein screen conformal half lightlike submanifold of Lorentz space forms endow with a conformal Killing co-screen distribution. A careful proof of [6] is even more involved than that of [5]. He proved a characterization theorem for such half lightlike submanifolds as it follow:

Theorem 1.1. *Let M be a screen conformal half lightlike submanifold of a Lorentz space form $\widetilde{M}^{m+3}(c)\,(m > 2)$ of constant curvature c equipped with a conformal Killing co-screen distribution of conformal factor δ. If M is Einstein, i.e., $Ric = \kappa g$, then M is locally a product manifold $\mathcal{C} \times M_1 \times M_2$, where $\mathcal{C}$ is a null curve and M_1 and M_2 are totally umbilical leaves of some distributions of M:*

(1) *If $\kappa \neq (m-1)(c+\delta^2)$, then either M_1 or M_2 is an m-dimensional Einstein Riemannian space form which is isometric to a sphere $(\kappa > 0)$ or a hyperbolic space $(\kappa < 0)$ and the other is a point on M.*
(2) *If $\kappa = (m-1)(c+\delta^2)$, then M_1 is an $(m-1)$ or m-dimensional Einstein Riemannian space form which is isometric to a sphere $(\kappa > 0)$ or a hyperbolic space $(\kappa < 0)$ or a Euclidean space $(\kappa = 0)$ and M_2 is a spacelike curve or a point on M.*

In particular, if the co-screen distribution is a Killing one, then $c = \delta = 0$ in the conditional paragraph of above two cases (1) *and* (2)*.*

The objective of this paper is to generalize the above characterization theorem for screen homothetic Einstein half lightlike submanifolds of a Lorentzian manifold of quasi-constant curvature. We prove a characterization theorem for screen homothetic half lightlike submanifolds M of a Lorentzian manifold $\widetilde{M}$ of quasi-constant curvature subject to the condition; (1) the curvature vector field of $\widetilde{M}$ is tangent to M, and (2) the co-screen distribution is a conformal Killing one.

2. Half Lightlike Submanifolds

It is well known that the radical distribution $Rad(TM) = TM \cap TM^{\perp}$ of half lightlike submanifolds M of a semi-Rimannian manifold $(\widetilde{M}, \widetilde{g})$ of codimension 2 is a vector subbundle of the tangent bundle TM and the normal bundle $TM^{\perp}$, of rank 1. Therefore there exist complementary non-degenerate distributions $S(TM)$ and $S(TM^{\perp})$ of $Rad(TM)$ in TM and $TM^{\perp}$ respectively, which called the *screen distribution* and *co-screen distribution* on M, such that

$$TM = Rad(TM) \oplus_{orth} S(TM), \quad TM^{\perp} = Rad(TM) \oplus_{orth} S(TM^{\perp}), \tag{2.1}$$

where $\oplus_{orth}$ denotes the orthogonal direct sum. We denote such a half lightlike submanifold by $M = (M, g, S(TM), S(TM^{\perp}))$. Denote by $F(M)$ the algebra of smooth functions on M and by $\Gamma(E)$ the $F(M)$ module of smooth sections of any vector bundle E over M. Consider the orthogonal complementary distribution $S(TM)^{\perp}$ to $S(TM)$ in $T\widetilde{M}$. Certainly $TM^{\perp}$ is a subbundle of $S(TM)^{\perp}$. As $S(TM^{\perp})$ is a non-degenerate subbundle of $S(TM)^{\perp}$, the orthogonal complementary distribution $S(TM^{\perp})^{\perp}$ of $S(TM^{\perp})$ in $S(TM)^{\perp}$ is also a non-degenerate distribution such that

$$S(TM)^{\perp} = S(TM^{\perp}) \oplus_{orth} S(TM^{\perp})^{\perp}.$$

Clearly $Rad(TM)$ is a vector subbundle of $S(TM^{\perp})^{\perp}$. Choose $L \in \Gamma(S(TM^{\perp}))$ as a unit vector field with $\widetilde{g}(L, L) = \epsilon = \pm 1$. For any null section ξ of $Rad(TM)$, there exists a uniquely defined null vector field $N \in \Gamma(S(TM^{\perp})^{\perp})$ satisfying

$$\widetilde{g}(\xi, N) = 1, \ \widetilde{g}(N, N) = \widetilde{g}(N, X) = \widetilde{g}(N, L) = 0, \ \forall X \in \Gamma(S(TM)).$$

Denote by $ltr(TM)$ the subbundle of $S(TM^{\perp})^{\perp}$ locally spanned by N. Then we show that $S(TM^{\perp})^{\perp} = Rad(TM) \oplus ltr(TM)$. Let $tr(TM) = S(TM^{\perp}) \oplus_{orth} ltr(TM)$. We call N, $ltr(TM)$ and $tr(TM)$ the *lightlike transversal vector field, lightlike transversal vector bundle* and *transversal vector bundle* of M with respect to the screen distribution $S(TM)$ respectively [3]. Then $T\widetilde{M}$ is decomposed as follow:

$$\begin{aligned} T\widetilde{M} &= TM \oplus tr(TM) = \{Rad(TM) \oplus tr(TM)\} \oplus_{orth} S(TM) \\ &= \{Rad(TM) \oplus ltr(TM)\} \oplus_{orth} S(TM) \oplus_{orth} S(TM^{\perp}). \end{aligned} \tag{2.2}$$

Let $\widetilde{\nabla}$ be the Levi-Civita connection of $\widetilde{M}$ and P the projection morphism of TM on $S(TM)$ with respect to the decomposition (2.1). Then the local Gauss and

Weingarten formulas of M and $S(TM)$ are given by

$$\widetilde{\nabla}_X Y = \nabla_X Y + B(X,Y)N + D(X,Y)L, \tag{2.3}$$

$$\widetilde{\nabla}_X N = -A_N X + \tau(X)N + \rho(X)L, \tag{2.4}$$

$$\widetilde{\nabla}_X L = -A_L X + \phi(X)N; \tag{2.5}$$

$$\nabla_X PY = \nabla^*_X PY + C(X,PY)\xi, \tag{2.6}$$

$$\nabla_X \xi = -A^*_\xi X - \tau(X)\xi, \quad \forall\, X,\, Y \in \Gamma(TM), \tag{2.7}$$

where ∇ and ∇^* are induced connections on TM and $S(TM)$ respectively, B and D are called the *local second fundamental forms* of M, C is called the *local second fundamental form* on $S(TM)$. A_N, A^*_ξ and A_L are linear operators on TM and τ, ρ and ϕ are 1-forms on TM. Since $\widetilde{\nabla}$ is torsion-free, the induced connection ∇ of M is also torsion-free and both B and D are symmetric. From the facts $B(X,Y) = \widetilde{g}(\widetilde{\nabla}_X Y,\, \xi)$ and $D(X,Y) = \epsilon\widetilde{g}(\widetilde{\nabla}_X Y,\, L)$, we know that B and D are independent of the choice of a screen distribution and

$$B(X,\xi) = 0, \quad D(X,\xi) = -\epsilon\phi(X). \tag{2.8}$$

The induced connection ∇ on M is not metric and satisfies

$$(\nabla_X g)(Y,Z) = B(X,Y)\,\eta(Z) + B(X,Z)\,\eta(Y), \tag{2.9}$$

where η is a 1-form on TM such that $\eta(X) = \widetilde{g}(X,N)$. But the connection ∇^* on M^* is metric. The above three local second fundamental forms of M and M^* are related to their shape operators by

$$B(X,Y) = g(A^*_\xi X, Y), \qquad \widetilde{g}(A^*_\xi X, N) = 0, \tag{2.10}$$

$$C(X,PY) = g(A_N X, PY), \qquad \widetilde{g}(A_N X, N) = 0, \tag{2.11}$$

$$\epsilon D(X,Y) = g(A_L X, Y) - \phi(X)\eta(Y), \quad \widetilde{g}(A_L X, N) = \epsilon\rho(X). \tag{2.12}$$

By (2.10) and (2.11), we show that A^*_ξ and A_N are $\Gamma(S(TM))$-valued shape operators related to B and C respectively and A^*_ξ is self-adjoint on TM and

$$A^*_\xi \xi = 0. \tag{2.13}$$

Denote by $\widetilde{R}$, R and R^* the curvature tensors of the Levi-Civita connection $\widetilde{\nabla}$ on $\widetilde{M}$, the induced connection ∇ on M and the induced connection ∇^* on $S(TM)$ respectively. Using the Gauss-Weingarten equations for M and $S(TM)$, for any

$X, Y, Z \in \Gamma(TM)$, we obtain the following Codazzi equations for M and $S(TM)$:

$$\widetilde{g}(\widetilde{R}(X,Y)Z,\,\xi) = (\nabla_X B)(Y,Z) - (\nabla_Y B)(X,Z) + B(Y,Z)\tau(X) - B(X,Z)\tau(Y) + D(Y,Z)\phi(X) - D(X,Z)\phi(Y), \tag{2.14}$$

$$\widetilde{g}(\widetilde{R}(X,Y)Z,\,N) = \widetilde{g}(R(X,Y)Z,\,N) + \epsilon\{D(X,Z)\rho(Y) - D(Y,Z)\rho(X)\}, \tag{2.15}$$

$$\widetilde{g}(\widetilde{R}(X,Y)\xi,\,N) = g(A^*_\xi X,\,A_{N} Y) - g(A^*_\xi Y,\,A_{N} X) - 2d\tau(X,Y) + \rho(X)\phi(Y) - \rho(Y)\phi(X), \tag{2.16}$$

$$\widetilde{g}(R(X,Y)PZ,\,N) = (\nabla_X C)(Y,PZ) - (\nabla_Y C)(X,PZ) + C(X,PZ)\tau(Y) - C(Y,PZ)\tau(X). \tag{2.17}$$

The *Ricci curvature tensor*, denoted by $\widetilde{Ric}$, of $\widetilde{M}$ is defined by

$$\widetilde{Ric}(X,Y) = trace\{Z \to \widetilde{R}(Z,X)Y\},$$

for any $X, Y \in \Gamma(T\widetilde{M})$. Let $\dim \widetilde{M} = m+3$. Locally, $\widetilde{Ric}$ is given by

$$\widetilde{Ric}(X,Y) = \sum_{i=1}^{m+3} \epsilon_i\, \widetilde{g}(\widetilde{R}(E_i,X)Y,\,E_i), \tag{2.18}$$

where $\{E_1, \ldots, E_{m+3}\}$ is an orthonormal frame field of $T\widetilde{M}$ and $\epsilon_i\,(=\pm 1)$ denotes the causal character of respective vector field E_i. Consider a quasi-orthonormal frame field $\{\xi; W_a\}$ on M such that $Rad(TM) = Span\{\xi\}$ and $S(TM) = Span\{W_a\}$, and let $E = \{\xi, W_a, N, L\}$ be the corresponding frame field on $\widetilde{M}$. Using this frame field, for all $X, Y \in \Gamma(TM)$, the equation (2.18) reduce to

$$\widetilde{Ric}(X,Y) = \sum_{a=1}^{m} \epsilon_a\, \widetilde{g}(\widetilde{R}(W_a,X)Y,\,W_a) + \widetilde{g}(\widetilde{R}(\xi,X)Y,\,N) + \epsilon\, \widetilde{g}(\widetilde{R}(L,X)Y,\,L) + \widetilde{g}(\widetilde{R}(N,X)Y,\,\xi). \tag{2.19}$$

Definition. A vector field X on a semi-Riemannian manifold $(\bar{M}, \bar{g})$ is said to be a *conformal Killing vector field* [5, 6] if $\bar{\mathcal{L}}_X \bar{g} = -2\delta\, \bar{g}$ for any non-vanishing smooth function δ, where $\bar{\mathcal{L}}_X$ denotes the Lie derivative with respect to X, that is,

$$(\bar{\mathcal{L}}_X \bar{g})(Y,Z) = X(\bar{g}(Y,Z)) - \bar{g}([X,Y],Z) - \bar{g}(Y,[X,Z]),\ \forall\, Y, Z \in \Gamma(T\bar{M}).$$

In particular, if $\delta = 0$, then X is called a *Killing vector field* [5]. A distribution $\mathcal{G}$ on $\bar{M}$ is called a *conformal Killing* (resp. *Killing*) *distribution* on $\bar{M}$ if each vector field belonging to $\mathcal{G}$ is a conformal Killing (resp. Killing) vector field on $\bar{M}$.

Theorem 2.1 ([5, 6]). *Let M be a half lightlike submanifold of a semi-Riemannian manifold $(\widetilde{M}, \widetilde{g})$. Then $S(TM^{\perp})$ is a conformal Killing distribution if and only if there exists a smooth function δ such that*

$$D(X,Y) = \epsilon\delta g(X,Y), \quad \forall\, X,\, Y \in \Gamma(TM). \tag{2.20}$$

Proof. By using (2.5) and (2.12), for any X, $Y \in \Gamma(TM)$, we have

$$(\widetilde{\mathcal{L}}_L \widetilde{g})(X,Y) = \widetilde{g}(\widetilde{\nabla}_X L, Y) + \widetilde{g}(X, \widetilde{\nabla}_Y L),$$
$$\widetilde{g}(\widetilde{\nabla}_X L, Y) = -g(A_L X, Y) + \phi(X)\eta(Y) = -\epsilon D(X,Y).$$

From $(\widetilde{\mathcal{L}}_L \widetilde{g})(X,Y) = -\,2\epsilon D(X,Y)$ we deduce our assertion. □

3. Main Theorem

Let M be a half lightlike submanifold of a semi-Riemannian manifold $\widetilde{M}$ of quasi-constant curvature. Assume that *the curvature vector field ζ of $\widetilde{M}$ is a unit spacelike vector field of M*. If ζ belongs to $Rad(TM)$, then $\zeta = e\xi$, where $e = \theta(N) \neq 0$. From this fact, we have $1 = \widetilde{g}(\zeta,\zeta) = e^2 g(\xi,\xi) = 0$. It is a contradiction. This enables one to choose a screen distribution $S(TM)$ which contains ζ. This implies that *if ζ is tangent to M, then it belongs to $S(TM)$* which we assume in this paper.

Definition. A half lightlike submanifold M of a semi-Riemannian manifold $\widetilde{M}$ is *screen conformal* [4, 5, 6] if the shape operators A_N and A^*_ξ of M and $S(TM)$ respectively are related by $A_N = \varphi A^*_\xi$, or equivalently, the second fundamental forms B and C of M and $S(TM)$ respectively satisfy

$$C(X, PY) = \varphi B(X,Y), \tag{3.1}$$

where φ is a non-vanishing smooth function on a coordinate neighborhood $\mathcal{U}$ in M. If φ is a non-zero constant, then we say that M is *screen homothetic.*

Theorem 3.1. *Let M be a screen conformal half lightlike submanifold of a semi-Riemannian manifold $(\widetilde{M}, \widetilde{g})$ of quasi-constant curvature. If ζ is tangent to M and $\phi = 0$, then the 1-form τ is closed, i.e., $d\tau = 0$, on TM.*

Proof. Replacing W by N to (1.1) and using the fact $\theta(N) = 0$, we have

$$\begin{aligned} \widetilde{g}(\widetilde{R}(X,Y)Z,\, N) \;&=\; \alpha\{\eta(X)g(Y,Z) - \eta(Y)g(X,Z)\} \\ &\quad+\; \beta\{\theta(Y)\eta(X) - \theta(X)\eta(Y)\}\theta(Z). \end{aligned} \tag{3.2}$$

Replacing Z by ξ to (3.2) and using $\theta(\xi) = 0$, we have $\widetilde{g}(\widetilde{R}(X,Y)\xi,\, N) = 0$.

Comparing this result with (2.16) and using the facts $A_N = \varphi A^*_\xi$ and $\phi = 0$, we show that the 1-form τ is closed, i.e., $d\tau = 0$, on TM. □

Note 1. In case $d\tau = 0$, by the cohomology theory there exist a smooth function l such that $\tau = dl$. Thus we get $\tau(X) = X(l)$. If we take $\tilde{\xi} = \gamma\xi$, then we have $\tau(X) = \tilde{\tau}(X) + X(\ln\gamma)$. Setting $\gamma = \exp(l)$ in this equation, we get $\tilde{\tau}(X) = 0$. We call the pair $\{\xi, N\}$ such that the corresponding 1-form τ vanishes the *canonical null pair* of M. Although $S(TM)$ is not unique but it is canonically isomorphic to the factor vector bundle $S(TM)^\sharp = TM/Rad(TM)$ due to Kupeli [9]. Thus all $S(TM)$ are mutually isomorphic. In the sequel, we deal with only half lightlike submanifolds M equipped with the canonical null pair.

Theorem 3.2. *Let M be a screen homothetic half lightlike submanifold of a semi-Riemannian manifold $\widetilde{M}$ of quasi-constant curvature such that the curvature vector field ζ of $\widetilde{M}$ is tangent to M.*

(1) *If $S(TM^\perp)$ is Killing, then the functions α and β, given by* (1.1)*, vanish identically, and $\widetilde{M}$ is a flat manifold.*
(2) *If $S(TM^\perp)$ is conformal Killing, then the functions β, given by* (1.1)*, vanishes identically, and $\widetilde{M}$ is a space of constant curvature α.*

Proof. Using (1.1), (2.18) and the facts $\theta(\xi) = \theta(N) = \theta(L) = 0$, we have

$$\widetilde{Ric}(X,Y) = \{(m+2)\alpha + \beta\}g(X,Y) + (m+1)\beta\,\theta(X)\theta(Y), \tag{3.3}$$

$$\widetilde{g}(\widetilde{R}(\xi,Y)X,\, N) = \alpha g(X,Y) + \beta\,\theta(X)\theta(Y), \tag{3.4}$$

$$\epsilon\widetilde{g}(\widetilde{R}(L,Y)X,\, L) = \alpha g(X,Y) + \beta\,\theta(X)\theta(Y), \quad \forall\, X,\, Y \in \Gamma(TM). \tag{3.5}$$

As $S(TM^\perp)$ is conformal Killing, from (2.8), (2.12) and (2.20) we have

$$D(X,Y) = \epsilon\delta g(X,Y), \quad \phi = 0, \quad A_L X = \delta\, PX + \epsilon\rho(X)\xi. \tag{3.6}$$

As $d\tau = 0$ by Theorem 3.1, we can take a canonical null pair such that $\tau = 0$ by Note 1. Replacing W by ξ to (1.1) and using (2.14) and the fact $\theta(\xi) = 0$, we have

$$(\nabla_X B)(Y,Z) - (\nabla_Y B)(X,Z) = 0, \quad \forall\, X,\, Y,\, Z \in \Gamma(TM). \tag{3.7}$$

As M is screen homothetic, substituting (3.1) into (2.17) and using (3.7), we get $\widetilde{g}(R(X,Y)PZ, N) = 0$. From this, (2.15) and the fact $\widetilde{g}(\widetilde{R}(X,Y)\xi, N) = 0$, we have

$$\widetilde{g}(\widetilde{R}(X,Y)Z, N) = \delta\{g(X,Z)\rho(Y) - g(Y,Z)\rho(X)\}.$$

Replacing X by ξ and Z by X to this and comparing with (3.4), we have

$$\beta\theta(X)\theta(Y) \;=\; -\,\{\alpha + \delta\rho(\xi)\}g(X,Y), \quad \forall\, X,\, Y \in \Gamma(TM). \tag{3.8}$$

Taking $X = Y = \zeta$ to (3.8), we get $\beta = -\{\alpha + \delta\rho(\xi)\}$. Substituting (3.8) into (3.3) and using the fact $\beta = -\{\alpha + \delta\rho(\xi)\}$, we obtain

$$\widetilde{Ric}(X,Y) = -(m+2)\delta\rho(\xi)g(X,Y) \quad \forall\, X,\, Y \in \Gamma(TM). \tag{3.9}$$

Substituting (3.8) into (1.1) and using the fact $\beta = -\{\alpha + \delta\rho(\xi)\}$, we have

$$\bar{g}(\widetilde{R}(X,Y)Z,\, W) = (\alpha + 2\delta\rho(\xi))\{g(X,Z)g(Y,W) - g(Y,Z)g(X,W)\}, \tag{3.10}$$

for all $X, Y, Z, W \in \Gamma(TM)$. Substituting (3.4), (3.5), (3.10) into (2.19), we have

$$\widetilde{Ric}(X,Y) = -\{(m-1)\alpha + (2m+1)\delta\rho(\xi)\}g(X,Y). \tag{3.11}$$

Comparing (3.9) and (3.11), we have $\alpha + \delta\rho(\xi) = 0$ as $m > 1$. Thus we have $\beta = 0$.

Case (1). If $S(TM^{\perp})$ is Killing distribution, then $\delta = 0$. In this case, we get $\alpha = \beta$. As $\beta = 0$, we obtain $\alpha = \beta = 0$. Therefore $\widetilde{M}$ is a flat manifold.

Case (2). If $S(TM^{\perp})$ is conformal Killing distribution, then $\delta \neq 0$. In this case, we get $\alpha = -\delta\rho(\xi)$ and $\beta = 0$. Therefore $\widetilde{M}$ is a space of constant curvature α. □

By Theorem 1.1, we have the following characterization theorem:

Theorem 3.3. *Let M be a screen homothetic half lightlike submanifold of a Lorentz manifold $\widetilde{M}^{m+3}(m > 2)$ of quasi-constant curvature. If the curvature vector field ζ of $\widetilde{M}$ is tangent to M, the co-screen distribution $S(TM^{\perp})$ is conformal Killing of conformal factor δ and M is Einstein, i.e., $Ric = \kappa g$, then M is locally a product manifold $\mathcal{C} \times M_1 \times M_2$, where $\mathcal{C}$ is a null curve tangent to the radical distribution, and M_1 and M_2 are totally umbilical leaves of some distributions of M:*

1. *If $\kappa \neq (m-1)(\alpha + \delta^2)$, then either M_1 or M_2 is an m-dimensional Einstein Riemannian space form which is isometric to a sphere $(\kappa > 0)$ or a hyperbolic space $(\kappa < 0)$ and the other is a point on M.*
2. *If $\kappa = (m-1)(\alpha + \delta^2)$, then M_1 is an $(m-1)$ or m-dimensional Einstein Riemannian space form which is isometric to a sphere $(\kappa > 0)$ or a hyperbolic space $(\kappa < 0)$ or a Euclidean space $(\kappa = 0)$ and M_2 is a spacelike curve or a point on M.*

Corollary 1. *Let M be a screen homothetic Einstein half lightlike submanifold of a Lorentzian manifold $\widetilde{M}$, $m > 2$, of quasi-constant curvature equipped with a Killing co-screen distribution. Then $\widetilde{M}$ is a flat manifold, and M is a locally product manifold $\mathcal{C} \times M_1 \times M_2$, where $\mathcal{C}$ is a null curve, and M_1 and M_2 are leaves of some distributions of M such that*

(1) *If* $\kappa \neq 0$, *then either* M_1 *or* M_2 *is an* m*-dimensional Einstein Riemannian space form which is isometric to a sphere* $(\kappa > 0)$ *or a hyperbolic space* $(\kappa < 0)$ *and the other is a point on* M.

(2) *If* $\kappa = 0$, M_1 *is an* $(m-1)$ *or an* m*-dimensional Euclidean space and* M_2 *is a spacelike curve or a point in* $\bar{M}$.

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