J. Korean Soc. Math. Educ. Ser. B: Pure Appl. Math.
http://dx.doi.org/10.7468/jksmeb.2014.21.2.113
Volume 21, Number 2 (May 2014), Pages 113–120

ISSN(Print) 1226-0657
ISSN(Online) 2287-6081

FILTER SPACES AND BASICALLY DISCONNECTED COVERS

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Abstract. In this paper, we first show that for any space X, there is a σ-complete Boolean subalgebra $Z(\Lambda_X)^\#$ of $\mathcal{R}(X)$ and that the subspace $\{\alpha \mid \alpha$ is a fixed $\sigma Z(X)^\#$-ultrafilter$\}$ of the Stone-space $S(Z(\Lambda_X)^\#)$ is the minimal basically disconnected cover of X. Using this, we will show that for any countably locally weakly Lindelöf space X, the set $\{M | M$ is a σ-complete Boolean subalgebra of $\mathcal{R}(X)$ containing $Z(X)^\#$ and $s_M^{-1}(X)$ is basically disconnected$\}$, when partially ordered by inclusion, becomes a complete lattice.

1. Introduction

All spaces in this paper are Tychonoff spaces and βX denotes the Stone-Čech compactification of a space X .

Vermeer([10]) showed that every space X has the minimal basically disconnected cover $(\Lambda X, \Lambda_X)$ and if X is a compact space, then ΛX is given by the Stone-space $S(\sigma Z(X)^\#)$ of a σ-complete Boolean subalgebra $\sigma Z(X)^\#$ of $\mathcal{R}(X)$. Henriksen, Vermeer and Woods([4])(Kim [7], resp.) showed that the

minimal basically disconnected cover of a weakly Lindelöf space (a locally weakly Lindelöf space, resp.) X is given by the subspace $\{\alpha \mid \alpha$ is a fixed $\sigma Z(X)^\#$-ultrafilter$\}$ of the Stone-space $S(\sigma Z(X)^\#)$.

In this paper, we first show that for any space X, there is a σ-complete Boolean subalgebra $Z(\Lambda_X)^\#$ of $\mathcal{R}(X)$ and that the subspace $\{\alpha \mid \alpha$ is a fixed $\sigma Z(X)^\#$-ultrafilter$\}$ of the Stone-space $S(Z(\Lambda_X)^\#)$ is the the minimal basically disconnected cover of X. Using this, we will show that $S(Z(\Lambda_X)^\#)$ and $\beta\Lambda X$ are homeomorphic. Moreover, we show that for any σ-complete Booeal subalgebra M of $\mathcal{R}(X)$ containing $Z(X)^\#$, the Stone-space $S(M)$ of M is a basically diconnected cover of X and that the subspace $\{\alpha \mid \alpha$ is a fixed M-ultrafilter$\}$ of the Stone-space $S(M)$ is the the

Received by the editors January 10, 2014. Revised March 4, 2014. Accepted March 10, 2014.
2010 *Mathematics Subject Classification.* 54G05, 54G99, 54C01.
Key words and phrases. basically disconnected cover, Stone-space, covering map.
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minimal basically disconnected cover of X if and only if it is a basically disconnected space and $M \subseteq Z(\Lambda_X)^{\#}$. Finally, we will show that for any countably locally weakly Lindelöf space X, the set $\{M | M$ is a σ-complete Boolean subalgebra of $\mathcal{R}(X)$ containg $Z(X)^{\#}$ and $s_M^{-1}(X)$ is basically disconnected$\}$, when partially ordered by inclusion, becomes a complete lattice.

For the terminology, we refer to [1] and [9].

2. Filter Spaces

The set $\mathcal{R}(X)$ of all regular closed sets in a space X, when partially ordered by inclusion, becomes a complete Boolean algebra, in which the join, meet, and complementation operations are defined as follows : for any $A \in \mathcal{R}(X)$ and any $\{A_i : i \in I\} \subseteq \mathcal{R}(X)$,

$\vee\{A_i : i \in I\} = cl_X(\cup\{A_i : i \in I\})$,

$\wedge\{A_i : i \in I\} = cl_X(int_X(\cap\{A_i : i \in I\}))$, and

$A' = cl_X(X - A)$

and a sublattice of $\mathcal{R}(X)$ is a subset of $\mathcal{R}(X)$ that contains $\emptyset$, X and is closed under finite joins and meets.

We recall that a map $f : Y \longrightarrow X$ is called *a covering map* if it is a continuous, onto, perfect, and irreducible map.

Lemma 2.1 ([5]).

(1) *Let $f : Y \longrightarrow X$ be a covering map. Then the map $\psi : \mathcal{R}(Y) \longrightarrow \mathcal{R}(X)$, defined by $\psi(A) = f(A) \cap X$, is a Boolean isomorphism and the inverse map ψ^{-1} of ψ is given by $\psi^{-1}(B) = cl_Y(f^{-1}(int_X(B))) = cl_Y(int_Y(f^{-1}(B)))$.*

(2) *Let X be a dense subspace of a space K. Then the map $\phi : \mathcal{R}(K) \longrightarrow \mathcal{R}(X)$, defined by $\phi(A) = A \cap X$, is a Boolean isomorphism and the inverse map ϕ^{-1} of ϕ is given by $\phi^{-1}(B) = cl_K(B)$.*

A lattice L is called *σ-complete* if every countable subset of L has the join and the meet. For any subset M of a Boolean algebra L, there is the smallest σ-complete Boolean subalgebra σM of L containing M. Let X be a space and $Z(X)$ the set of all zero-sets in X. Then $Z(X)^{\#} = \{cl_X(int_X(Z)) \mid Z \in Z(X)\}$ is a sublattice of $\mathcal{R}(X)$.

We recall that a subspace X of a space Y is *C^*-embedded in Y* if for any real-valued continuous map $f : X \longrightarrow \mathbb{R}$, there is a continuous map $g : Y \longrightarrow \mathbb{R}$ such that $g|_X = f$.

Let X be a space. Since X is C^*-embedded in βX, by Lemma 2.1., $\sigma Z(X)^\#$ and $\sigma Z(\beta X)^\#$ are Boolean isomorphic.

Let X be a space and $\mathcal{B}$ a Boolean subalgebra of $\mathcal{R}(X)$. Let $S(\mathcal{B}) = \{\alpha \mid \alpha$ is a $\mathcal{B}$-ultrafilter$\}$ and for any $B \in \mathcal{B}$, $\Sigma_B^{\mathcal{B}} = \{\alpha \in S(\mathcal{B}) \mid B \in \alpha\}$. Then the space $S(\mathcal{B})$, equipped with the topology for which $\{\Sigma_B^{\mathcal{B}} \mid B \in \mathcal{B}\}$ is a base, called *the Stone-space of* $\mathcal{B}$. Then $S(\mathcal{B})$ is a compact, zero-dimensional space and the map $s_{\mathcal{B}} : S(\mathcal{B}) \longrightarrow \beta X$, defined by $s_{\mathcal{B}}(\alpha) = \cap\{cl_{\beta X}(A) \mid A \in \mathcal{B}\}$, is a covering map ([7]).

Definition 2.2. A space X is called *basically disconnected* if for any zero-set Z in X, $int_X(Z)$ is closed in X, equivalently, every cozero-set in X is C^*-embedded in X.

A space X is a basically disconnected space if and only if βX is a basically disconnected space. If X is a basically disconnected space, every element in $Z(X)^\#$ is clopen in X and so X is a basically disconnected space if and only if $Z(X)^\#$ is a σ-complete Boolean algebra.

Definition 2.3. Let X be a space. Then a pair (Y, f) is called

(1) *a cover of* X if $f : Y \longrightarrow X$ is a covering map,

(2) *a basically disconnected cover of* X if (Y, f) is a cover of X and Y is a basically disconnected space, and

(3) *a minimal basically disconnected cover of* X if (Y, f) is a basically disconnected cover of X and for any basically disconnected cover (Z, g) of X, there is a covering map $h : Z \longrightarrow Y$ such that $f \circ h = g$.

Vermeer([10]) showed that every space X has a minimal basically disconnected cover $(\Lambda X, \Lambda_X)$ and that if X is a compact space, then ΛX is the Stone-space $S(\sigma Z(X)^\#)$ of $\sigma Z(X)^\#$ and $\Lambda_X(\alpha) = \cap\{A \mid A \in \alpha\}$ $(\alpha \in \Lambda X)$.

Let X be a space. Since $\sigma Z(X)^\#$ and $\sigma Z(\beta X)^\#$ are Boolean isomorphic, $S(\sigma Z(X)^\#)$ and $S(\sigma Z(\beta X)^\#)$ are homeomorphic.

Let X, Y be spaces and $f : Y \longrightarrow X$ a map. For any $U \subseteq X$, let $f_U : f^{-1}(U) \longrightarrow U$ denote the restriction and co-restriction of f with respect to $f^{-1}(U)$ and U, respectively.

In the following, for any space X, $(\Lambda\beta X, \Lambda_\beta)$ denotes the minimal basically disconnected cover of βX.

Lemma 2.4 ([7]). *Let X be a space. If $\Lambda_\beta^{-1}(X)$ is a basically disconnected space, then $(\Lambda_\beta^{-1}(X), \Lambda_{\beta_X})$ is the minimal basically disconnected cover of X.*

For any covering map $f : Y \longrightarrow X$, let $Z(f)^{\#} = \{cl_Y(int_X(f(Z))) \mid Z \in Z(Y)^{\#}\}$. Since $\mathcal{R}(\Lambda X)$ and $\mathcal{R}(X)$ are Boolean isomorphic and $Z(\Lambda X)^{\#}$ is a σ-complete Boolean subalgebra of $\mathcal{R}(\Lambda X)$, by Lemma 2.1, $Z(\Lambda_X)^{\#}$ is a σ-complete Boolean subalgebra of $\mathcal{R}(X)$.

Definition 2.5. Let X be a space and $\mathcal{B}$ a sublattice of $\mathcal{R}(X)$. Then a $\mathcal{B}$-filter $\mathcal{F}$ is called *fixed* if $\{F \mid F \in \mathcal{F}\} \neq \emptyset$.

Let X be a space and for any $Z(\Lambda_X)^{\#}$-ultrafilter α, let $\alpha_\lambda = \{A \in Z(\Lambda X)^{\#} \mid \Lambda_X(A) \in \alpha\}$.

Proposition 2.6. *Let X be a space and α a fixed $Z(\Lambda_X)^{\#}$-ultrafilter. Then α_λ is a fixed $Z(\Lambda X)^{\#}$-ultrafilter and $\mid \cap\{A \mid A \in \alpha_\lambda\} \mid = 1$.*

Proof. Clearly, α_λ is a $Z(\Lambda X)^{\#}$-filter. Suppose that $A \in Z(\Lambda X)^{\#} - \alpha_\lambda$. Then $\Lambda_X(A) \in Z(\Lambda_X)^{\#} - \ \alpha$. Since α is a $Z(\Lambda_X)^{\#}$-ultrafilter, there is a $C \in \alpha$ such that $C \wedge \Lambda_X(A) = \emptyset$ and hence $A \wedge cl_{\Lambda X}(\Lambda_X^{-1}(int_X(C))) = \emptyset$. Since $\Lambda_X(cl_{\Lambda X}(\Lambda_X^{-1}(int_X(C)))) = C \in \alpha$, $cl_{\Lambda X}(\Lambda_X^{-1}(int_X(C))) \in \alpha_\lambda$ and hence α_λ is a $Z(\Lambda X)^{\#}$-ultrafilter. Since α is fixed, there is an $x \in \cap\{B \mid B \in \alpha\}$. Then $\{A \cap \Lambda_X^{-1}(x) \mid A \in \alpha_\lambda\}$ has a family of closed sets in $\Lambda_X^{-1}(x)$ with the finite intersection property. Since $\Lambda_X^{-1}(x)$ is a compact subset of ΛX, $\cap\{A \cap \Lambda_X^{-1}(x) \mid A \in \alpha_\lambda\} \neq \emptyset$ and hence $\cap\{A \mid A \in \alpha_\lambda\} \neq \emptyset$. Since $Z(\Lambda X)^{\#}$ is a base for ΛX and α_λ is a $Z(\Lambda X)^{\#}$-ultrafilter, $\mid \cap\{A \mid A \in \alpha_\lambda\} \mid = 1$. □

Let X be a space and $FX = \{\alpha \mid \alpha$ is a fixed $Z(\Lambda_X)^{\#}$-ultrafilter$\}$ the subspace of the Stone space $S(Z(\Lambda_X)^{\#})$. Define a map $h_X : FX \longrightarrow \Lambda X$ by $h_X(\alpha) = \cap\{A \mid A \in \alpha_\lambda\}$. In the following, let $\Sigma_B = \Sigma_B^{Z(\Lambda_X)^{\#}}$ for any $B \in Z(\Lambda_X)^{\#}$.

Theorem 2.7. *Let X be a space. Then $h_X : FX \longrightarrow \Lambda X$ is a homeomorphism.*

Proof. Take any α, δ in FX with $\alpha \neq \delta$. Since α and δ are $Z(\Lambda_X)^{\#}$-ultrafilters, there are A, B in $Z(\Lambda X)^{\#}$ such that $\Lambda_X(A) \in \alpha$, $\Lambda_X(B) \in \delta$ such that $\Lambda_X(A) \wedge \Lambda_X(B) = \emptyset$. Then $A \in \alpha_\lambda$, $B \in \delta_\lambda$ and $A \wedge B = \emptyset$. By Lemma 2.1, $cl_{\Lambda X}(A) \cap cl_{\Lambda X}(B) = \emptyset$ and $h_X(\alpha) = \cap\{G \mid G \in \alpha_\lambda\} \neq \cap\{H \mid H \in \delta_\lambda\} = h_X(\delta)$. Thus h_X is an one-to-one map.

Let $y \in \Lambda X$ and $\gamma = \{\Lambda_X(C) \mid y \in C \in Z(\Lambda X)^{\#}\}$. Since every element of $Z(\Lambda X)^{\#}$ is a clopen set in ΛX, $\gamma \in FX$ and $h_X(\gamma) = y$ and hence h_X is an onto map.

Let $E \in Z(\Lambda X)^{\#}$. Suppose that $\mu \in FX - h_X^{-1}(E)$. Since $\Lambda_X(E) \notin \mu$, $\mu \notin \Sigma_{\Lambda_X(E)}$ and so $\Sigma_{\Lambda_X}(E) \subseteq h^{-1}(E)$. Suppose that $\theta \in h_X^{-1}(E)$. Then $h_X(\theta) \in E$ and hence for any $A \in \theta_\lambda$, $A \cap E \neq \emptyset$. Since θ_λ is a $Z(\Lambda X)^{\#}$-ultrafilter, $E \in \theta_\lambda$ and so $E \in \Sigma_{\Lambda X(E)}$

and $h_X(\theta) \in E$. Hence $\Sigma_{\Lambda_X(E)} = h_X^{-1}(E)$ and since h_X is one-to-one and onto, h_X is a homeomorphism. □

Corollary 2.8. *Let X be a space and $F_X = \Lambda_X \circ h_X$. Then (FX, F_X) is the minimal basically disconnected cover of X and $F(\alpha) = \cap\{A \mid A \in \alpha\}$ for all $\alpha \in FX$.*

It is well-known that a space X is C^*-embedded in its compactification Y if and only if $\beta X = Y$.

Theorem 2.9. *Let X be a space. Then there is a homeomorphism $k : \beta\Lambda X \longrightarrow S(Z(\Lambda_X)^\#)$ such that $k \circ \beta_{\Lambda X} \circ h_X = j$, where $j : FX \longrightarrow S(Z(\Lambda_X)^\#)$ is the inclusion map.*

Proof. By Theorem 2.7., $\beta FX = \beta\Lambda X$ and $S(Z(\Lambda_X)^\#)$ is a compactification of FX. Hence there is a continuous map $k : \beta\Lambda X \longrightarrow S(Z(\Lambda_X)^\#)$ such that $k \circ \beta_{\Lambda X} \circ h_X = j$, where $j : \Lambda X \longrightarrow S(Z(\Lambda_X)^\#)$ is the dense embedding. Let $T = S(Z(\Lambda_X)^\#)$ and A, B be disjoint zero-sets in FX. Then there are disjoint zero-sets C, D in FX such that $A \subseteq int_{FX}(C)$ and $B \subseteq int_{FX}(D)$. Since $h_X : FX \longrightarrow \Lambda X$ is a homeomorphism, $cl_{FX}(int_{FX}(C)) = \Sigma_{F_X(cl_{FX}(int_{FX}(C)))} \cap FX$ and since FX is dense in T, $cl_T(cl_{FX}(int_{FX}(C))) = \Sigma_{F_X(cl_{FX}(int_{FX}(C)))}$. Similarly,

$$cl_T(cl_{FX}(int_{FX}(D))) = \Sigma_{F_X(cl_{FX}(int_{FX}(D)))}.$$

Since $cl_{FX}(int_{FX}(C))) \wedge cl_{FX}(int_{FX}(D))) = \emptyset$,

$$F_X(cl_{FX}(int_{FX}(C))) \wedge F_X(cl_{FX}(int_{FX}(D))) = \emptyset.$$

Hence

$$\begin{aligned} & cl_T(cl_{FX}(int_{FX}(C))) \cap cl_T(cl_{FX}(int_{FX}(D))) \\ & = \Sigma_{F_X(cl_{FX}(int_{FX}(C)))} \cap \Sigma_{F_X(cl_{FX}(int_{FX}(D)))} \\ & = \Sigma_{F_X(cl_{\Lambda X}(int_{\Lambda X}(C))) \wedge F_X(cl_{FX}(int_{FX}(D)))} \\ & = \emptyset. \end{aligned}$$

By the Uryshon's extension theorem, FX is C^*-embedded in T and so k is a homeomorphism. □

It is known that $\beta\Lambda X = \Lambda\beta X$ if and only if $\{\Lambda_X(A) \mid A \in Z(\Lambda X)^\#\} = \sigma Z(X)^\#$([5]). Hence we have the following :

Corollary 2.10. *Let X be a space. Then $\beta\Lambda X = \Lambda\beta X$ if and only if $Z(\Lambda_X)^\# = \sigma Z(X)^\#$.*

3. Basically Disconnected Covers

Let X be a space and M a σ-complete Boolean subalgebra of $\mathcal{R}(X)$ containg $Z(X)^{\#}$. By the dfinition of $\sigma Z(X)^{\#}$, $\sigma Z(X)^{\#} \subseteq M$.

Proposition 3.1. *Let X be a space and M a σ-complete Boolean subalgebra of $\mathcal{R}(X)$ containg $Z(X)^{\#}$. Then $S(M)$ is a basically disconnected space.*

Proof. Let D be a cozero-set in $S(M)$. Since $S(M)$ is a compact space, D is a Lindelöf space and hence there is a sequense (A_n) in M such that $D = \cup\{\Sigma^M_{A_n} \mid n \in N\}$. Clearly, $cl_{S(M)}(D) \subseteq \Sigma^M_{\vee\{A_n|n\in N\}}$. Let $\alpha \in S(M) - cl_{S(M)}(\cup\{\Sigma^M_{A_n} \mid n \in N\})$. Then there is a $B \in M$ such that $\alpha \in \Sigma^M_B$ and $(\cup\{\Sigma^M_{A_n} \mid n \in N\}) \cap \Sigma^M_B = \emptyset$. Hence for any $n \in N$, $\Sigma^M_{A_n} \cap \Sigma^M_B = \Sigma_{A_n \wedge B} = \emptyset$ and hence $A_n \wedge B = \emptyset$. So, $\vee\{A_n \wedge B \mid n \in N\} = (\vee\{A_n \mid n \in N\}) \wedge B = \emptyset$. Since $B \in \alpha$, $\vee\{A_n \mid n \in N\} \notin \alpha$ and so $\alpha \notin \Sigma_{\vee\{A_n|n\in N\}}$. Hence $cl_{S(M)}(D)$ is open in $S(M)$ and thus $S(M)$ is a basically disconnected space. □

Let X be a space and M a σ-complete Boolean subalgebra of $\mathcal{R}(X)$ containg $Z(X)^{\#}$. By Theorem 3.1, there is a covering map $t : S(M) \longrightarrow \Lambda\beta X$ such that $\Lambda_\beta \circ t = s_M$.

Theorem 3.2. *Let X be a space and M a σ-complete Boolean subalgebra of $\mathcal{R}(X)$ containg $Z(X)^{\#}$. Then we have the following :*

(1) *There is a covering map $g : S(M) \longrightarrow \beta\Lambda X$ such that $s_{Z(\Lambda_X)^{\#}} \circ g = s_M$ if and only if $Z(\Lambda_X)^{\#} \subseteq M$.*

(2) *There is a covering map $f : \beta\Lambda X \longrightarrow S(M)$ such that $s_M \circ f = s_{Z(\Lambda_X)^{\#}}$ if and only if $M \subseteq Z(\Lambda_X)^{\#}$.*

(3) *$(s_M^{-1}(X), s_{M_X})$ is the minimal basically disconnected cover of X if and only if $(s_M^{-1}(X), s_{M_X})$ is a basically disconnected cover of X and $M \subseteq Z(\Lambda_X)^{\#}$.*

Proof. (1) ($\Rightarrow$) Take any $Z \in Z(\Lambda X)^{\#}$. Then there is an $A \in Z(\beta\Lambda X)^{\#}$ such that $Z = A \cap \Lambda X$. Since $\beta\Lambda X$ is basically disconnected, $g^{-1}(A)$ is a clopen-set in $S(M)$. Since $S(M)$ is compact, there is a $D \in M$ such that $g^{-1}(A) = \Sigma^M_D$. Since s_M and $s_{Z(\Lambda_X)^{\#}}$ are covering maps, $cl_{\beta X}(D) = s_M(g^{-1}(A)) = s_{Z(\Lambda_X)^{\#}}(A)$. By Lemma 2.1, $D = s_M(g^{-1}(A)) \cap X = s_{Z(\Lambda_X)^{\#}}(A) \cap X = \Lambda_X(A \cap \Lambda X) = \Lambda_X(Z)$ and hence $\Lambda_X(Z) \in M$.

($\Leftarrow$) It is trivial([9]).

Similarly, we have (2)

(3) ($\Rightarrow$) Suppose that $(s_M^{-1}(X), s_{M_X})$ is the minimal basically disconnected cover of X. Then there is a homeomorhpism $l : s_M^{-1}(X) \longrightarrow \Lambda X$ such that $\Lambda_X \circ l = s_{M_X}$. Hence there is a covering map $f : \beta\Lambda X \longrightarrow S(M)$ such that $f \circ \beta_{\Lambda X} \circ l = j$, where $j : s_M^{-1}(X) \longrightarrow S(M)$ is the inclusion map. Take any $D \in M$. Then $f^{-1}(\Sigma_D^M)$ is a clopen set in $\beta\Lambda X$ and since $\beta\Lambda X$ is a compact space, there is an $A \in Z(\Lambda_X)^\#$ such that $\Sigma_A = f^{-1}(\Sigma_D^M)$. Hence $s_{Z(\Lambda_X)^\#}(\Sigma_A) = cl_{\beta X}(A) = s_{Z(\Lambda_X)^\#}(f^{-1}(\Sigma_D^M))$. Since $s_M \circ f \circ \beta_{\Lambda X} \circ l = s_M \circ j = \beta_X \circ \Lambda_X \circ l = s_{Z(\Lambda_X)^\#} \circ \beta_{\Lambda X} \circ l$ and $\beta_{\Lambda X} \circ l$ is a dense embedding, $s_M \circ f = s_{Z(\Lambda_X)^\#}$. By (2), we have the result.

($\Leftarrow$) Since $s_M^{-1}(X)$ is a basically disconnected space, there is a covering map $l : s_M^{-1}(X) \longrightarrow \Lambda X$ such that $\Lambda_X \circ l = s_{M_X}$. Since $M \subseteq Z(\Lambda_X)^\#$, by (2), there is a covering map $f : \beta\Lambda X \longrightarrow S(M)$ such that $s_M \circ f = s_{Z(\Lambda_X)^\#}$. Since $s_M \circ f \circ \beta_{\Lambda X} = s_{Z(\Lambda_X)^\#} \circ \beta_{\Lambda X} = \beta_X \circ \Lambda_X$, there is a covering $m : \Lambda X \longrightarrow s_M^{-1}(X)$ such that $s_{M_X} \circ m = \Lambda_X$ and $j \circ m = f \circ \beta_{\Lambda X}$. Since $\Lambda_X \circ l \circ m = s_{M_X} \circ m = \Lambda_X = \Lambda_X \circ 1_{\Lambda X}$ and $\Lambda_X, l \circ m$ are coevring maps, $l \circ m = 1_{\Lambda X}$. Hence $s_M^{-1}(X)$ and ΛX are homeomorphic. $\square$

We recall that a space X is called *a weakly Lindelöf space* if for any open cover $\mathcal{U}$, there is a countable subset $\mathcal{V}$ of $\mathcal{U}$ such that $\cup\mathcal{V}$ is dense in X and that X is called *a countably locally weakly Lindelöf space* if for any countable set $\{\mathcal{U}_n | n \in \mathbb{N}\}$ of open covers of X and any $x \in X$, there is a neighborhood G of x in X and for any $n \in \mathbb{N}$, there is a countable subset $\mathcal{V}_n$ of $\mathcal{U}_n$ such that $G \subseteq cl_X(\cup\mathcal{V}_n)$.

It was shown that for any countably locally weakly Lindelöf space X, $\Lambda_\beta^{-1}(X)$ is a basically disconnected space([8]). Using Lemma 2.4 and Theorem 3.2, we have the following corollary :

Corollary 3.3. *Let X be a countably locally weakly Lindelöf space. Then the set $\{M | M$ is a σ-complete Boolean subalgebra of $\mathcal{R}(X)$ containg $Z(X)^\#$ and $s_M^{-1}(X)$ is basically disconnected$\}$, when partially ordered by inclusion, becomes a complete lattice. Moreover, $\sigma Z(X)^\#$ is the bottom element and $Z(\Lambda_X)^\#$ is the top element.*

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