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# TRIPLED FIXED POINT THEOREM FOR HYBRID PAIR OF MAPPINGS UNDER GENERALIZED NONLINEAR CONTRACTION

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Abstract. In this paper, we introduce the concept of $w-$compatibility and weakly commutativity for hybrid pair of mappings $F : X \times X \times X \to 2^X$ and $g : X \to X$ and establish a common tripled fixed point theorem under generalized nonlinear contraction. An example is also given to validate our result. We improve, extend and generalize various known results.

## 1. Introduction and Preliminaries

Let $(X,\ d)$ be a metric space and $CB(X)$ be the set of all nonempty closed bounded subsets of $X$. Let $D(x,\ A)$ denote the distance from $x$ to $A \subset X$ and $H$ denote the Hausdorff metric induced by $d$, that is,

$$\begin{aligned} D(x,A) &= \inf_{a\in A} d(x,a) \\ \text{and } H(A,B) &= \max\{\sup_{a\in A} D(a,B), \sup_{b\in B} D(b,A)\}, \text{ for all } A,B \in CB(X). \end{aligned}$$

Markin [23] initiated the study of fixed points for multivalued contractions and non-expansive maps using the Hausdorff metric. Fixed points existence for various multivalued contractive mappings has been studied by several authors under different conditions. For details, we refer the reader to [1, 2, 12, 13, 14, 15, 16, 18, 19, 20, 21, 25, 26, 27] and the reference therein. Multivalued maps theory has application in control theory, convex optimization, differential equations and economics.

Bhaskar and Lakshmikantham [10], established some coupled fixed point theorems and apply these to study the existence and uniqueness of solution for periodic

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boundary value problems. Lakshmikantham and Ciric [22] proved coupled coincidence and common coupled fixed point theorems for nonlinear contractive mappings in partially ordered complete metric spaces and extended the results of Bhaskar and Lakshmikantham [10].

Berinde and Borcut [8] introduced the concept of tripled fixed point for single valued mappings in partially ordered metric spaces. In [8], Berinde and Borcut established the existence of tripled fixed point of single-valued mappings in partially ordered metric spaces. For more details on tripled fixed point theory, we also refer the reader to [3, 4, 5, 6, 7, 9, 11]. Samet and Vetro [24] introduced the notion of fixed point of N order in case of single-valued mappings. In particular for N=3 (tripled case), we have the following definition:

**Definition 1.1** ([24])**.** Let $X$ be a non-empty set and $F : X \times X \times X \to X$ be a given mapping. An element $(x,\, y,\, z) \in X \times X \times X$ is called a *tripled fixed point* of the mapping $F$ if

$$F(x, y, z) = x, F(y, z, x) = y \text{ and } F(z, x, y) = z.$$

In this paper, we prove a common tripled fixed point for hybrid pair of mappings under generalized nonlinear contraction. We improve, extend and generalize the results of Ding, Li and Radenovic [17] and Abbas, Ciric, Damjanovic and Khan [2]. The effectiveness of the present work is validated with the help of suitable example.

## 2. Main Results

First we introduce the following:

**Definition 2.1.** Let $X$ be a nonempty set, $F : X \times X \times X \to 2^X$ (a collection of all nonempty subsets of $X$) and $g$ be a self-map on $X$. An element $(x,\, y,\, z) \in X \times X \times X$ is called

(1) a *tripled fixed point* of $F$ if $x \in F(x,\, y,\, z)$, $y \in F(y,\, z,\, x)$ and $z \in F(z,\, x,\, y)$.
(2) a *tripled coincidence point* of hybrid pair $\{F,\, g\}$ if $g(x) \in F(x,\, y,\, z)$, $g(y) \in F(y,\, z,\, x)$ and $g(z) \in F(z,\, x,\, y)$.
(3) a *common tripled fixed point* of hybrid pair $\{F,\, g\}$ if $x = g(x) \in F(x,\, y,\, z)$, $y = g(y) \in F(y,\, z,\, x)$ and $z = g(z) \in F(z,\, x,\, y)$.

We denote the set of tripled coincidence points of mappings $F$ and $g$ by $C(F,\, g)$. Note that if $(x,\, y,\, z) \in C(F,\, g)$, then $(y,\, z,\, x)$ and $(z,\, x,\, y)$ are also in $C(F,\, g)$.

**Definition 2.2.** Let $F : X \times X \times X \to 2^X$ be a multivalued mapping and $g$ be a self-map on $X$. The hybrid pair $\{F, g\}$ is called *$w$-compatible* if $g(F(x, y, z)) \subseteq F(gx, gy, gz)$ whenever $(x, y, z) \in C(F, g)$.

**Definition 2.3.** Let $F : X \times X \times X \to 2^X$ be a multivalued mapping and $g$ be a self-map on $X$. The mapping $g$ is called *$F$-weakly commuting* at some point $(x, y, z) \in X^3$ if $g^2x \in F(gx, gy, gz)$, $g^2y \in F(gy, gz, gx)$ and $g^2z \in F(gz, gx, gy)$.

**Lemma 2.1.** *Let $(X, d)$ be a metric space. Then, for each $a \in X$ and $B \in CB(X)$, there is $b_0 \in B$ such that $D(a, B) = d(a, b_0)$, where $D(a, B) = \inf_{b \in B} d(a, b)$.*

*Proof.* Let $a \in X$ and $B \in CB(X)$. Since the function $d$ is continuous. Thus, by the closedness of $B$, there exists $b_0 \in B$ such that $\inf_{b\in B} d(a, b) = d(a, b_0)$, that is, $D(a, B) = d(a, b_0)$.

Let $\Phi$ denote the set of all functions $\varphi : [0, \infty) \to [0, \infty)$ satisfying

$(i_\varphi)$ $\varphi$ is non-decreasing,

$(ii_\varphi)$ $\lim_{n\to\infty} \varphi^n(t) = 0$ for all $t > 0$.

It is clear that $\varphi(t) < t$ for each $t > 0$. In fact, if $\varphi(t_0) \geq t_0$ for some $t_0 > 0$, then, since $\varphi$ is non-decreasing, $\varphi^n(t_0) \geq t_0$ for all $n \in \mathbb{N}$, which contradicts with $\lim_{n\to\infty} \varphi^n(t_0) = 0$. In addition, it is easy to see that $\varphi(0) = 0$. ☐

**Theorem 2.1.** *Let $(X, d)$ be a metric space. Assume $F : X \times X \times X \to CB(X)$ and $g : X \to X$ be two mappings satisfying*

$$\begin{aligned} &H(F(x, y, z), F(u, v, w)) \\ &\leq \varphi\left[\max\left\{\begin{array}{c} d(gx, gu), D(gx, F(x, y, z)), D(gu, F(u, v, w)), \\ d(gy, gv), D(gy, F(y, z, x)), D(gv, F(v, w, u)), \\ d(gz, gw), D(gz, F(z, x, y)), D(gw, F(w, u, v)), \\ \frac{1}{2}\left[D(gx, F(u, v, w)) + D(gu, F(x, y, z))\right], \\ \frac{1}{2}\left[D(gy, F(v, w, u)) + D(gv, F(y, z, x))\right], \\ \frac{1}{2}\left[D(gz, F(w, u, v)) + D(gw, F(z, x, y))\right] \end{array}\right\}\right], \end{aligned} \tag{2.1}$$

*for all $x, y, z, u, v, w \in X$, where $\varphi \in \Phi$. Furthermore assume that $F(X \times X \times X) \subseteq g(X)$ and $g(X)$ is a complete subset of $X$. Then $F$ and $g$ have a tripled coincidence point. Moreover, $F$ and $g$ have a common tripled fixed point, if one of the following conditions holds:*

(a) *$F$ and $g$ are $w$-compatible.* $\lim_{n\to\infty} g^n x = u$, $\lim_{n\to\infty} g^n y = v$ *and* $\lim_{n\to\infty} g^n z = w$ *for some $(x, y, z) \in C(F, g)$ and for some $u, v, w \in X$ and $g$ is continuous at $u$, $v$ and $w$.*

(b) *$g$ is $F$-weakly commuting for some $(x,\, y,\, z) \in C(F,\, g)$ and $gx$, $gy$ and $gz$ are fixed points of $g$, that is, $g^2x = gx$, $g^2y = gy$ and $g^2z = gz$.*

(c) *$g$ is continuous at $x$, $y$ and $z$. $\lim_{n\to\infty} g^n u = x$, $\lim_{n\to\infty} g^n v = y$ and $\lim_{n\to\infty} g^n w = z$ for some $(x,\, y,\, z) \in C(F,\, g)$ and for some $u,\, v,\, w \in X$.*

(d) *$g(C(g,\, F))$ is singleton subset of $C(g,\, F)$.*

*Proof.* Let $x_0,\, y_0,\, z_0 \in X$ be arbitrary. Then $F(x_0,\, y_0,\, z_0)$, $F(y_0,\, z_0,\, x_0)$ and $F(z_0,\, x_0,\, y_0)$ are well defined. Choose $gx_1 \in F(x_0,\, y_0,\, z_0)$, $gy_1 \in F(y_0,\, z_0,\, x_0)$ and $gz_1 \in F(z_0,\, x_0,\, y_0)$, because $F(X\times X\times X) \subseteq g(X)$. Since $F : X\times X\times X \to CB(X)$, therefore by Lemma 2.1, there exist $u_1 \in F(x_1,\, y_1,\, z_1)$, $u_2 \in F(y_1,\, z_1,\, x_1)$ and $u_3 \in F(z_1,\, x_1,\, y_1)$ such that

$$\begin{aligned}
d(gx_1, u_1) &\leq H(F(x_0, y_0, z_0), F(x_1, y_1, z_1)),\\
d(gy_1, u_2) &\leq H(F(y_0, z_0, x_0), F(y_1, z_1, x_1)),\\
d(gz_1, u_3) &\leq H(F(z_0, x_0, y_0), F(z_1, x_1, y_1)).
\end{aligned}$$

Since $F(X\times X\times X) \subseteq g(X)$, there exist $x_2,\, y_2,\, z_2 \in X$ such that $u_1 = gx_2$, $u_2 = gy_2$ and $u_3 = gz_2$. Thus

$$\begin{aligned}
d(gx_1, gx_2) &\leq H(F(x_0, y_0, z_0), F(x_1, y_1, z_1)),\\
d(gy_1, gy_2) &\leq H(F(y_0, z_0, x_0), F(y_1, z_1, x_1)),\\
d(gz_1, gz_2) &\leq H(F(z_0, x_0, y_0), F(z_1, x_1, y_1)).
\end{aligned}$$

Continuing this process, we obtain sequences $\{x_n\}$, $\{y_n\}$ and $\{z_n\}$ in $X$ such that for all $n \in \mathbb{N}$, we have $gx_{n+1} \in F(x_n,\, y_n,\, z_n)$, $gy_{n+1} \in F(y_n,\, z_n,\, x_n)$ and $gz_{n+1} \in F(z_n,\, x_n,\, y_n)$ such that

$$\begin{aligned}
& d(gx_n, gx_{n+1})\\
\leq\ & H(F(x_{n-1}, y_{n-1}, z_{n-1}), F(x_n, y_n, z_n))
\end{aligned}$$

$$\leq \varphi\left[\max\left\{\begin{array}{c}
d(gx_{n-1}, gx_n), D(gx_{n-1}, F(x_{n-1}, y_{n-1}, z_{n-1})),\\
D(gx_n, F(x_n, y_n, z_n)), d(gy_{n-1}, gy_n),\\
D(gy_{n-1}, F(y_{n-1}, z_{n-1}, x_{n-1})), D(gy_n, F(y_n, z_n, x_n)),\\
d(gz_{n-1}, gz_n), D(gz_{n-1}, F(z_{n-1}, x_{n-1}, y_{n-1})),\\
D(gz_n, F(z_n, x_n, y_n)),\\
\frac{D(gx_{n-1}, F(x_n, y_n, z_n)) + D(gx_n, F(x_{n-1}, y_{n-1}, z_{n-1}))}{2},\\
\frac{D(gy_{n-1}, F(y_n, z_n, x_n)) + D(gy_n, F(y_{n-1}, z_{n-1}, x_{n-1}))}{2},\\
\frac{D(gz_{n-1}, F(z_n, x_n, y_n)) + D(gz_n, F(z_{n-1}, x_{n-1}, y_{n-1}))}{2}
\end{array}\right\}\right]$$

$$\leq \varphi\left[\max\left\{\begin{array}{c} d(gx_{n-1},gx_n),d(gx_{n-1},gx_n),d(gx_n,gx_{n+1}),\\ d(gy_{n-1},gy_n),d(gy_{n-1},gy_n),d(gy_n,gy_{n+1}),\\ d(gz_{n-1},gz_n),d(gz_{n-1},gz_n),d(gz_n,gz_{n+1}),\\ \frac{d(gx_{n-1},gx_{n+1})+d(gx_n,gx_n)}{2},\frac{d(gy_{n-1},gy_{n+1})+d(gy_n,gy_n)}{2},\\ \frac{d(gz_{n-1},gz_{n+1})+d(gz_n,gz_n)}{2} \end{array}\right\}\right]$$

$$\leq \varphi\left[\max\left\{\begin{array}{c} d(gx_{n-1},gx_n),d(gy_{n-1},gy_n),d(gz_{n-1},gz_n),\\ d(gx_n,gx_{n+1}),d(gy_n,gy_{n+1}),d(gz_n,gz_{n+1}),\\ \frac{d(gx_{n-1},gx_{n+1})}{2},\frac{d(gy_{n-1},gy_{n+1})}{2},\frac{d(gz_{n-1},gz_{n+1})}{2} \end{array}\right\}\right].$$

Thus

$$d(gx_n,gx_{n+1})\leq\varphi\left[\max\left\{\begin{array}{c} d(gx_{n-1},gx_n),d(gy_{n-1},gy_n),\\ d(gz_{n-1},gz_n),d(gx_n,gx_{n+1}),\\ d(gy_n,gy_{n+1}),d(gz_n,gz_{n+1}),\\ \frac{d(gx_{n-1},gx_{n+1})}{2},\frac{d(gy_{n-1},gy_{n+1})}{2},\frac{d(gz_{n-1},gz_{n+1})}{2} \end{array}\right\}\right]. \tag{2.2}$$

Similarly

$$d(gy_n,gy_{n+1})\leq\varphi\left[\max\left\{\begin{array}{c} d(gx_{n-1},gx_n),d(gy_{n-1},gy_n),\\ d(gz_{n-1},gz_n),d(gx_n,gx_{n+1}),\\ d(gy_n,gy_{n+1}),d(gz_n,gz_{n+1}),\\ \frac{d(gx_{n-1},gx_{n+1})}{2},\frac{d(gy_{n-1},gy_{n+1})}{2},\frac{d(gz_{n-1},gz_{n+1})}{2} \end{array}\right\}\right], \tag{2.3}$$

$$d(gz_n,gz_{n+1})\leq\varphi\left[\max\left\{\begin{array}{c} d(gx_{n-1},gx_n),d(gy_{n-1},gy_n),\\ d(gz_{n-1},gz_n),d(gx_n,gx_{n+1}),\\ d(gy_n,gy_{n+1}),d(gz_n,gz_{n+1}),\\ \frac{d(gx_{n-1},gx_{n+1})}{2},\frac{d(gy_{n-1},gy_{n+1})}{2},\frac{d(gz_{n-1},gz_{n+1})}{2} \end{array}\right\}\right]. \tag{2.4}$$

Combining (2.2), (2.3) and (2.4), we get

$$\max\{d(gx_n,gx_{n+1}),d(gy_n,gy_{n+1}),d(gz_n,gz_{n+1})\}$$

$$\leq \varphi\left[\max\left\{\begin{array}{c} d(gx_{n-1},gx_n),d(gy_{n-1},gy_n),d(gz_{n-1},gz_n),\\ d(gx_n,gx_{n+1}),d(gy_n,gy_{n+1}),d(gz_n,gz_{n+1}),\\ \frac{d(gx_{n-1},gx_{n+1})}{2},\frac{d(gy_{n-1},gy_{n+1})}{2},\frac{d(gz_{n-1},gz_{n+1})}{2} \end{array}\right\}\right]$$

$$\leq \varphi\left[\max\left\{\begin{array}{c} d(gx_{n-1},gx_n),d(gy_{n-1},gy_n),d(gz_{n-1},gz_n),\\ d(gx_n,gx_{n+1}),d(gy_n,gy_{n+1}),d(gz_n,gz_{n+1}),\\ \frac{d(gx_{n-1},gx_n)+d(gx_n,gx_{n+1})}{2},\\ \frac{d(gy_{n-1},gy_n)+d(gy_n,gy_{n+1})}{2},\\ \frac{d(gz_{n-1},gz_n)+d(gz_n,gz_{n+1})}{2} \end{array}\right\}\right]$$

$$\leq \varphi\left[\max\left\{\begin{array}{c} d(gx_{n-1},gx_n),d(gy_{n-1},gy_n),d(gz_{n-1},gz_n),\\ d(gx_n,gx_{n+1}),d(gy_n,gy_{n+1}),d(gz_n,gz_{n+1}), \end{array}\right\}\right].$$

Thus

$$\begin{aligned} &\max\{d(gx_n, gx_{n+1}), d(gy_n, gy_{n+1}), d(gz_n, gz_{n+1})\} \\ &\leq \varphi\left[\max\left\{\begin{array}{c} d(gx_{n-1}, gx_n), d(gy_{n-1}, gy_n), d(gz_{n-1}, gz_n), \\ d(gx_n, gx_{n+1}), d(gy_n, gy_{n+1}), d(gz_n, gz_{n+1}) \end{array}\right\}\right]. \end{aligned} \tag{2.5}$$

If we suppose that

$$\begin{aligned} & \max\left\{\begin{array}{c} d(gx_{n-1}, gx_n), d(gy_{n-1}, gy_n), d(gz_{n-1}, gz_n), \\ d(gx_n, gx_{n+1}), d(gy_n, gy_{n+1}), d(gz_n, gz_{n+1}) \end{array}\right\} \\ = \quad & \max\{d(gx_n, gx_{n+1}), d(gy_n, gy_{n+1}), d(gz_n, gz_{n+1})\}, \end{aligned}$$

then by (2.5), $(i_\varphi)$ and $(ii_\varphi)$, we have

$$\begin{aligned} & \max\{d(gx_n, gx_{n+1}), d(gy_n, gy_{n+1}), d(gz_n, gz_{n+1})\} \\ \leq \quad & \varphi[\max\{d(gx_n, gx_{n+1}), d(gy_n, gy_{n+1}), d(gz_n, gz_{n+1})\}] \\ < \quad & \max\{d(gx_n, gx_{n+1}), d(gy_n, gy_{n+1}), d(gz_n, gz_{n+1})\}, \end{aligned}$$

which is a contradiction. Thus, we must have

$$\begin{aligned} & \max\left\{\begin{array}{c} d(gx_{n-1}, gx_n), d(gy_{n-1}, gy_n), d(gz_{n-1}, gz_n), \\ d(gx_n, gx_{n+1}), d(gy_n, gy_{n+1}), d(gz_n, gz_{n+1}) \end{array}\right\} \\ = \quad & \max\{d(gx_{n-1}, gx_n), d(gy_{n-1}, gy_n), d(gz_{n-1}, gz_n)\}. \end{aligned}$$

Hence by (2.5), we have for all $n \in \mathbb{N}$,

$$\begin{aligned} & \max\{d(gx_n, gx_{n+1}), d(gy_n, gy_{n+1}), d(gz_n, gz_{n+1})\} \\ \leq \quad & \varphi[\max\{d(gx_{n-1}, gx_n), d(gy_{n-1}, gy_n), d(gz_{n-1}, gz_n)\}] \\ \leq \quad & \varphi^n[\max\{d(gx_0, gx_1), d(gy_0, gy_1), d(gz_0, gz_1)\}]. \end{aligned}$$

Thus

$$\max\{d(gx_n, gx_{n+1}), d(gy_n, gy_{n+1}), d(gz_n, gz_{n+1})\} \leq \varphi^n(\delta), \tag{2.6}$$

where

$$\delta = \max\{d(gx_0, gx_1), d(gy_0, gy_1), d(gz_0, gz_1)\}.$$

Without loss of generality, one can assume that $\max\{d(gx_0, gx_1), d(gy_0, gy_1), d(gz_0, gz_1)\} \neq 0$. In fact, if this is not true, then $gx_0 = gx_1 \in F(x_0, y_0, z_0)$, $gy_0 = gy_1 \in F(y_0, z_0, x_0)$ and $gz_0 = gz_1 \in F(z_0, x_0, y_0)$, that is, $(x_0, y_0, z_0)$ is a tripled coincidence point of $F$ and $g$.

Thus, for $m$, $n \in \mathbb{N}$ with $m > n$, by triangle inequality and (2.6), we get

$$\begin{aligned}
& d(gx_n, gx_{m+n}) \\
\leq\ & d(gx_n, gx_{n+1}) + d(gx_{n+1}, gx_{n+2}) + ... + d(gx_{n+m-1}, gx_{m+n}) \\
\leq\ & \max\{d(gx_n, gx_{n+1}), d(gy_n, gy_{n+1}), d(gz_n, gz_{n+1})\} \\
& + \max\{d(gx_{n+1}, gx_{n+2}), d(gy_{n+1}, gy_{n+2}), d(gz_{n+1}, gz_{n+2})\} \\
& + ... + \max\{d(gx_{n+m-1}, gx_{n+m+1}), d(gy_{n+m-1}, gy_{n+m}), d(gz_{n+m-1}, gz_{n+m})\} \\
\leq\ & \varphi^n(\delta) + \varphi^{n+1}(\delta) + ... + \varphi^{n+m-1}(\delta) \\
\leq\ & \sum_{i=n}^{n+m-1} \varphi^i(\delta),
\end{aligned}$$

which implies, by $(ii_\varphi)$, that $\{gx_n\}$ is a Cauchy sequence in $g(X)$. Similarly we obtain that $\{gy_n\}$ and $\{gz_n\}$ are Cauchy sequences in $g(X)$. Since $g(X)$ is complete, there exist $x$, $y$, $z \in X$ such that

$$\lim_{n\to\infty} gx_n = gx,\ \lim_{n\to\infty} gy_n = gy \text{ and } \lim_{n\to\infty} gz_n = gz. \tag{2.7}$$

Now, since $gx_{n+1} \in F(x_n, y_n, z_n)$, $gy_{n+1} \in F(y_n, z_n, x_n)$ and $gz_{n+1} \in F(z_n, x_n, y_n)$, therefore by using condition (2.1), we get

$$D(gx_{n+1}, F(x, y, z)) \leq H(F(x_n, y_n, z_n), F(x, y, z)) \leq \varphi[\Delta_n], \tag{2.8}$$

$$D(gy_{n+1}, F(y, z, x)) \leq H(F(y_n, z_n, x_n), F(y, z, x)) \leq \varphi[\Delta_n], \tag{2.9}$$

$$D(gz_{n+1}, F(z, x, y)) \leq H(F(z_n, x_n, y_n), F(z, x, y)) \leq \varphi[\Delta_n], \tag{2.10}$$

where

$$\Delta_n = \max\left\{\begin{array}{c} d(gx_n, gx), d(gx_n, gx_{n+1}), D(gx, F(x, y, z)), \\ d(gy_n, gy), d(gy_n, gy_{n+1}), D(gy, F(y, z, x)), \\ d(gz_n, gz), d(gz_n, gz_{n+1}), D(gz, F(z, x, y)), \\ \frac{1}{2}\left[D(gx_n, F(x, y, z)) + d(gx, gx_{n+1})\right], \\ \frac{1}{2}\left[D(gy_n, F(y, z, x)) + d(gy, gy_{n+1})\right], \\ \frac{1}{2}\left[D(gz_n, F(z, x, y)) + d(gz, gz_{n+1})\right] \end{array}\right\}.$$

Since $\lim_{n\to\infty} gx_n = gx$, $\lim_{n\to\infty} gy_n = gy$ and $\lim_{n\to\infty} gz_n = gz$, there exists $n_0 \in \mathbb{N}$ such that for all $n > n_0$,

$$\Delta_n = \max\{D(gx, F(x, y, z)), D(gy, F(y, z, x)), D(gz, F(z, x, y))\}.$$

Combining this with (2.8), (2.9) and (2.10), we get for all $n > n_0$,

$$\begin{aligned} &\max\left\{ \begin{array}{c} D(gx_{n+1}, F(x,y,z)), D(gy_{n+1}, F(y,z,x)), \\ D(gz_{n+1}, F(z,x,y)) \end{array} \right\} \\ &\leq \varphi\left[\max\left\{ \begin{array}{c} D(gx, F(x,y,z)), D(gy, F(y,z,x)), \\ D(gz, F(z,x,y)) \end{array} \right\}\right]. \end{aligned} \tag{2.11}$$

Now, we claim that

$$\max\{D(gx, F(x,y,z)), D(gy, F(y,z,x)), D(gz, F(z,x,y))\} = 0. \tag{2.12}$$

If this is not true, then

$$\max\{D(gx, F(x,y,z)), D(gy, F(y,z,x)), D(gz, F(z,x,y))\} > 0.$$

Thus, by (2.11), $(i_\varphi)$ and $(ii_\varphi)$, we get for all $n > n_0$,

$$\begin{aligned} &\max\left\{ \begin{array}{c} D(gx_{n+1}, F(x,y,z)), D(gy_{n+1}, F(y,z,x)), \\ D(gz_{n+1}, F(z,x,y)) \end{array} \right\} \\ \leq\ &\varphi\left[\max\left\{ \begin{array}{c} D(gx, F(x,y,z)), D(gy, F(y,z,x)), \\ D(gz, F(z,x,y)) \end{array} \right\}\right] \\ <\ &\max\left\{ \begin{array}{c} D(gx, F(x,y,z)), D(gy, F(y,z,x)), \\ D(gz, F(z,x,y)) \end{array} \right\}. \end{aligned}$$

Thus

$$\begin{aligned} &\max\left\{ \begin{array}{c} D(gx_{n+1}, F(x,y,z)), D(gy_{n+1}, F(y,z,x)), \\ D(gz_{n+1}, F(z,x,y)) \end{array} \right\} \\ &< \max\left\{ \begin{array}{c} D(gx, F(x,y,z)), D(gy, F(y,z,x)), \\ D(gz, F(z,x,y)) \end{array} \right\} \end{aligned} \tag{2.13}$$

Letting $n \to \infty$ in (2.13), by using (2.7), we obtain

$$\begin{aligned} &\max\{D(gx, F(x,y,z)), D(gy, F(y,z,x)), D(gz, F(z,x,y))\} \\ <\ &\max\{D(gx, F(x,y,z)), D(gy, F(y,z,x)), D(gz, F(z,x,y))\}, \end{aligned}$$

which is a contradiction. So (2.12) holds. Thus, it follows that

$$gx \in F(x,y,z),\ gy \in F(y,z,x) \text{ and } gz \in F(z,x,y),$$

that is, $(x, y, z)$ is a tripled coincidence point of $F$ and $g$. Hence $C(F, g)$ is nonempty.

Suppose now that (a) holds. Assume that for some $(x, y, z) \in C(F, g)$,

$$\lim_{n\to\infty} g^n x = u,\ \lim_{n\to\infty} g^n y = v \text{ and } \lim_{n\to\infty} g^n z = w, \tag{2.14}$$

where $u$, $v$, $w \in X$. Since $g$ is continuous at $u$, $v$ and $w$. We have, by (2.14), that $u$, $v$ and $w$ are fixed points of $g$, that is,

$$gu = u,\ gv = v \text{ and } gw = w. \tag{2.15}$$

As $F$ and $g$ are $w$-compatible, so for all $n \geq 1$,

$$\begin{aligned} g^n x &\in F(g^{n-1}x,\, g^{n-1}y,\, g^{n-1}z), \\ g^n y &\in F(g^{n-1}y,\, g^{n-1}z,\, g^{n-1}x), \\ g^n x &\in F(g^{n-1}z,\, g^{n-1}x,\, g^{n-1}y). \end{aligned} \tag{2.16}$$

Now, by using (2.1) and (2.16), we obtain

$$\begin{aligned} D(g^n x,\, F(u,\, v,\, w)) &\leq H(F(g^{n-1}x,\, g^{n-1}y,\, g^{n-1}z),\, F(u,\, v,\, w)) \leq \varphi[\nabla_n], \\ D(g^n y,\, F(v,\, w,\, u)) &\leq H(F(g^{n-1}y,\, g^{n-1}z,\, g^{n-1}x),\, F(v,\, w,\, u)) \leq \varphi[\nabla_n], \\ D(g^n z,\, F(w,\, u,\, v)) &\leq H(F(g^{n-1}z,\, g^{n-1}x,\, g^{n-1}y),\, F(w,\, u,\, v)) \leq \varphi[\nabla_n], \end{aligned} \tag{2.17}$$

where

$$\nabla_n = \max\left\{ \begin{array}{c} d(g^n x, gu), D(gu, F(u,v,w)), \frac{D(g^n x, F(u,v,w)) + d(gu, g^n x)}{2}, \\ d(g^n y, gv), D(gv, F(v,w,u)), \frac{D(g^n y, F(v,w,u)) + d(gv, g^n y)}{2}, \\ d(g^n z, gw), D(gw, F(w,u,v)), \frac{D(g^n z, F(w,u,v)) + d(gw, g^n z)}{2} \end{array} \right\}.$$

By (2.14) and (2.15), there exists $n_0 \in \mathbb{N}$ such that for all $n > n_0$,

$$\nabla_n = \max\left\{ D(gu, F(u,v,w)), D(gv, F(v,w,u)), D(gw, F(w,u,v)) \right\}.$$

Combining this with (2.17), we get for all $n > n_0$,

$$\max\left\{ \begin{array}{c} D(g^n x, F(u,v,w)), \\ D(g^n y, F(v,w,u)), \\ D(g^n z, F(w,u,v)) \end{array} \right\} \leq \varphi\left[ \max\left\{ \begin{array}{c} D(gu, F(u,v,w)), \\ D(gv, F(v,w,u)), \\ D(gw, F(w,u,v)) \end{array} \right\} \right]. \tag{2.18}$$

Now, we claim that

$$\max\left\{ D(gu, F(u,v,w)), D(gv, F(v,w,u)), D(gw, F(w,u,v)) \right\} = 0. \tag{2.19}$$

If this is not true, then

$$\max\left\{ D(gu, F(u,v,w)), D(gv, F(v,w,u)), D(gw, F(w,u,v)) \right\} > 0.$$

Thus, by (2.18), $(i_\varphi)$ and $(ii_\varphi)$, we get for all $n > n_0$,

$$\max\left\{ \begin{array}{c} D(g^n x, F(u,v,w)), \\ D(g^n y, F(v,w,u)), \\ D(g^n z, F(w,u,v)) \end{array} \right\} < \max\left\{ \begin{array}{c} D(gu, F(u,v,w)), \\ D(gv, F(v,w,u)), \\ D(gw, F(w,u,v)) \end{array} \right\}. \tag{2.20}$$

On taking limit as $n \to \infty$ in (2.20), by using (2.14) and (2.15), we get

$$\begin{aligned} & \max\left\{ D(gu, F(u,v,w)), D(gv, F(v,w,u)), D(gw, F(w,u,v)) \right\} \\ < \ & \max\left\{ D(gu, F(u,v,w)), D(gv, F(v,w,u)), D(gw, F(w,u,v)) \right\}, \end{aligned}$$

which is a contradiction. So (2.19) holds. Thus, it follows that

$$gu \in F(u,v,w),\ gv \in F(v,w,u) \text{ and } gw \in F(w,u,v). \tag{2.21}$$

Now, from (2.15) and (2.21), we have

$$u = gu \in F(u, v, w),\ v = gv \in F(v, w, u) \text{ and } w = gw \in F(w, u, v),$$

that is, $(u,\ v,\ w)$ is a common tripled fixed point of $F$ and $g$.

Suppose now that $(b)$ holds. Assume that for some $(x,\ y,\ z) \in C(F,\ g)$, $g$ is $F$-weakly commuting, that is, $g^2x \in F(gx,\ gy,\ gz)$, $g^2y \in F(gy,\ gz,\ gx)$, $g^2z \in F(gz,\ gx,\ gy)$ and $g^2x = gx$, $g^2y = gy$, $g^2z = gz$. Thus $gx = g^2x \in F(gx,\ gy,\ gz)$, $gy = g^2y \in F(gy,\ gz,\ gx)$ and $gz = g^2z \in F(gz,\ gx,\ gy)$, that is, $(gx,\ gy,\ gz)$ is a common tripled fixed point of $F$ and $g$.

Suppose now that $(c)$ holds. Assume that for some $(x,\ y,\ z) \in C(F,\ g)$ and for some $u,\ v,\ w \in X$, $\lim_{n\to\infty} g^n u = x$, $\lim_{n\to\infty} g^n v = y$ and $\lim_{n\to\infty} g^n w = z$. Since $g$ is continuous at $x$, $y$ and $z$. We have that $x$, $y$ and $z$ are fixed point of $g$, that is, $gx = x$, $gy = y$ and $gz = z$. Since $(x,\ y,\ z) \in C(F,\ g)$, therefore, we obtain

$$x = gx \in F(x, y, z),\ \ y = gy \in F(y, z, x)$$

and

$$z = gz \in F(z, x, y),$$

that is, $(x,\ y,\ z)$ is a common tripled fixed point of $F$ and $g$.

Finally, suppose that $(d)$ holds. Let $g(C(F,\ g)) = \{(x,\ x,\ x)\}$. Then $\{x\} = \{gx\} = F(x,\ x,\ x)$. Hence $(x,\ x,\ x)$ is tripled fixed point of $F$ and $g$. $\square$

**Example 2.1.** Suppose that $X = [0,\ 1]$, equipped with the metric $d : X \times X \to [0, +\infty)$ defined by $d(x,\ y) = \max\{x,\ y\}$ and $d(x,\ x) = 0$ for all $x,\ y \in X$. Let $F : X \times X \times X \to CB(X)$ be defined as

$$F(x,y,z) = \begin{cases} \{0\}, & \text{for } x,y,z = 1 \\ \left[0, \frac{x^2+y^2+z^2}{6}\right], & \text{for } x,y,z \in [0,1) \end{cases}$$

and $g : X \to X$ be defined as

$$g(x) = x^2, \text{ for all } x \in X.$$

Define $\varphi : [0,\ \infty) \to [0,\ \infty)$ by

$$\varphi(t) = \begin{cases} \frac{t}{2}, & \text{for } t \neq 1 \\ \frac{3}{4}, & \text{for } t = 1. \end{cases}$$

Now, for all $x,\ y,\ z,\ u,\ v,\ w \in X$ with $x,\ y,\ z,\ u,\ v,\ w \in [0,\ 1)$, we have

Case $(a)$ If $x^2 + y^2 + z^2 = u^2 + v^2 + w^2$, then

$$\begin{aligned}
& H(F(x,y,z),F(u,v,w)) \\
= \;& \frac{u^2+v^2+w^2}{6} \\
\leq \;& \frac{1}{6}\max\left\{x^2,u^2\right\}+\frac{1}{6}\max\left\{y^2,v^2\right\}+\frac{1}{6}\max\left\{z^2,w^2\right\} \\
\leq \;& \frac{1}{6}d(gx,gu)+\frac{1}{6}d(gy,gv)+\frac{1}{6}d(gz,gw) \\
\leq \;& \frac{1}{2}\left[\max\left\{\begin{array}{c} d(gx,gu),D(gx,F(x,y,z)),D(gu,F(u,v,w)), \\ d(gy,gv),D(gy,F(y,z,x)),D(gv,F(v,w,u)), \\ d(gz,gw),D(gz,F(z,x,y)),D(gw,F(w,u,v)), \\ \frac{1}{2}\left[D(gx,F(u,v,w))+D(gu,F(x,y,z))\right], \\ \frac{1}{2}\left[D(gy,F(v,w,u))+D(gv,F(y,z,x))\right], \\ \frac{1}{2}\left[D(gz,F(w,u,v))+D(gw,F(z,x,y))\right] \end{array}\right\}\right] \\
\leq \;& \varphi\left[\max\left\{\begin{array}{c} d(gx,gu),D(gx,F(x,y,z)),D(gu,F(u,v,w)), \\ d(gy,gv),D(gy,F(y,z,x)),D(gv,F(v,w,u)), \\ d(gz,gw),D(gz,F(z,x,y)),D(gw,F(w,u,v)), \\ \frac{1}{2}\left[D(gx,F(u,v,w))+D(gu,F(x,y,z))\right], \\ \frac{1}{2}\left[D(gy,F(v,w,u))+D(gv,F(y,z,x))\right], \\ \frac{1}{2}\left[D(gz,F(w,u,v))+D(gw,F(z,x,y))\right] \end{array}\right\}\right].
\end{aligned}$$

Case $(b)$ If $x^2+y^2+z^2 \neq u^2+v^2+w^2$ with $x^2+y^2+z^2 < u^2+v^2+w^2$, then

$$\begin{aligned}
& H(F(x,y,z),F(u,v,w)) \\
= \;& \frac{u^2+v^2+w^2}{6} \\
\leq \;& \frac{1}{6}\max\left\{x^2,u^2\right\}+\frac{1}{6}\max\left\{y^2,v^2\right\}+\frac{1}{6}\max\left\{z^2,w^2\right\} \\
\leq \;& \frac{1}{6}d(gx,gu)+\frac{1}{6}d(gy,gv)+\frac{1}{6}d(gz,gw) \\
\leq \;& \frac{1}{2}\left[\max\left\{\begin{array}{c} d(gx,gu),D(gx,F(x,y,z)),D(gu,F(u,v,w)), \\ d(gy,gv),D(gy,F(y,z,x)),D(gv,F(v,w,u)), \\ d(gz,gw),D(gz,F(z,x,y)),D(gw,F(w,u,v)), \\ \frac{1}{2}\left[D(gx,F(u,v,w))+D(gu,F(x,y,z))\right], \\ \frac{1}{2}\left[D(gy,F(v,w,u))+D(gv,F(y,z,x))\right], \\ \frac{1}{2}\left[D(gz,F(w,u,v))+D(gw,F(z,x,y))\right] \end{array}\right\}\right] \\
\leq \;& \varphi\left[\max\left\{\begin{array}{c} d(gx,gu),D(gx,F(x,y,z)),D(gu,F(u,v,w)), \\ d(gy,gv),D(gy,F(y,z,x)),D(gv,F(v,w,u)), \\ d(gz,gw),D(gz,F(z,x,y)),D(gw,F(w,u,v)), \\ \frac{1}{2}\left[D(gx,F(u,v,w))+D(gu,F(x,y,z))\right], \\ \frac{1}{2}\left[D(gy,F(v,w,u))+D(gv,F(y,z,x))\right], \\ \frac{1}{2}\left[D(gz,F(w,u,v))+D(gw,F(z,x,y))\right] \end{array}\right\}\right].
\end{aligned}$$

Similarly, we obtain the same result for $u^2 + v^2 + w^2 < x^2 + y^2 + z^2$. Thus the contractive condition (2.1) is satisfied for all $x, y, z, u, v, w \in X$ with $x, y, z, u, v, w \in [0, 1)$. Again, for all $x, y, z, u, v, w \in X$ with $x, y, z \in [0, 1)$ and $u, v, w = 1$, we have

$$
\begin{aligned}
& H(F(x,y,z), F(u,v,w)) \\
= \ & \frac{x^2+y^2+z^2}{6} \\
\leq \ & \frac{1}{6}\max\left\{x^2, u^2\right\} + \frac{1}{6}\max\left\{y^2, v^2\right\} + \frac{1}{6}\max\left\{z^2, w^2\right\} \\
\leq \ & \frac{1}{6}d(gx, gu) + \frac{1}{6}d(gy, gv) + \frac{1}{6}d(gz, gw) \\
\leq \ & \frac{1}{2}\left[\max\left\{\begin{array}{c} d(gx,gu), D(gx,F(x,y,z)), D(gu,F(u,v,w)), \\ d(gy,gv), D(gy,F(y,z,x)), D(gv,F(v,w,u)), \\ d(gz,gw), D(gz,F(z,x,y)), D(gw,F(w,u,v)), \\ \frac{1}{2}\left[D(gx,F(u,v,w)) + D(gu,F(x,y,z))\right], \\ \frac{1}{2}\left[D(gy,F(v,w,u)) + D(gv,F(y,z,x))\right], \\ \frac{1}{2}\left[D(gz,F(w,u,v)) + D(gw,F(z,x,y))\right] \end{array}\right\}\right] \\
\leq \ & \varphi\left[\max\left\{\begin{array}{c} d(gx,gu), D(gx,F(x,y,z)), D(gu,F(u,v,w)), \\ d(gy,gv), D(gy,F(y,z,x)), D(gv,F(v,w,u)), \\ d(gz,gw), D(gz,F(z,x,y)), D(gw,F(w,u,v)), \\ \frac{1}{2}\left[D(gx,F(u,v,w)) + D(gu,F(x,y,z))\right], \\ \frac{1}{2}\left[D(gy,F(v,w,u)) + D(gv,F(y,z,x))\right], \\ \frac{1}{2}\left[D(gz,F(w,u,v)) + D(gw,F(z,x,y))\right] \end{array}\right\}\right].
\end{aligned}
$$

Thus the contractive condition (2.1) is satisfied for all $x, y, z, u, v, w \in X$ with $x, y, z \in [0, 1)$ and $u, v, w = 1$. Similarly, we can see that the contractive condition (2.1) is satisfied for all $x, y, z, u, v, w \in X$ with $x, y, z, u, v, w = 1$. Hence, the hybrid pair $\{F, g\}$ satisfies the contractive condition (2.1), for all $x, y, z, u, v, w \in X$. In addition, all the other conditions of Theorem 2.1 are satisfied and $z = (0, 0, 0)$ is a common tripled fixed point of hybrid pair $\{F, g\}$. The function $F : X \times X \times X \to CB(X)$ involved in this example is not continuous at the point $(1, 1, 1) \in X \times X \times X$.

**Remark 2.1.** We improve, extend and generalize the result of Ding, Li and Radenovic [17] in the following sense:

(i) We prove our result in the settings of multivalued mapping and for hybrid pair of mappings while Ding, Li and Radenovic [17] proved result for single valued mappings.

(ii) We prove tripled coincidence and common tripled fixed point theorem while Ding, Li and Radenovic [17] proved coupled coincidence and common coupled fixed point theorems.

(iii) To prove the result we consider non complete metric space and the space is also not partially ordered.

(iv) The mapping $F : X \times X \times X \to CB(X)$ is discontinuous and not satisfying mixed g-monotone property.

(v) The function $\varphi : [0, \infty) \to [0, \infty)$ involved in our theorem and example is discontinuous.

(vi) Our proof is simple and different from the other results in the existing literature.

If we put $g = I$ ($I$ is the identity mapping) in Theorem 2.1, then we have the following result:

**Corollary 2.2.** *Let $(X, d)$ be a complete metric space, $F : X \times X \times X \to CB(X)$ be a mapping satisfying*

$$\begin{aligned} & H(F(x,y,z), F(u,v,w)) \\ \leq \quad & \varphi \left[ \max \left\{ \begin{array}{c} d(x,u), D(x, F(x,y,z)), D(u, F(u,v,w)), \\ d(y,v), D(y, F(y,z,x)), D(v, F(v,w,u)), \\ d(z,w), D(z, F(z,x,y)), D(w, F(w,u,v)), \\ \frac{1}{2}\left[D(x, F(u,v,w)) + D(u, F(x,y,z))\right], \\ \frac{1}{2}\left[D(y, F(v,w,u)) + D(v, F(y,z,x))\right], \\ \frac{1}{2}\left[D(z, F(w,u,v)) + D(w, F(z,x,y))\right] \end{array} \right\} \right], \end{aligned}$$

*for all $x$, $y$, $z$, $u$, $v$, $w \in X$, where $\varphi \in \Phi$. Then $F$ has a tripled fixed point.*

If we put $\varphi(t) = kt$ where $0 < k < 1$ in Theorem 2.1, then we have the following result:

**Corollary 2.3.** *Let $(X, d)$ be a metric space. Assume $F : X \times X \times X \to CB(X)$ and $g : X \to X$ be two mappings satisfying*

$$\begin{aligned} & H(F(x,y,z), F(u,v,w)) \\ \leq \quad & k \max \left\{ \begin{array}{c} d(gx,gu), D(gx, F(x,y,z)), D(gu, F(u,v,w)), \\ d(gy,gv), D(gy, F(y,z,x)), D(gv, F(v,w,u)), \\ d(gz,gw), D(gz, F(z,x,y)), D(gw, F(w,u,v)), \\ \frac{1}{2}\left[D(gx, F(u,v,w)) + D(gu, F(x,y,z))\right], \\ \frac{1}{2}\left[D(gy, F(v,w,u)) + D(gv, F(y,z,x))\right], \\ \frac{1}{2}\left[D(gz, F(w,u,v)) + D(gw, F(z,x,y))\right] \end{array} \right\}, \end{aligned}$$

*for all* $x,\ y,\ z,\ u,\ v,\ w \in X$, *where* $0 < k < 1$. *Furthermore assume that* $F(X \times X \times X) \subseteq g(X)$ *and* $g(X)$ *is a complete subset of* $X$. *Then* $F$ *and* $g$ *have a tripled coincidence point. Moreover,* $F$ *and* $g$ *have a common tripled fixed point, if one of the following conditions holds:*

(a) $F$ *and* $g$ *are* $w$*-compatible.* $\lim_{n\to\infty} g^n x = u$, $\lim_{n\to\infty} g^n y = v$ *and* $\lim_{n\to\infty} g^n z = w$ *for some* $(x,\ y,\ z) \in C(F,\ g)$ *and for some* $u,\ v,\ w \in X$ *and* $g$ *is continuous at* $u,\ v$ *and* $w$.

(b) $g$ *is* $F$*-weakly commuting for some* $(x,\ y,\ z) \in C(F,\ g)$ *and* $gx,\ gy$ *and* $gz$ *are fixed points of* $g$, *that is,* $g^2x = gx,\ g^2y = gy$ *and* $g^2z = gz$.

(c) $g$ *is continuous at* $x,\ y$ *and* $z$. $\lim_{n\to\infty} g^n u = x$, $\lim_{n\to\infty} g^n v = y$ *and* $\lim_{n\to\infty} g^n w = z$ *for some* $(x,\ y,\ z) \in C(F,\ g)$ *and for some* $u,\ v,\ w \in X$.

(d) $g(C(g,\ F))$ *is singleton subset of* $C(g,\ F)$.

If we put $g = I$ ($I$ is the identity mapping) in Corollary 2.3, then we have the following result:

**Corollary 2.4.** *Let* $(X,\ d)$ *be a complete metric space,* $F : X \times X \times X \to CB(X)$ *be a mapping satisfying*

$$\begin{aligned} & H(F(x,y,z),F(u,v,w)) \\ \leq\ & k \max \left\{ \begin{array}{c} d(x,u), D(x,F(x,y,z)), D(u,F(u,v,w)), \\ d(y,v), D(y,F(y,z,x)), D(v,F(v,w,u)), \\ d(z,w), D(z,F(z,x,y)), D(w,F(w,u,v)), \\ \frac{1}{2}\left[D(x,F(u,v,w)) + D(u,F(x,y,z))\right], \\ \frac{1}{2}\left[D(y,F(v,w,u)) + D(v,F(y,z,x))\right], \\ \frac{1}{2}\left[D(z,F(w,u,v)) + D(w,F(z,x,y))\right] \end{array} \right\}, \end{aligned}$$

*for all* $x,\ y,\ z,\ u,\ v,\ w \in X$. *Then* $F$ *has a tripled fixed point.*

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