J. Korean Soc. Math. Educ. Ser. B: Pure Appl. Math. ISSN(Print) 1226-0657
https://doi.org/10.7468/jksmeb.2017.24.1.33 ISSN(Online) 2287-6081
Volume 24, Number 1 (February 2017), Pages 33–34

# MODULE LEFT $(m, n)$-DERIVATIONS

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Abstract. Fošner [1] defined a module left $(m, n)$-derivation and proved the Hyers-Ulam stability of module left $(m, n)$-derivations.

In this note, we prove that every module left $(m, n)$-derivation is trival if the algebra is unital and $m \neq n$.

## 1. Stability of Module Left $(m, n)$-derivations

Let $A$ be an algebra and $M$ be a left $A$-module. An additive mapping $d : A \to M$ is called a *module left derivation* if $d(xy) = x \cdot d(y) + y \cdot d(x)$ for all $x, y \in A$.

**Definition 1.1** ([1])**.** Let $A$ be an algebra and $M$ be a left $A$-module. An additive mapping $d : A \to M$ is called a *module left* $(m, n)$*-derivation* if

$$(m + n)d(xy) = 2mx \cdot d(y) + 2ny \cdot d(x) \tag{1}$$

for all $x, y \in A$. Here $m$ and $n$ are nonnegative integers with $m + n \neq 0$.

**Theorem 1.2.** *Let $A$ be a unital algebra with unit $e$ and $M$ be a left $A$-module. Assume that $m$ and $n$ are nonnegative integers with $m + n \neq 0$ and $m \neq n$. Then each module left $(m, n)$-derivation $d : A \to M$ is trival.*

Assume that $e \cdot x = x$ for all $x \in M$.

*Proof.* Letting $x = y = e$ in (1), we get $(m+n)d(e) = 2(m+n)d(e)$ and so $d(e) = 0$.

Letting $y = e$ in (1), we get

$$(m + n)d(x) = 2mx \cdot d(e) + 2nd(x) = 2nd(x)$$

for all $x \in A$.

Received by the editors November 08, 2016. Accepted February 03, 2017.
2010 *Mathematics Subject Classification.* 16W25, 16D60, 39B62.
*Key words and phrases.* normed algebra, Banach left $A$-module, module left $(m, n)$-derivation.
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Letting $x = e$ and replacing $y$ by $x$ in (1), we get

$$(m+n)d(x) = 2m \cdot d(x) + 2nx \cdot d(e) = 2md(x)$$

for all $x \in A$. So $2nd(x) = 2md(x)$ for all $x \in A$. Since $m \neq n$, $d(x) = 0$ for all $x \in A$, as desired. $\square$

**Remark 1.3.** When $m = n$, the module left $(m, n)$-derivation is just a module left derivation. In [2], Jung proved the Hyers-Ulam stability of module left derivations $d : A \to M$.

**Problem 1.4.** *Let $A$ be a non-unital algebra and $M$ be a left $A$-module. Assume that $m$ and $n$ are nonnegative integers with $m + n \neq 0$ and $m \neq n$.*

(1) *Is there a non-trivial module left $(m, n)$-derivation $d : A \to M$?*

(2) *Construct a non-trival module left $(m, n)$-derivation $d : A \to M$.*

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