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ON THE GAUSS MAP OF GENERALIZED SLANT CYLINDRICAL SURFACES

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Abstract. In this article, we study the Gauss map of generalized slant cylindrical surfaces (GSCS's) in the 3-dimensional Euclidean space $\mathbb{E}^3$. Surfaces of revolution, cylindrical surfaces and tubes along a plane curve are special cases of GSCS's. Our main results state that the only GSCS's with Gauss map G satisfying $\Delta G = AG$ for some 3×3 matrix A are the planes, the spheres and the circular cylinders.

1. Introduction and Preliminaries

The notion of finite type submanifolds in Euclidean or pseudo-Euclidean space, introduced by B.-Y. Chen during the late 1970's, has become a useful tool for investigating and characterizing many important submanifolds (cf. [3, 4]). In [2, 6] the notion of finite type was extended to differential maps, in particular, to Gauss map of submanifolds.

Let M be a surface of the Euclidean 3-space $\mathbb{E}^3$. The map $G : M \to S^2 \subset \mathbb{E}^3$ which sends each point of M to the unit normal vector to M at the point is called the *Gauss map* of the surface M, where S^2 is the unit sphere in $\mathbb{E}^3$ centered at the origin.

For the matrix $g = (g_{ij})$ consisting of the components of the metric on M, we denote by $g^{-1} = (g^{ij})$ (resp. $\mathcal{G}$) the inverse matrix (resp. the determinant) of the matrix (g_{ij}). The Laplacian Δ on M is, in turn, given by

$$\Delta = -\frac{1}{\sqrt{\mathcal{G}}} \sum_{i,j} \frac{\partial}{\partial x^i}\Big(\sqrt{\mathcal{G}}\; g^{ij} \frac{\partial}{\partial x^j}\Big). \tag{1.1}$$

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If a submanifold M of Euclidean or pseudo-Euclidean space has 1-type Gauss map G, then G satisfies

$$\Delta G = \lambda(G + C) \tag{1.2}$$

for some $\lambda \in \mathbb{R}$ and some constant vector C, where Δ is the Laplace operator corresponding to the induced metric on M. Generalizing (1.1), many authors studied various surfaces with Gauss map G satisfying

$$\Delta G = f(G + C) \tag{1.3}$$

for some constant vector C and some smooth function f ([5, 8, 9, 10, 11, 13]). Gauss map of a surface satisfying (1.2) is called a *pointwise 1-type Gauss map.*

On the other hand, Dillen et al. studied surfaces of revolution with Gauss map satisfying

$$\Delta G = AG \tag{1.4}$$

for some 3×3 matrix A, which was inspired by a theorem of Ruh and Vilms on surfaces of constant mean curvature ([14]). As a result, they proved ([7]).

Proposition 1. *Among the surfaces of revolution in $\mathbb{E}^3$, the only ones whose Gauss map satisfies (1.4) are the planes, the spheres and the circular cylinders.*

Baikoussis and Blair also studied ruled surfaces and proved ([1]).

Proposition 2. *Among the ruled surfaces in $\mathbb{E}^3$, the only ones whose Gauss map satisfies (1.4) are the planes and the circular cylinders.*

In [12], the first author with Y.H. Kim introduced the family of *generalized slant cylinders* (GSCS's). Surfaces of revolution, cylindrical surfaces and tubes along a plane curve are special cases of GSCS's. See Section 2 for the definition and properties of GSCS's.

Here, we give examples of GSCS's with Gauss map satisfying (1.4).

Examples. (1) Plane: $z = 0$. In this case, $G = (0, 0, 1)$ so $\Delta G = 0$ and the plane satisfies (1.4) with

$$A = \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ * & * & 0 \end{pmatrix}.$$

(2) Cylinder: $(x-a)^2 + (y-b)^2 = r^2$. In this case, we have $G = \frac{1}{r}(x-a, y-b, 0)$

so the cylinder satisfies (1.4) with

$$A = \begin{pmatrix} \frac{1}{r^2} & 0 & * \\ 0 & \frac{1}{r^2} & * \\ 0 & 0 & * \end{pmatrix}.$$

(2) Sphere: $(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$. In this case, we have $G = \frac{1}{r}(x-a, y-b, z-c)$ so the sphere satisfies (1.4) with $A = \frac{2}{r^2}I$, where I denotes the identity matrix.

In this paper, we study the GSCS's with Gauss map satisfying (1.4). As a result, we establish

Theorem 3.5. *Let M denote a generalized slant cylindrical surface in the 3-dimensional Euclidean space $\mathbb{E}^3$. Suppose that the Gauss map G of M satisfies $\Delta G = AG$ for some 3×3 matrix A. Then M is part of a plane, a sphere or a circular cylinder.*

Hereafter, all objects are assumed to be connected and smooth, unless mentioned otherwise.

2. Generalized Slant Cylindrical Surfaces

For a fixed unit speed plane curve $\alpha(s) = (x(s), y(s), 0)$, let $T(s) = \alpha'(s)$ and $N(s) = (-y'(s), x'(s), 0)$ denote the unit tangent and principal normal vector, respectively. The curvature $\kappa(s)$ of $\alpha(s)$ is defined by $T'(s) = \kappa(s)N(s)$ and we have $T(s) \times N(s) = V$, where V denotes the unit vector $(0, 0, 1)$. For a constant θ, we let $\beta_s(t) = t(\cos\theta N(s) + \sin\theta V)$. Then the ruled surface M defined by

$$X(s,t) = \alpha(s) + \beta_s(t) \tag{2.1}$$

is regular at (s,t) where $1 - \cos\theta\kappa(s)t$ does not vanish. This ruled surface M is called a *slant cylindrical surface* (SCS) over $\alpha(s)$. The SCS with $\sin\theta = 0$ or $\cos\theta = 0$ is nothing but a parametrization of either a plane or a cylindrical surface.

In general, we consider another unit speed plane curve $\beta(t) = (z(t), w(t))$. If we let $\beta_s(t) = z(t)N(s) + w(t)V$, then the parametrized surface defined by

$$X(s,t) = \alpha(s) + \beta_s(t) \tag{2.2}$$

is regular at (s,t) where $1 - \kappa(s)z(t)$ does not vanish. This parametrized surface M is called a *generalized slant cylindrical surface* (GSCS) over $\alpha(s)$.

If $\beta(t)$ is a straight line (resp., a circle), then the GSCS $X(s,t)$ is nothing but an SCS (resp., a tube) along a plane curve α. If $\alpha(s)$ is a straight line, then the GSCS $X(s,t)$ is a cylindrical surface over a plane curve. Furthermore, we have the following ([12]).

Proposition 2.1. *If $\alpha(s)$ is a circle, then a GSCS M over $\alpha(s)$ is a surface of revolution.*

Therefore we see that cylindrical surfaces, tubes along a plane curve and surfaces of revolution are special cases of GSCS's.

We also have the following characterizations ([10]):

Proposition 2.2. *Let M denote a GSCS given by (2.2). Then we have the following.*
(1) *If the mean curvature H is constant, then M is a surface of revolution.*
(2) *If the Gaussian curvature K is constant, then M is either a surface of revolution or an SCS.*

3. GAUSS MAP OF GSCS'S

Let $\alpha(s) = (x(s), y(s), 0)$ be a unit speed plane curve with the Frenet frame $T(s)$ and $N(s)$ which is defined on an interval I. We consider a GSCS M parametrized by

$$X(s,t) = \alpha(s) + \beta_s(t), \quad (s,t) \in I \times J, \tag{3.1}$$

where $\beta(t) = (z(t), w(t))$ is a unit speed plane curve, $\beta_s(t) = z(t)N(s) + w(t)V$, and $V = (0,0,1)$. Without loss of generality, we may assume that $\beta(0) = 0$, hence we have $X(s,0) = \alpha(s)$ for all $s \in I$.

Then $X(s,t)$ is regular at (s,t) where $q(s,t) = 1 - \kappa(s)z(t)$ does not vanish and we get

$$\begin{aligned} &X_s = q(s,t)T(s),\ X_t = z'(t)N(s) + w'(t)V, \\ &G(s,t) = -w'(t)N(s) + z'(t)V. \end{aligned} \tag{3.2}$$

The Laplacian Δ on M is given by for $f \in C^\infty(M)$

$$\Delta f = -q^{-3}\{\kappa'(s)z(t)f_s + qf_{ss} - q^2\kappa(s)z'(t)f_t + q^3 f_{tt}\}. \tag{3.3}$$

Hence it follows from (3.2) and (3.3) that

$$\begin{aligned} -q^3\Delta G = {}& \kappa'(s)w'(t)T(s) + q\{\kappa(s)^2 w'(t) + q\kappa(s)z'(t)w''(t) \\ & - q^2 w'''(t)\}N(s) + q^2\{-\kappa(s)z'(t)z''(t) + qz'''(t)\}V. \end{aligned} \tag{3.4}$$

First, we show that a slant cylindrical surface with Gauss map G satisfying (1.4) is an open part of either a plane or a circular cylinder. Since an SCS is a ruled surface, Proposition 3.1 can be deduced from the results in [1]. But, for conveniences, we give a proof.

Proposition 3.1. *Let M be an SCS given by (3.1) with $z(t) = t\cos\theta$ and $w(t) = t\sin\theta$. Suppose that the Gauss map G of M satisfies $\Delta G = AG$ for some 3×3 matrix A. Then M is an open part of either a plane or a circular cylinder.*

Proof. Since $z(t) = t\cos\theta$ and $w(t) = t\sin\theta$, it follows from (3.2) and (3.4) that

$$G(s,t) = -\sin\theta N(s) + \cos\theta V = (\sin\theta y'(s), -\sin\theta x'(s), \cos\theta) \tag{3.5}$$

and

$$-q^3\Delta G = \kappa'(s)\sin\theta T(s) + q\kappa(s)^2\sin\theta N(s). \tag{3.6}$$

By the assumption, we obtain

$$\sin\theta\{x'\kappa' - y'\kappa^2 q\} = -q^3\{(a_{11}y' - a_{12}x')\sin\theta + a_{13}\cos\theta\}, \tag{3.7}$$

$$\sin\theta\{y'\kappa' + x'\kappa^2 q\} = -q^3\{(a_{21}y' - a_{22}x')\sin\theta + a_{23}\cos\theta\} \tag{3.8}$$

and

$$a_{31}\sin\theta y'(s) - a_{32}\sin\theta x'(s) + a_{33}\cos\theta = 0. \tag{3.9}$$

It follows from $x'(s)\times(3.7) + y'(s)\times(3.8)$ and $x'(s)\times(3.8) - y'(s)\times(3.7)$ that

$$\begin{aligned} \sin\theta\kappa'(s) &= q^3 f(s) = (1 - t\cos\theta\kappa(s))^3 f(s),\\ \sin\theta\kappa(s)^2 &= q^2 g(s) = (1 - t\cos\theta\kappa(s))^2 g(s), \end{aligned} \tag{3.10}$$

where $f(s)$ and $g(s)$ are given by

$$\begin{aligned} f(s) &= \sin\theta\{a_{12}(x')^2 + (a_{22} - a_{11})x'y' - a_{21}(y')^2\} - \cos\theta\{a_{13}x' + a_{23}y'\},\\ g(s) &= \sin\theta\{a_{22}(x')^2 - (a_{12} + a_{21})x'y' + a_{11}(y')^2\} - \cos\theta\{a_{23}x' - a_{13}y'\}. \end{aligned} \tag{3.11}$$

We divide by two cases as follows.

Case 1. $\cos\theta \neq 0$.

If $\sin\theta = 0$ or $\kappa(s)$ vanishes identically, then M is a part of a plane. Hence we may suppose that $\sin\theta \neq 0$ and $\kappa(s) \neq 0$ on an interval I_0. Then, the second equation in (3.10) shows that t is a function of $s \in I_0$, which is a contradiction.

Case 2. $\cos\theta = 0$.

In this case, M is a cylindrical surface over $\alpha(s)$. We may assume that $\sin\theta = 1$. Hence, from (3.10) and (3.11) we get

$$\kappa'(s) = a_{12}x'(s)^2 + (a_{22} - a_{11})x'(s)y'(s) - a_{21}y'(s)^2 \tag{3.12}$$

and

$$\kappa(s)^2 = a_{22}x'(s)^2 - (a_{12} + a_{21})x'(s)y'(s) + a_{11}y'(s)^2. \tag{3.13}$$

By differentiating (3.13) and using $y'y'' = -x'x''$, we obtain

$$2\kappa'\kappa = 2(a_{22} - a_{11})x'x'' - (a_{12} + a_{21})\{x''y' + x'y''\}. \tag{3.14}$$

Since $\kappa(s)y'(s) = -x''(s)$, multiplying the both sides of (3.14) by $y'(s)$ and then using (3.12), we get

$$x''(s)\{4\kappa'(s) + (a_{21} - a_{12})\} = 0. \tag{3.15}$$

Suppose that $I_0 = \{s \in I | x''(s) \neq 0\}$ is nonempty. Then on I_0, $\kappa'(s)$ is constant. Hence, by differentiating (3.12) and then multiplying y', on I_0 we obtain

$$(a_{22} - a_{11})\{y'(s)^2 - x'(s)^2\} + 2(a_{12} + a_{21})x'(s)y'(s) = 0. \tag{3.16}$$

Hence, it follows from (3.13) that on I_0

$$2\kappa(s)^2 = a_{11} + a_{22}, \tag{3.17}$$

which is a nonzero constant because $\kappa(s)y'(s) = -x''(s) \neq 0$ on the interval I_0.

If the complement I_0^c of I_0 has nonempty interior, then $\kappa(s) = 0$ there. Thus, by the continuity of $\kappa(s)$ we see that $\kappa(s)$ is constant on the whole domain I of α. Therefore, M is an open portion of either a plane or a circular cylinder.

Combining Cases 1 and 2 completes the proof of Proposition 3.1. □

Now, suppose that the Gauss map G of a GSCS M defined by (3.1) satisfies (1.4) for some 3×3 matrix A. Then, from (3.2) and (3.4) we get

$$x'\kappa'w' - pqy' = -q^3\{a_{11}y'w' - a_{12}x'w' + a_{13}z'\}, \tag{3.18}$$

$$y'\kappa'w' + pqx' = -q^3\{a_{21}y'w' - a_{22}x'w' + a_{23}z'\}, \tag{3.19}$$

and

$$-\kappa z'z'' + qz''' = -q\{(a_{31}y' - a_{32}x')w' + a_{33}z'\}, \tag{3.20}$$

where

$$p(s,t) = \kappa(s)^2w'(t) + q\kappa(s)z'(t)w''(t) - q^2w'''(t). \tag{3.21}$$

It follows from $x'(s) \times (3.18) + y'(s) \times (3.19)$ and $x'(s) \times (3.19) - y'(s) \times (3.18)$ that

$$\kappa' w' = -q^3[\{(a_{11} - a_{22})x'y' - a_{12}(x')^2 - a_{21}(y')^2\}w' + (a_{13}x' + a_{23}y')z'] \tag{3.22}$$

and

$$p = q^2[\{(a_{11}(y')^2 - (a_{12} + a_{21})x'y' + a_{22}(x')^2\}w' + (a_{13}y' - a_{23}x')z']. \tag{3.23}$$

First, we prove

Lemma 3.2. *Let M be a GSCS given by (3.1). Suppose that the Gauss map G of M satisfies $\Delta G = AG$ for some 3×3 matrix A with $a_{13} = a_{23} = 0$. Then M is open part of either an SCS or a surface of revolution.*

Proof. It follows from (3.22) that

$$w'(t)\{\kappa'(s) - q^3 f(s)\} = 0, \tag{3.24}$$

where $f(s)$ is a function of s.

Let's denote by $J_0 = \{t \in J | w'(t) \neq 0\}$. If J_0 is empty, then M is part of a plane parallel to the xy-plane. Otherwise, we have $\kappa'(s) = q^3 f(s)$ for all $s \in I$. Recall that $q = 1 - \kappa(s)z(t)$.

Suppose that $I_0 = \{s \in I | \kappa'(s) \neq 0\}$ is nonempty. Then, it follows from (3.24) that on $I_0 \times J_0$ we obtain

$$\kappa(s)z(t) = 1 - \left(\frac{\kappa'(s)}{f(s)}\right)^{1/3}, \tag{3.25}$$

which shows that $z(t)$ is constant on J_0. This shows that J_0 is the whole domain J of β and hence β is a straight line perpendicular to the xy-plane. Thus on $I_0 \times J$, M is a cylindrical surface over α. Due to Proposition 3.1, we see that on I_0, α has constant curvature κ, which is a contradiction. Therefore $\kappa(s)$ is constant on the whole domain I.

The above discussion implies that when $w'(t) \neq 0$ for some t, α is either a straight line or a circle. If α is a straight line, then M is a cylindrical surface over a plane curve. That is, M is an SCS. If α is a circle, then M is a surface of revolution.

This completes the proof of Lemma 3.2. □

Next, we show

Lemma 3.3. *Let M be a GSCS given by (3.1). Suppose that the Gauss map G of M satisfies $\Delta G = AG$ for some 3×3 matrix A with $a_{31} = a_{32} = 0$. Then M is an open part of an SCS or a surface of revolution.*

Proof. It follows from (3.20) that

$$\kappa(s)\{z(t)(z'''(t) + a_{33}z'(t)) + z'(t)z''(t)\} = z'''(t) + a_{33}z'(t). \tag{3.26}$$

If $z'''(t) + a_{33}z'(t)$ is nonzero for some t, then $\kappa(s)$ is a nonzero constant. Hence M is a surface of revolution.

Otherwise, that is, $z'''(t) + a_{33}z'(t)$ vanishes identically, then (3.26) implies that

$$\kappa(s)z'(t)z''(t) = 0. \tag{3.27}$$

This shows that $\kappa(s) = 0$ or $z'(t)$ is constant, that is, α or β is a straight line. Hence M is a slant cylindrical surface.

This completes the proof of Lemma 3.3. □

Finally, we prove the following.

Lemma 3.4. *Let M be a GSCS given by (3.1). Suppose that the Gauss map G of M satisfies $\Delta G = AG$ for some arbitrary 3×3 matrix A. Then M is an open part of either an SCS or a surface of revolution.*

Proof. From (3.20), we get

$$\kappa(s)z'(t)z''(t) - qz'''(t) = q\{f(s)w'(t) + a_{33}z'(t)\}, \tag{3.28}$$

where

$$f(s) = a_{31}y'(s) - a_{32}x'(s). \tag{3.29}$$

Note that $\beta(0) = (z(0), w(0)) = 0$. We divide by two cases.
Case 1. Suppose that $w'(0) \neq 0$. In this case, since $q(0) = 1$, putting $t = 0$ in (3.28), we obtain

$$f(s)w'(0) = \kappa(s)z'(0)z''(0) - (z'''(0) + a_{33}z'(0)). \tag{3.30}$$

This shows that

$$f(s) = a\kappa(s) + b \tag{3.31}$$

for some constants a and b given by

$$a = z'(0)z''(0)/w'(0), b = -(z'''(0) + a_{33}z'(0))/w'(0). \tag{3.32}$$

If $a = 0$, then as in the proof of Lemma 3.3, we may prove that M is an open part of either an SCS or a surface of revolution. Hence we may assume that a is nonzero.

Since $q = 1 - \kappa(s)z(t)$, substituting $f(s)$ in (3.31) into (3.28), we see that the curvature function $\kappa(s)$ of α satisfies the following quadratic polynomial:

$$g(t)\kappa(s)^2 + h(t)\kappa(s) + k(t) = 0, \tag{3.33}$$

where

$$\begin{aligned} &g(t) = az(t)w'(t), \quad k(t) = -z'''(t) - a_{33}z'(t), \\ &h(t) = bz(t)w'(t) + z'(t)z''(t) - aw'(t) - z(t)k(t). \end{aligned} \tag{3.34}$$

Therefore, it follows from (3.33) that $\kappa(s)$ is constant, unless the coefficients $g(t), h(t)$ and $k(t)$ identically vanish. In this case, M is either a surface of revolution or a cylindrical surface.

If the coefficients $g(t), h(t)$ and $k(t)$ identically vanish, then it follows from (3.34) that $z(t)z'(t)z''(t)$ vanishes identically. This shows that $\beta(t)$ is a straight line. Hence, we see that M is an SCS.

Case 2. Suppose that $w'(0) = 0$. In this case, we may assume that β is not a straight line because otherwise, M is an SCS. Then, for some t_0, we have $w'(t_0) \neq 0$.

Putting $\bar{z}(t) = z(t) - c$ with $c = z(t_0)$, we consider the parametrization $\bar{X}$ of M given by

$$\bar{X}(s,t) = \bar{\alpha}(s) + \bar{z}(t)N(s) + w(t)V, \tag{3.35}$$

where the base curve $\bar{\alpha}$ is a parallel curve of α defined by $\bar{\alpha}(s) = \alpha(s) + cN(s)$.

For an arc length parameter u of $\bar{\alpha}$, M has the reparametrization $\bar{X}(u,t) = \bar{\alpha}(u) + \bar{z}(t)N(u) + w(t)V$ of $X(s,t)$ with $\bar{z}(t_0) = 0$ and $w'(t_0) \neq 0$. Hence, we can proceed as in the proof of Case 1 to show that M is an open part of either a surface of revolution or an SCS. This completes the proof of Case 2.

Combining Cases 1 and 2 completes the proof of Lemma 3.4. □

Now, we combine Lemma 3.4, Proposition 3.1 and Proposition 1 in Section 1. Then, we get the following theorem.

Theorem 3.5. *Let M denote a generalized slant cylindrical surface in the 3-dimensional Euclidean space $\mathbb{E}^3$. Suppose that the Gauss map G of M satisfies $\Delta G = AG$ for some 3×3 matrix A. Then M is an open part of a plane, a sphere or a circular cylinder.*

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