

A GENERALIZATION OF FUZZY SUBSEMIGROUPS IN SEMIGROUPS

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ABSTRACT. As a generalization of fuzzy subsemigroups, the notion of ε -generalized fuzzy subsemigroups is introduced, and several properties are investigated. A condition for an ε -generalized fuzzy subsemigroup to be a fuzzy subsemigroup is considered. Characterizations of ε -generalized fuzzy subsemigroups are established, and we show that the intersection of two ε -generalized fuzzy subsemigroups is also an ε -generalized fuzzy subsemigroup. A condition for an ε -generalized fuzzy subsemigroup to be ε -fuzzy idempotent is discussed. Using a given ε -generalized fuzzy subsemigroup, a new ε -generalized fuzzy subsemigroup is constructed. Finally, the fuzzy extension of an ε -generalized fuzzy subsemigroup is considered.

1. INTRODUCTION

The concept of a fuzzy set was first introduced by Zadeh [16] and this concept was adapted by Rosenfeld [14] to define fuzzy subgroups and fuzzy ideals. Based on this pioneering work, Kuroki [6, 7, 8, 9, 10] introduced fuzzy semigroups and various kinds of fuzzy ideals in semigroups and characterized certain semigroups using those fuzzy ideals. Since then the literature of various fuzzy algebraic concepts has been growing very rapidly. In the literature, the relationships between the fuzzy sets and semigroups have been considered by many authors (see [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 15]).

The aim of this paper is to consider more general form of fuzzy subsemigroups. We define the notion of ε -generalized fuzzy subsemigroups, and investigate several properties. We provide a condition for an ε -generalized fuzzy subsemigroup to be a fuzzy subsemigroup. We establish characterizations of ε -generalized fuzzy subsemigroups. We show that the intersection of two ε -generalized fuzzy subsemigroups is also an ε -generalized fuzzy subsemigroup. We consider a condition for an ε -generalized fuzzy

Received by the editors February 12, 2013. Accepted April 22, 2013.

2010 *Mathematics Subject Classification.* 20M12, 08A72.

Key words and phrases. ε -generalized fuzzy subsemigroup, ε -product, ε -Cartesian product, fuzzy α -translation, $(\varepsilon_1, \varepsilon_2)$ -fuzzy extension.

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subsemigroup to be ε -fuzzy idempotent. Using a given ε -generalized fuzzy subsemigroup, we establish a new ε -generalized fuzzy subsemigroup. Finally, we discuss fuzzy extension of an ε -generalized fuzzy subsemigroup.

2. PRELIMINARIES

Let S be a semigroup. Let A and B be subsets of S . Then the multiplication of A and B is defined as follows:

$$AB = \{ab \in S \mid a \in A \text{ and } b \in B\}.$$

Let S be a semigroup. By a *subsemigroup* of S we mean a nonempty subset A of S such that $A^2 \subseteq A$. A fuzzy subset μ of S is called a *fuzzy subsemigroup* of S if it satisfies:

$$(\forall x, y \in S)(\mu(xy) \geq \min\{\mu(x), \mu(y)\}).$$

Let μ and ν be fuzzy subsets of S . The *product* $\mu \circ \nu$ of μ and ν is defined by

$$\mu \circ \nu(x) = \begin{cases} \sup_{x=yz} \min\{\mu(y), \nu(z)\} & \text{if } \exists y, z \in S \text{ such that } x = yz, \\ 0 & \text{otherwise.} \end{cases}$$

We refer the reader to the book [12] for further information regarding fuzzy semigroups.

3. A GENERALIZATION OF FUZZY SUBSEMIGROUPS

In what follows, let $\varepsilon \in \mathbb{R}^+$ where \mathbb{R}^+ is the set of all positive real numbers unless otherwise specified.

Definition 3.1. A fuzzy subset μ of S is called an ε -generalized fuzzy subsemigroup of S if the following assertions is valid:

$$(3.1) \quad (\forall x, y \in S) (\mu(xy) \geq \min\{\mu(x), \mu(y), \varepsilon\}).$$

If $\varepsilon \geq 1$, the ε -generalized fuzzy subsemigroup of S is a fuzzy subsemigroup of S .

Example 3.2. Let S be a semigroup of four elements $\{a, b, c, d\}$ with the following multiplication table:

	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	b	a
d	a	a	b	b

Let μ be a fuzzy subset of S defined by

$$\mu : S \rightarrow [0, 1], x \mapsto \begin{cases} 0.8 & \text{if } x = a, \\ 0.5 & \text{if } x = b, \\ 0.7 & \text{if } x = c, \\ 0.3 & \text{if } x = d. \end{cases}$$

Note that μ is not a fuzzy subsemigroup of S . If $\varepsilon \leq 0.5$, then μ is an ε -generalized fuzzy subsemigroup of S . If $0.5 < \varepsilon \leq 0.7$, then μ is not an ε -generalized fuzzy subsemigroup of S since

$$\mu(cc) = \mu(b) = 0.5 < \varepsilon = \min\{\mu(c), \mu(c), \varepsilon\}.$$

Obviously, every fuzzy subsemigroup is an ε -generalized fuzzy subsemigroup for any ε , but the converse is not true in general. For example, Example 3.2 shows that there exists ε such that μ is an ε -generalized fuzzy subsemigroup of S which is not a fuzzy subsemigroup of S .

Example 3.3. Let S be a semigroup of four elements $\{e, a, b, c\}$ with the following multiplication table:

	e	a	b	c
e	e	a	b	c
a	a	e	b	c
b	b	c	b	c
c	c	b	b	c

Let μ be a fuzzy subset of S defined by

$$\mu : S \rightarrow [0, 1], x \mapsto \begin{cases} 0.9 & \text{if } x = e, \\ 0.7 & \text{if } x = a, \\ 0.6 & \text{if } x = b, \\ 0.6 & \text{if } x = c. \end{cases}$$

It is routine to verify that μ is an ε -generalized fuzzy subsemigroup of S for every ε .

Proposition 3.4. *Let μ be an ε -generalized fuzzy subsemigroup of S . If $\varepsilon \geq \mu(x)$ for all $x \in S$, then μ is a fuzzy subsemigroup of S .*

Proof. It is straightforward. □

Proposition 3.5. *Let ε_1 and ε_2 be any elements of \mathbb{R}^+ . If $\varepsilon_1 \leq \varepsilon_2$, then every ε_2 -generalized fuzzy subsemigroup is an ε_1 -generalized fuzzy subsemigroup.*

Proof. Straightforward. □

The following examples shows that there exist $\varepsilon_1, \varepsilon_2 \in \mathbb{R}^+$ with $\varepsilon_1 \leq \varepsilon_2$ such that

any ε_1 -generalized fuzzy subsemigroup may not be an ε_2 -generalized fuzzy subsemigroup.

Example 3.6. Let $S = \{e, a, b, c\}$ be a semigroup which is given in Example 3.3 and let μ be a fuzzy subset of S defined by

$$\mu : S \rightarrow [0, 1], x \mapsto \begin{cases} 0.8 & \text{if } x = e, \\ 0.5 & \text{if } x = a, \\ 0.3 & \text{if } x = b, \\ 0.6 & \text{if } x = c. \end{cases}$$

If we take $\varepsilon_1 \in [0, 0.3]$ and $\varepsilon_2 \in (0.3, 0.5]$, then μ is an ε_1 -generalized fuzzy subsemigroup of S . Since $\mu(ca) = \mu(b) = 0.3 < \varepsilon_2 = \{\mu(c), \mu(a), \varepsilon_2\}$, μ is not an ε_2 -generalized fuzzy subsemigroup of S .

Theorem 3.7. *Let μ be a fuzzy subset of S . Then μ is an ε -generalized fuzzy subsemigroup of S if and only if μ_t is a subsemigroup of S for all $t \in [0, 1]$ with $t \leq \varepsilon$.*

Proof. Assume that μ is an ε -generalized fuzzy subsemigroup of S . Let $x, y \in S$ be such that $x, y \in \mu_t$ for all $t \in [0, 1]$ with $t \leq \varepsilon$. Then

$$\mu(xy) \geq \min\{\mu(x), \mu(y), \varepsilon\} \geq \min\{t, \varepsilon\} = t,$$

and so $xy \in \mu_t$. Therefore μ_t is a subsemigroup of S for all $t \in [0, 1]$ with $t \leq \varepsilon$.

Conversely, suppose that μ_t is a subsemigroup of S for all $t \in [0, 1]$ with $t \leq \varepsilon$. Assume that μ is not an ε -generalized fuzzy subsemigroup of S . Then there exist $a, b \in S$ such that

$$\mu(ab) < \min\{\mu(a), \mu(b), \varepsilon\}.$$

Putting k equal to $\min\{\mu(a), \mu(b), \varepsilon\}$, we have $\mu(ab) < k$, $a, b \in \mu_k$ and $k \in [0, 1]$ with $k \leq \varepsilon$. It follows that $ab \in \mu_k$ which is a contradiction. Hence

$$\mu(xy) \geq \min\{\mu(x), \mu(y), \varepsilon\}$$

for all $x, y \in S$. Thus μ is an ε -generalized fuzzy subsemigroup of S . □

Let μ and ν be fuzzy subsets of S . The ε -product $\mu \circ_\varepsilon \nu$ of μ and ν is defined by

$$\mu \circ_\varepsilon \nu(x) = \begin{cases} \sup_{x=yz} \min\{\mu(y), \nu(z), \varepsilon\} & \text{if } \exists y, z \in S \text{ such that } x = yz, \\ 0 & \text{otherwise.} \end{cases}$$

Theorem 3.8. *For a fuzzy subset μ of S , μ is an ε -generalized fuzzy subsemigroup of S if and only if $\mu \circ_\varepsilon \mu \subseteq \mu$.*

Proof. Assume that μ is an ε -generalized fuzzy subsemigroup of S . Let $x \in S$. If $x \neq yz$ for all $x, y \in S$, then clearly $\mu \circ_\varepsilon \mu \subseteq \mu$. If $x = ab$ for some $a, b \in S$, then

$$\mu \circ_\varepsilon \mu(x) = \sup_{x=ab} \min\{\mu(a), \mu(b), \varepsilon\} \leq \sup_{x=ab} \mu(ab) = \mu(x).$$

Therefore $\mu \circ_\varepsilon \mu \subseteq \mu$.

Conversely, suppose that $\mu \circ_\varepsilon \mu \subseteq \mu$. Let $x, y \in S$. Set $a = xy$. Then we have

$$\mu(xy) = \mu(a) \geq (\mu \circ_\varepsilon \mu)(a) = \sup_{a=bc} \min\{\mu(b), \mu(c), \varepsilon\} \geq \min\{\mu(x), \mu(y), \varepsilon\},$$

and so μ is an ε -generalized fuzzy subsemigroup of S . \square

If we take $\varepsilon = 1$ in Theorem 3.8, then we have the following corollary.

Corollary 3.9 ([12]). *For a fuzzy subset μ of S , μ is a fuzzy subsemigroup of S if and only if $\mu \circ \mu \subseteq \mu$.*

Theorem 3.10. *Let S be a semigroup with identity e and let μ be a fuzzy subset of S such that*

$$(\forall x \in S) (\mu(x) \leq \mu(e) \leq \varepsilon).$$

If μ is an ε -generalized fuzzy subsemigroup of S , then it is ε -fuzzy idempotent, that is, $\mu \circ_\varepsilon \mu = \mu$.

Proof. Let $x \in S$. Then

$$\begin{aligned} (\mu \circ_\varepsilon \mu)(x) &= \sup_{x=yz} \min\{\mu(y), \mu(z), \varepsilon\} \\ &\geq \min\{\mu(e), \mu(x), \varepsilon\} = \mu(x), \end{aligned}$$

and so $\mu \circ_\varepsilon \mu \subseteq \mu$. It follows from Theorem 3.8 that $\mu \circ_\varepsilon \mu = \mu$, that is, μ is ε -fuzzy idempotent. \square

Corollary 3.11. *Let S be a semigroup with identity e . Then every fuzzy subsemigroup μ of S is fuzzy idempotent, that is $\mu \circ \mu = \mu$.*

Theorem 3.12. *For any $\varepsilon_1, \varepsilon_2 \in \mathbb{R}^+$, let μ and ν be an ε_1 -generalized fuzzy subsemigroup and an ε_2 -generalized fuzzy subsemigroup of S respectively. Then $\mu \cap \nu$ is an ε -generalized fuzzy subsemigroup of S where $\varepsilon = \min\{\varepsilon_1, \varepsilon_2\}$.*

Proof. Let $\varepsilon = \min\{\varepsilon_1, \varepsilon_2\}$ and let x and y be any elements of S . Then

$$\begin{aligned} (\mu \cap \nu)(xy) &= \min\{\mu(xy), \nu(xy)\} \\ &\geq \min\{\min\{\mu(x), \mu(y), \varepsilon_1\}, \min\{\nu(x), \nu(y), \varepsilon_2\}\} \\ &\geq \min\{\min\{\mu(x), \mu(y), \varepsilon\}, \min\{\nu(x), \nu(y), \varepsilon\}\} \end{aligned}$$

$$\begin{aligned}
&= \min \{ \min \{ \mu(x), \nu(x) \}, \min \{ \mu(y), \nu(y) \}, \varepsilon \} \\
&= \min \{ (\mu \cap \nu)(x), (\mu \cap \nu)(y), \varepsilon \}.
\end{aligned}$$

Therefore $\mu \cap \nu$ is an ε -generalized fuzzy subsemigroup of S . \square

Corollary 3.13. *Let μ and ν be two ε -generalized fuzzy subsemigroups of S . Then $\mu \cap \nu$ is an ε -generalized fuzzy subsemigroup of S .*

The following example shows that the union of two ε -generalized fuzzy subsemigroups is not an ε -generalized fuzzy subsemigroup.

Example 3.14. Consider the semigroup $S = \{e, a, b, c\}$ which is given in Example 3.3. Let μ and ν be fuzzy subsets of S defined by

$$\mu : S \rightarrow [0, 1], \quad x \mapsto \begin{cases} 0.6 & \text{if } x \in \{e, a\}, \\ 0.5 & \text{if } x \in \{b, c\}, \end{cases}$$

and

$$\nu : S \rightarrow [0, 1], \quad x \mapsto \begin{cases} 0.7 & \text{if } x \in \{e, b\}, \\ 0.5 & \text{if } x \in \{a, c\}, \end{cases}$$

respectively. Routine calculations, we know that μ and ν are ε -generalized fuzzy subsemigroups of S . The union of μ and ν is given as follows:

$$\mu \cup \nu : S \rightarrow [0, 1], \quad x \mapsto \begin{cases} 0.7 & \text{if } x \in \{e, b\}, \\ 0.6 & \text{if } x = a, \\ 0.5 & \text{if } x = c. \end{cases}$$

If we put $\varepsilon = 0.6$ and take $t \in (0.5, 0.6]$, then $(\mu \cup \nu)_t = \{e, a, b\}$ is not a subalgebra of S . It follows from Theorem 3.7 that $\mu \cup \nu$ is not an ε -generalized fuzzy subsemigroup of S .

Theorem 3.15. *Let μ be an ε -generalized fuzzy subsemigroup of S . If $k = \sup_{x \in S} \mu(x)$, then the set*

$$S_\mu := \{x \in S \mid \mu(x) \geq \min\{k, \varepsilon\}\}$$

is a subsemigroup of S .

Proof. Let $x, y \in S_\mu$. Then $\mu(x) \geq \min\{k, \varepsilon\}$ and $\mu(y) \geq \min\{k, \varepsilon\}$. It follows from (3.1) that

$$\mu(xy) \geq \min\{\mu(x), \mu(y), \varepsilon\} \geq \min\{k, \varepsilon\}.$$

Hence $xy \in S_\mu$, and thus S_μ is a subsemigroup of S . \square

Let S and T be semigroups. For a map $f : S \rightarrow T$ and a fuzzy subset μ of T , define a fuzzy subset μ_f^ε of S by

$$\mu_f^\varepsilon : S \rightarrow [0, 1], \quad x \mapsto \min\{\mu(f(x)), \varepsilon\}.$$

Theorem 3.16. *Let $f : S \rightarrow T$ be a homomorphism of semigroups. If μ is an ε -generalized fuzzy subsemigroup of T , then μ_f^ε is an ε -generalized fuzzy subsemigroup of S .*

Proof. Let $x, y \in S$. Then

$$\begin{aligned} \mu_f^\varepsilon(xy) &= \min\{\mu(f(xy)), \varepsilon\} = \min\{\mu(f(x)f(y)), \varepsilon\} \\ &\geq \min\{\min\{\mu(f(x)), \mu(f(y)), \varepsilon\}, \varepsilon\} \\ &= \min\{\min\{\mu(f(x)), \varepsilon\}, \min\{\mu(f(y)), \varepsilon\}, \varepsilon\} \\ &= \min\{\mu_f^\varepsilon(x), \mu_f^\varepsilon(y), \varepsilon\}. \end{aligned}$$

Hence μ_f^ε is an ε -generalized fuzzy subsemigroup of S . □

For two ε -generalized fuzzy subsemigroups μ and ν of S , we define the ε -Cartesian product of μ and ν as follows:

$$\mu \times_\varepsilon \nu : S \times S \rightarrow [0, 1], \quad (x, y) \mapsto \min\{\mu(x), \nu(y), \varepsilon\}.$$

Theorem 3.17. *If μ and ν are ε -generalized fuzzy subsemigroups of S , then the ε -Cartesian product of μ and ν is an ε -generalized fuzzy subsemigroup of $S \times S$.*

Proof. Let $(x, y), (a, b) \in S \times S$. Then

$$\begin{aligned} (\mu \times_\varepsilon \nu)((x, y)(a, b)) &= (\mu \times_\varepsilon \nu)((xa, yb)) = \min\{\mu(xa), \nu(yb), \varepsilon\} \\ &\geq \min\{\min\{\mu(x), \mu(a), \varepsilon\}, \min\{\nu(y), \nu(b), \varepsilon\}, \varepsilon\} \\ &= \min\{\min\{\mu(x), \nu(y), \varepsilon\}, \min\{\mu(a), \nu(b), \varepsilon\}, \varepsilon\} \\ &= \min\{(\mu \times_\varepsilon \nu)(x, y), (\mu \times_\varepsilon \nu)(a, b), \varepsilon\}, \end{aligned}$$

and so $\mu \times_\varepsilon \nu$ is an ε -generalized fuzzy subsemigroup of $S \times S$. □

For a fuzzy subset μ of S , we define a new fuzzy subset μ^* of S as follows:

$$\mu^* : S \rightarrow [0, 1], \quad x \mapsto \mu(x) + 1 - k$$

where $k = \sup_{x \in S} \mu(x)$.

Theorem 3.18. *If μ is an ε -generalized fuzzy subsemigroup of S , then so is μ^* .*

Proof. Let $x, y \in S$. Note that $\varepsilon + 1 - k \geq \varepsilon$. Hence

$$\begin{aligned}\mu^*(xy) &= \mu(xy) + 1 - k \geq \min\{\mu(x), \mu(y), \varepsilon\} + 1 - k \\ &= \min\{\mu(x) + 1 - k, \mu(y) + 1 - k, \varepsilon + 1 - k\} \\ &= \min\{\mu^*(x), \mu^*(y), \varepsilon\},\end{aligned}$$

and so μ^* is an ε -generalized fuzzy subsemigroup of S . \square

For any fuzzy subset μ of S , we denote $\top := 1 - \sup\{\mu(x) \mid x \in S\}$. For any $\alpha \in [0, \top]$, a mapping $\mu_\alpha^t : S \rightarrow [0, 1]$ is called a *fuzzy α -translation* of μ if it satisfies:

$$(\forall x \in S) (\mu_\alpha^t(x) = \mu(x) + \alpha).$$

Theorem 3.19. *Let μ be an ε -generalized fuzzy subsemigroup of S and let $\alpha \in [0, \top]$ with $\alpha + \varepsilon \leq 1$. Then the fuzzy α -translation μ_α^t of μ is an $\alpha + \varepsilon$ -generalized fuzzy subsemigroup of S .*

Proof. Let $x, y \in S$. Then

$$\begin{aligned}\mu_\alpha^t(xy) &= \mu(xy) + \alpha \geq \min\{\mu(x), \mu(y), \varepsilon\} + \alpha \\ &= \min\{\mu(x) + \alpha, \mu(y) + \alpha, \alpha + \varepsilon\} \\ &= \min\{\mu_\alpha^t(x), \mu_\alpha^t(y), \alpha + \varepsilon\}.\end{aligned}$$

Therefore μ_α^t is an $\alpha + \varepsilon$ -generalized fuzzy subsemigroup of S . \square

Theorem 3.20. *Let μ be a fuzzy subset of S such that the fuzzy α -translation μ_α^t of μ is an $\alpha + \varepsilon$ -generalized fuzzy subsemigroup of S for some $\alpha \in [0, \top]$ with $\alpha + \varepsilon \leq 1$. Then μ is an ε -generalized fuzzy subsemigroup of S .*

Proof. Assume that μ_α^t is an $\alpha + \varepsilon$ -generalized fuzzy subsemigroup of S for some $\alpha \in [0, \top]$ with $\alpha + \varepsilon \leq 1$. Let $x, y \in S$. Then

$$\begin{aligned}\mu(xy) + \alpha &= \mu_\alpha^t(xy) \geq \min\{\mu_\alpha^t(x), \mu_\alpha^t(y), \alpha + \varepsilon\} \\ &= \min\{\mu(x) + \alpha, \mu(y) + \alpha, \alpha + \varepsilon\} \\ &= \min\{\mu(x), \mu(y), \varepsilon\} + \alpha,\end{aligned}$$

and so $\mu(xy) \geq \min\{\mu(x), \mu(y), \varepsilon\}$. Consequently, μ is an ε -generalized fuzzy subsemigroup of S . \square

Let μ_1 and μ_2 be fuzzy subsets of S and $\varepsilon_1, \varepsilon_2 \in \mathbb{R}^+$. We say that μ_2 is an $(\varepsilon_1, \varepsilon_2)$ -fuzzy extension of μ_1 if

$$(1) (\forall x \in S) (\mu_1(x) \leq \mu_2(x)),$$

- (2) If μ_1 is an ε_1 -generalized fuzzy subsemigroup of S , then μ_2 is an ε_2 -generalized fuzzy subsemigroup of S .

Let S be the semigroup and μ be the fuzzy subset of S as in Example 3.6. Define a fuzzy subset ν of S as follows:

$$\nu : S \rightarrow [0, 1], x \mapsto \begin{cases} 0.9 & \text{if } x = e, \\ 0.7 & \text{if } x = a, \\ 0.4 & \text{if } x = b, \\ 0.8 & \text{if } x = c. \end{cases}$$

Then $\mu(x) \leq \nu(x)$ for all $x \in S$ and note that μ is an ε_1 -generalized fuzzy subsemigroup of S with $\varepsilon_1 = 0.2$ (see Example 3.6). If we take $\varepsilon_2 \in (0.2, 0.3]$, then ν is an ε_2 -generalized fuzzy subsemigroup of S . Hence ν is an $(\varepsilon_1, \varepsilon_2)$ -fuzzy extension of μ . But, if we take $\varepsilon_3 \in (0.4, 0.7]$, then ν is not an ε_2 -generalized fuzzy subsemigroup of S since $\nu_t = \{e, a, c\}$ is not a subsemigroup of S for $t \in (0.4, \varepsilon_3]$. Therefore ν is not an $(\varepsilon_1, \varepsilon_3)$ -fuzzy extension of μ .

By means of the definition of fuzzy α -translation, we know that $\mu_\alpha^t(x) \geq \mu(x)$ for all $x \in S$. Hence the following theorem is implied by Theorem 3.19.

Theorem 3.21. *Let μ be an ε -generalized fuzzy subsemigroup of S and $\alpha \in [0, \top]$ with $\alpha + \varepsilon \leq 1$. Then the fuzzy α -translation μ_α^t of μ is an $(\varepsilon, \alpha + \varepsilon)$ -fuzzy extension of μ .*

The following example illustrates Theorem 3.21.

Example 3.22. Let S be a semigroup of four elements $\{e, a, b, c\}$ with the following multiplication table:

	e	a	b	c
e	e	a	b	c
a	a	e	b	c
b	b	c	b	c
c	c	b	b	c

Let μ be a fuzzy subset of S defined by

$$\mu : S \rightarrow [0, 1], x \mapsto \begin{cases} 0.85 & \text{if } x = e, \\ 0.8 & \text{if } x = a, \\ 0.6 & \text{if } x = b, \\ 0.55 & \text{if } x = c. \end{cases}$$

Then μ is an ε -generalized fuzzy subsemigroup of S for $\varepsilon = 0.4$. For any $\alpha = 0.02 \in [0, 0.15]$, the fuzzy α -translation μ_α^t of μ is given as follows:

$$\mu_\alpha^t : S \rightarrow [0, 1], x \mapsto \begin{cases} 0.87 & \text{if } x = e, \\ 0.82 & \text{if } x = a, \\ 0.62 & \text{if } x = b, \\ 0.57 & \text{if } x = c, \end{cases}$$

which is an $(\varepsilon, \alpha + \varepsilon)$ -fuzzy extension of μ .

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