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GENERALIZED MODULE LEFT (m, n)-DERIVATIONS

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Abstract. Fošner [4] defined a generalized module left (m, n)-derivation and proved the Hyers-Ulam stability of generalized module left (m, n)-derivations.

In this note, we prove that every generalized module left (m, n)-derivation is trival if the algebra is unital and $m \neq n$.

1. Stability of Module Left (m, n)-derivations

Let A be an algebra and M be a left A-module. An additive mapping $d : A \to M$ is called a *module left derivation* if $d(xy) = x \cdot d(y) + y \cdot d(x)$ for all $x, y \in A$.

Definition 1.1 ([3]). Let A be an algebra and M be a left A-module. An additive mapping $d : A \to M$ is called a *module left (m, n)-derivation* if

$$(m + n)d(xy) = 2mx \cdot d(y) + 2ny \cdot d(x) \tag{1.1}$$

for all $x, y \in A$. Here m and n are nonnegative integers with $m + n \neq 0$.

Let A be an algebra and M be a left A-module. An additive mapping $g : A \to M$ is called a *generalized module left derivation* if there exists a module left derivation $d : A \to M$ such that $g(xy) = x \cdot g(y) + y \cdot d(x)$ for all $x, y \in A$.

Definition 1.2 ([4]). Let A be an algebra and M be a left A-module. An additive mapping $g : A \to M$ is called a *generalized module left (m, n)-derivation* if there exists a module left (m, n)-derivation $d : A \to M$ such that

$$(m + n)g(xy) = 2mx \cdot g(y) + 2ny \cdot d(x) \tag{1.2}$$

for all $x, y \in A$. Here m and n are nonnegative integers with $m + n \neq 0$.

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Proposition 1.3 ([2])**.** *Let A be a unital algebra with unit e and M be a left A-module. Assume that m and n are nonnegative integers with $m + n \neq 0$ and $m \neq n$. Then each module left (m, n)-derivation $d : A \to M$ is trival.*

Assume that $e \cdot x = x$ for all $x \in M$.

Theorem 1.4. *Let A be a unital algebra with unit e and M be a left A-module. Assume that m and n are nonnegative integers with $m + n \neq 0$ and $m \neq n$. Then each generalized module left (m, n)-derivation $g : A \to M$ is trival.*

Proof. By Proposition 1.3, $d(e) = 0$.

Letting $x = y = e$ in (1.2), we get $(m + n)g(e) = 2mg(e) + 2nd(e) = 2mg(e)$ and so $g(e) = 0$, since $m \neq n$.

Letting $y = e$ in (1.2), we get

$$(m + n)g(x) = 2mx \cdot g(e) + 2nd(x) = 2nd(x) = 0$$

for all $x \in A$, since $d(x) = 0$ and $g(e) = 0$. Since $m + n \neq 0$, $g(x) = 0$ for all $x \in A$, as desired. □

Remark 1.5. When $m = n$, the generalized module left (m, n)-derivation is just a generalized module left derivation. In [1], Cao et al. proved the Hyers-Ulam stability of generalized module left derivations $d : A \to M$.

Problem 1.6. Let A be a non-unital algebra and M be a left A-module. Assume that m and n are nonnegative integers with $m + n \neq 0$ and $m \neq n$.

(1) Is there a non-trivial generalized module left (m, n)-derivation $d : A \to M$?

(2) Construct a non-trival generalized module left (m, n)-derivation $d : A \to M$.

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