J. Korean Soc. Math. Educ. Ser. B: Pure Appl. Math. ISSN(Print) 1226-0657
https://doi.org/10.7468/jksmeb.2018.25.2.115 ISSN(Online) 2287-6081
Volume 25, Number 2 (May 2018), Pages 115–125

STUDY ON CLEAN ORDERED RINGS DERIVED FROM CLEAN ORDERED KRASNER HYPERRINGS

Saber Omidi [a] and Bijan Davvaz [b,*]

Abstract. In this paper, we introduce the notion of a clean ordered Krasner hyperring and investigate some properties of it. Now, let $(R, +, \cdot, \leq)$ be a clean ordered Krasner hyperring. The following is a natural question to ask: Is there a strongly regular relation σ on R for which R/σ is a clean ordered ring? Our motivation to write the present paper is reply to the above question.

1. Introduction

The algebraic hyperstructure theory was first introduced by Marty [14] in 1934. Algebraic hyperstructures are a suitable generalization of classical algebraic structures. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. Several books have been written on this topic; here, we just mention the books of Corsini and Leoreanu [5], Davvaz [6], Davvaz and Leoreanu-Fotea [7] and Vougiouklis [21]. Marty introduced hypergroups as a generalization of groups. He published some notes on hypergroups, using them in different contexts: algebraic functions, rational fractions, non-commutative groups. The concept of an ordered semihypergroup was first given by Heidari and Davvaz [12]. The concept of ordered semihypergroups is a generalization of the concept of ordered semigroups. In 2015, Davvaz, Corsini and Changphas [10] introduced the concept of a pseudoorder relation in ordered semihypergroups. Using this notion, they obtained an ordered semigroup from an ordered semihypergroup. The work on ordered semihypergroup theory can be found in [10, 11, 17].

Let us introduce a background of our study. The notion of a clean ring was first

Received by the editors July 18, 2017. Accepted March 08, 2018.
2010 *Mathematics Subject Classification.* 16Y99.
Key words and phrases. clean element, clean ordered Krasner hyperring, pseudoorder.
*Corresponding author.

introduced by Nicholson [15] in 1977. Later on, Anderson and Camillo studied clean rings in more details in [3]. Let $(R, +, \cdot)$ be a ring with 1. Then R is *clean* if every $a \in R$ can be written as $a = u + e$, where u is an invertible and e is idempotent. There are different types of hyperrings. A well-known type of a hyperring, called the Krasner hyperring [13]. Let $(R, +, \cdot)$ be a commutative hyperring with identity in the sense of Krasner. Following Amouzegar and Talebi [2], an element a of a hyperring R is said to be *clean* if $a \in u + e$, where u is an invertible and e is idempotent. If every element of R is clean, then R is called a *clean hyperring*. We invite the readers to [1] to see more about the clean multiplicative hyperrings.

2. Basic Concepts

In this Section, we recall some notions that will be useful in the development of the paper.

A *Krasner hyperring* [9, 13] is an algebraic hypersructure $(R, +, \cdot)$ which satisfies the following axioms:

(1) $(R, +)$ is a canonical hypergroup, i.e., (i) for any $x, y, z \in R, x + (y + z) = (x + y) + z$, (ii) for any $x, y \in R, x + y = y + x$, (iii) there exists $0 \in R$ such that $0 + x = x + 0 = x$, for any $x \in R$, (iv) for every $x \in R$, there exists a unique element $x' \in R$, such that $0 \in x + x'$ (we shall write $-x$ for x' and we call it the opposite of x), (v) $z \in x + y$ implies that $y \in -x + z$ and $x \in z - y$, that is $(R, +)$ is reversible;

(2) $(R, \cdot)$ is a semigroup having zero as a bilaterally absorbing element, i.e., $x \cdot 0 = 0 \cdot x = 0$;

(3) The multiplication is distributive with respect to the hyperoperation $+$.

A Krasner hyperring $(R, +, \cdot)$ is called *commutative* if $(R, \cdot)$ is a commutative semigroup. A Krasner hyperring R is called with identity if there exists an element, say $1 \in R$, such that $1 \cdot x = x \cdot 1 = x$. An element x of a Krasner hyperring R is called a *unit* if there exists $y \in R$ such that $x \cdot y = y \cdot x = 1$. For the definitions of subhyperring and hyperideal of a Krasner hyperring, we refer to Section 2 of the paper [9] by Davvaz and Salasi.

Let σ be an equivalence relation on a Krasner hyperring $(R, +, \cdot)$. If A and B are non-empty subsets of R, then $A\overline{\overline{\sigma}}B$ means that for all $a \in A$ and for all $b \in B$, we have $a\sigma b$. An equivalence relation σ on R is said to be *strongly regular* if for all $a, b, x \in R$, we have (i) $a\sigma b \Rightarrow (a+x)\overline{\overline{\sigma}}(b+x)$; (ii) $a\sigma b \Rightarrow (a\cdot x)\overline{\overline{\sigma}}(b\cdot x)$ and $(x\cdot a)\overline{\overline{\sigma}}(x\cdot b)$.

Theorem 2.1. *Let $(R, +, \cdot)$ be a Krasner hyperring and σ an equivalence relation on R. If we define the following hyperoperations on the set of all equivalence classes with respect to σ, that is, $R/\sigma = \{\sigma(x) \mid x \in R\}$:*

$$\sigma(x) \oplus \sigma(y) = \{\sigma(z) \mid z \in x + y\},$$

$$\sigma(x) \odot \sigma(y) = \sigma(x \cdot y),$$

then σ is strongly regular if and only if $(R/\sigma, \oplus, \odot)$ is a ring.

In 2016, Omidi et al. [18] introduced the concept of ordered Krasner hyperrings and investigated some related properties, also see [20]. Recently, Davvaz and Omidi studied the notion of ordered (semi)hyperrings [8, 16, 19].

Definition 2.2 ([18])**.** Let $(R, +, \cdot)$ be a Krasner hyperring. We say that $(R, +, \cdot, \leq)$ is an *ordered Krasner hyperring* if the following axioms are fulfilled:

(1) $(R, \leq)$ is a partially ordered set.
(2) For every $a, b, c \in R$, $a \leq b$ implies $a + c \leq b + c$, meaning that for any $x \in a + c$, there exists $y \in b + c$ such that $x \leq y$.
(3) For every $a, b, c \in R$, $a \leq b$ and $0 \leq c$ imply $a \cdot c \leq b \cdot c$ and $c \cdot a \leq c \cdot b$.

3. Examples of Clean Ordered Krasner Hyperrings

Let $(R, +, \cdot, \leq)$ be a commutative ordered hyperring with identity in the sense of Krasner. Denote the set of all invertible elements in R by $U(R)$ and the set of all idempotent elements in R by $Id(R)$. We start with the following definition.

Definition 3.1. Let $(R, +, \cdot, \leq)$ be an ordered Krasner hyperring. Then an element $a \in R$ is said to be *clean* if $a \leq u + e$, where $u \in U(R)$ and $e \in Id(R)$. Also, we say that R is *clean ordered Krasner hyperring*, if all of elements in R are clean elements.

In the following, we present several examples of clean ordered Krasner hyperrings with different covering relations.

Example 3.2. Every clean Krasner hyperring induces a clean ordered Krasner hyperring. Indeed: Let $(R, +, \cdot)$ be a clean Krasner hyperring. Define the order on R by $\leq := \{(a, b) \mid a = b\}$. Then $(R, +, \cdot, \leq)$ is a clean ordered Krasner hyperring.

Example 3.3. Consider the hyperring $R = \{0, 1, -1\}$ with the hyperaddition $+$ and the multiplication $\cdot$ defined as follows:

$+$	0	1	-1
0	0	1	-1
1	1	1	R
-1	-1	R	-1

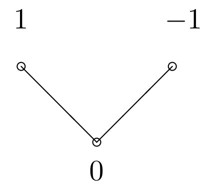
$\cdot$	0	1	-1
0	0	0	0
1	0	1	-1
-1	0	-1	1

We have $(R, +, \cdot, \leq)$ is an ordered Krasner hyperring, where the order relation $\leq$ is defined by:

$$\leq := \{(0,0), (1,1), (-1,-1), (0,1), (0,-1)\}.$$

The covering relation and the figure of R are given by:

$$\prec = \{(0,1), (0,-1)\}.$$



Now, it is easy to see that R is a clean ordered Krasner hyperring.

Example 3.4. Let $R = \{0, 1, a\}$. Consider the following tables:

$+$	0	1	a
0	0	1	a
1	1	R	1
a	a	1	$\{0,a\}$

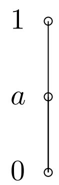
$\cdot$	0	1	a
0	0	0	0
1	0	1	a
a	0	a	0

Then $(R, +, \cdot)$ is a Krasner hyperring. We have $(R, +, \cdot, \leq)$ is an ordered Krasner hyperring where the order relation $\leq$ is defined by:

$$\leq := \{(0,0), (1,1), (a,a), (0,1), (0,a), (a,1)\}.$$

The covering relation and the figure of R are given by:

$$\prec = \{(0,a), (a,1)\}.$$



We can easily verify that R is a clean ordered Krasner hyperring.

Example 3.5. Let $R = \{0, a, b, c\}$ be a set with the hyperoperation $+$ and the multiplication $\cdot$ defined as follows:

$+$	0	a	b	c
0	0	a	b	c
a	a	$\{0, b\}$	$\{a, c\}$	b
b	b	$\{a, c\}$	$\{0, b\}$	a
c	c	b	a	0

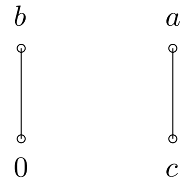
$\cdot$	0	a	b	c
0	0	0	0	0
a	0	a	b	c
b	0	b	b	0
c	0	c	0	c

Then $(R, +, \cdot)$ is a Krasner hyperring [4]. We have $(R, +, \cdot, \leq)$ is an ordered Krasner hyperring where the order relation $\leq$ is defined by:

$$\leq := \{(0, 0), (a, a), (b, b), (c, c), (0, b), (c, a)\}.$$

The covering relation and the figure of R are given by:

$$\prec = \{(0, b), (c, a)\}.$$



The following can easily be verified: $0 \leq a + c$, $a \leq b + c$, $b \leq a + c$ and $c \leq b + c$, where $a, b \in Id(R)$ and $c \in U(R)$. Hence, R is a clean ordered Krasner hyperring.

Example 3.6. Let $(R, +, \cdot, \leq)$ be a clean ordered Krasner hyperring. We consider

$$\mathbf{M} = \left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \mid a, b \in R \right\},$$

and define the hyperoperation $\boxplus$ and operation $\boxdot$ on $\mathbf{M}$ as

$$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \boxplus \begin{pmatrix} c & 0 \\ 0 & d \end{pmatrix} = \left\{ \begin{pmatrix} p & 0 \\ 0 & q \end{pmatrix} \mid p \in a + c, q \in b + d \right\},$$

$$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \boxdot \begin{pmatrix} c & 0 \\ 0 & d \end{pmatrix} = \begin{pmatrix} ac & 0 \\ 0 & bd \end{pmatrix},$$

where $A = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ and $B = \begin{pmatrix} c & 0 \\ 0 & d \end{pmatrix}$ are two arbitrary elements of $\mathbf{M}$. We define $A \preceq B$ if and only if $a \leq c$ and $b \leq d$. Then, $(\mathbf{M}, \boxplus, \boxdot, \preceq)$ is an ordered Krasner hyperring. Let $A = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \in \mathbf{M}$. Since R is clean, it follows that $a \leq u + e$ and

$b \le v+f$, where $u,v \in U(R)$ and $e,f \in Id(R)$. Thus there exist $x \in u+e$ and $y \in v+f$ such that $a \le x$ and $b \le y$. This means that

$$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \preceq \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} \in \begin{pmatrix} u & 0 \\ 0 & v \end{pmatrix} \boxplus \begin{pmatrix} e & 0 \\ 0 & f \end{pmatrix}$$

Now, we have $A \preceq U \boxplus E$, where $U = \begin{pmatrix} u & 0 \\ 0 & v \end{pmatrix} \in U(\mathbf{M})$ and $E = \begin{pmatrix} e & 0 \\ 0 & f \end{pmatrix} \in Id(\mathbf{M})$. Hence, $(\mathbf{M}, \boxplus, \boxdot, \preceq)$ is a clean ordered Krasner hyperring.

4. Main Results

Theorem 4.1. *Let $(R_i, +_i, \cdot_i, \le_i)$ be a clean ordered Krasner hyperring for all $i \in I$. Then $\prod_{i\in I} R_i = \{(r_i)_{i\in I} \mid r_i \in R_i\}$ is a clean ordered Krasner hyperring.*

Proof. For all $(x_i)_{i\in I}, (y_i)_{i\in I} \in \prod_{i\in I} R_i$ we define

(1) $(x_i)_{i\in I} + (y_i)_{i\in I} = \{(z_i)_{i\in I} \mid z_i \in x_i +_i y_i\}$,

(2) $(x_i)_{i\in I} \cdot (y_i)_{i\in I} = (x_i \cdot_i y_i)_{i\in I}$,

(3) $(x_i)_{i\in I} \le (y_i)_{i\in I}$ if and only if $x_i \le_i y_i$ for all $i \in I$.

First we show that $(\prod_{i\in I} R_i, +, \cdot, \le)$ is an ordered Krasner hyperring. Suppose that $(x_i)_{i\in I} \le (y_i)_{i\in I}$ for $(x_i)_{i\in I}, (y_i)_{i\in I} \in \prod_{i\in I} R_i$. If $(t_i)_{i\in I} \in (a_i)_{i\in I} + (x_i)_{i\in I}$, where $(a_i)_{i\in I} \in \prod_{i\in I} R_i$, then $t_i \in a_i +_i x_i$. Since $(x_i)_{i\in I} \le (y_i)_{i\in I}$, it follows that $x_i \le_i y_i$ for all $i \in I$. By hypothesis, we have $t_i \in a_i +_i x_i \le_i a_i +_i y_i$. So there exists $s_i \in a_i +_i y_i$ such that $t_i \le_i s_i$. Thus we have $(t_i)_{i\in I} \le (s_i)_{i\in I}$. This implies that $(a_i)_{i\in I} + (x_i)_{i\in I} \le (a_i)_{i\in I} + (y_i)_{i\in I}$. Also, we have $a_i \cdot_i x_i \le_i a_i \cdot_i y_i$, where $(0) \le (a_i)_{i\in I}$. This means that $(a_i)_{i\in I} \cdot (x_i)_{i\in I} \le (a_i)_{i\in I} \cdot (y_i)_{i\in I}$. Therefore, $\prod_{i\in I} R_i$ is an ordered Krasner hyperring.

Now, let $\{R_i\}_{i\in I}$ be clean for each $i \in I$ and $(a_i)_{i\in I} \in \prod_{i\in I} R_i$. We have $a_i \le_i u_i +_i e_i$, where $u_i \in U(R_i)$ and $e_i \in Id(R_i)$. Thus there exists $b_i \in u_i +_i e_i$ such that $a_i \le_i b_i$ This implies that $(a_i)_{i\in I} \le (b_i)_{i\in I}$, where $(b_i)_{i\in I} \in (u_i)_{i\in I} + (e_i)_{i\in I}$. Then $(a_i)_{i\in I} \le (u_i)_{i\in I} + (e_i)_{i\in I}$, where $(u_i)_{i\in I} \in U(\prod_{i\in I} R_i)$ and $(e_i)_{i\in I} \in Id(\prod_{i\in I} R_i)$. Hence, $\prod_{i\in I} R_i$ is a clean ordered Krasner hyperring. $\square$

Let $(R, +, \cdot, \le)$ and $(T, \boxplus, \boxdot, \preceq)$ be two ordered Krasner hyperring. A map $\varphi : R \to T$ is called a *homomorphism* if for all a, b in R: (1) $\varphi(a+b) \subseteq \varphi(a) \boxplus \varphi(b)$; (2)

$\varphi(a \cdot b) = \varphi(a) \boxdot \varphi(b)$ and (3) φ is isotone, that is, for any $a, b \in R$, $a \leq b$ implies $\varphi(a) \preceq \varphi(b)$.

Theorem 4.2. *Any homomorphic image of a clean ordered Krasner hyperring is a clean ordered Krasner hyperring.*

Proof. Suppose that φ is a surjective homomorphism from an ordered Krasner hyperring $(R, +, \cdot, \leq)$ into an ordered Krasner hyperring $(T, \boxplus, \boxdot, \preceq)$. Take any $t \in T$; then there exists $x \in R$ such that $\varphi(x) = t$. Since R is clean, we have $x \leq u + e$, where $u \in U(R)$ and $e \in Id(R)$. Thus there exists $y \in u + e$ such that $x \leq y$. So, we have

$$\varphi(x) \preceq \varphi(y) \in \varphi(u + e) \subseteq \varphi(u) \boxplus \varphi(e),$$

where $\varphi(u) \in U(T)$ and $\varphi(e) \in Id(T)$. This completes the proof. □

Theorem 4.3. *A clean ordered Krasner hyperring* $(R, +, \cdot, \leq)$ *is a clean ordered ring if and only if* $1 + (-1) = \{0\}$.

Proof. The necessity follows easily, so that we will concentrate on the sufficiency. To that aim, Suppose that $x, y \in R$. Let $u, v \in x + y$. Then we have

$$\begin{aligned} u - v \ &\subseteq (a + b) - (a + b) \\ &= (a + b) - a - b \\ &= (a + (-a)) + (b + (-b)) \\ &= a \cdot (1 + (-1)) + b \cdot (1 + (-1)) \\ &= a \cdot \{0\} + b \cdot \{0\} \\ &= 0 + 0 \\ &= \{0\}. \end{aligned}$$

Thus $u - v = \{0\}$ and hence $u = v$. It follows that $a + b = \{u\}$, and so $+$ is a binary operation. Therefore, $(R, +, \cdot, \leq)$ is an ordered ring. By hypothesis, every $a \in R$ can be written as $a \leq u + e$, where $u \in U(R)$, $e \in Id(R)$ and $u + e$ is a sigelton set. Thus R is a clean ordered ring. □

The concept of pseudoorder on an ordered semihypergroup $(S, \circ, \leq)$ was introduced and studied by Davvaz et al. [10]. Now, we extend this notion for ordered Krasner hyperrings.

Definition 4.4. Let $(R, +, \cdot, \leq)$ be an ordered Krasner hyperring. A relation σ on R is called *pseudoorder* if the following conditions hold:

(1) $\leq \subseteq \sigma$;

(2) $a\sigma b$ and $b\sigma c$ imply $a\sigma c$;
(3) $a\sigma b$ implies $a + c\overline{\overline{\sigma}}b + c$, for all $c \in R$;
(4) $a\sigma b$ implies $a \cdot c\sigma b \cdot c$, for all $c \in R$.

We now give the main result of this paper as bellow.

Theorem 4.5. *Let $(R, +, \cdot, \leq)$ be a clean ordered Krasner hyperring and σ a pseudoorder on R. Then, there exists a strongly regular equivalence relation $\sigma^* = \{(a, b) \in R \times R \mid a\sigma b \text{ and } b\sigma a\}$ on R such that $(R/\sigma^*, \oplus, \odot, \preceq_{\sigma^*})$ is a clean ordered ring, where $\preceq_{\sigma^*} := \{(\sigma^*(x), \sigma^*(y)) \in R/\sigma^* \times R/\sigma^* \mid \exists a \in \sigma^*(x), \exists b \in \sigma^*(y) \text{ such that } (a, b) \in \sigma\}$.*

Proof. We divide the proof into three steps.

STEP 1. We first construct an ordered ring from an ordered Krasner hyperring.

Suppose that σ^* is the relation on R defined as follows:

$$\sigma^* = \{(a, b) \in R \times R \mid a\sigma b \text{ and } b\sigma a\}.$$

Clearly, σ^* is a strongly regular relation on $(R, +)$ and $(R, \cdot)$. Hence, By Theorem 2.1, R/σ^* with the following operations is a ring:

$$\sigma^*(x) \oplus \sigma^*(y) = \sigma^*(z), \text{ for all } z \in x + y;$$

$$\sigma^*(x) \odot \sigma^*(y) = \sigma^*(x \cdot y).$$

Now, for each $\sigma^*(x), \sigma^*(y) \in R/\sigma^*$, define the order relation $\preceq_{\sigma^*}$ on R/σ^* by:

$$\preceq_{\sigma^*} := \{(\sigma^*(x), \sigma^*(y)) \in R/\sigma^* \times R/\sigma^* \mid \exists a \in \sigma^*(x), \exists b \in \sigma^*(y) \text{ such that } (a, b) \in \sigma\}.$$

We have

$$\sigma^*(x) \preceq_{\sigma^*} \sigma^*(y) \Leftrightarrow x\sigma y.$$

Now, we prove that $(R/\sigma^*, \oplus, \odot, \preceq_{\sigma^*})$ is an ordered ring. Let $a, b, c \in R$. Since $(a, a) \in \leq \subseteq \sigma$, we have $\sigma^*(a) \preceq_{\sigma^*} \sigma^*(a)$. If $\sigma^*(a) \preceq_{\sigma^*} \sigma^*(b)$ and $\sigma^*(b) \preceq_{\sigma^*} \sigma^*(a)$, then $(a, b) \in \sigma$ and $(b, a) \in \sigma$. This means that $(a, b) \in \sigma^*$, and so $\sigma^*(a) = \sigma^*(b)$. Let $\sigma^*(a) \preceq_{\sigma^*} \sigma^*(b)$ and $\sigma^*(b) \preceq_{\sigma^*} \sigma^*(c)$. Then, $(a, b) \in \sigma$ and $(b, c) \in \sigma$. This means that $(a, c) \in \sigma$, and so $\sigma^*(a) \preceq_{\sigma^*} \sigma^*(c)$. Therefore, $\preceq_{\sigma^*}$ is an order on R/σ^*. Now, let $\sigma^*(x) \preceq_{\sigma^*} \sigma^*(y)$ and $\sigma^*(z) \in R/\sigma^*$. Then $x\sigma y$ and $z \in R$. Since σ is a pseudoorder on R, we have $x + z\overline{\overline{\sigma}}y + z$. So, for all $a \in x + z$ and $b \in y + z$, we have $a\sigma b$. This implies that $\sigma^*(a) \preceq_{\sigma^*} \sigma^*(b)$. Hence, $\sigma^*(x) \oplus \sigma^*(z) \preceq_{\sigma^*} \sigma^*(y) \oplus \sigma^*(z)$. Similarly, we have $\sigma^*(x) \odot \sigma^*(z) \preceq_{\sigma^*} \sigma^*(y) \odot \sigma^*(z)$.

STEP 2. The following hold for an ordered Krasner hyperring R:

(1) If $e \in Id(R)$, then $\sigma^*(e) \in Id(R/\sigma^*)$.

(2) If $u \in U(R)$, then $\sigma^*(u) \in U(R/\sigma^*)$.

STEP 3. We finally show that R/σ^* is clean.

Suppose that $(R, +, \cdot, \leq)$ is a clean ordered Krasner hyperring. Let $\sigma^*(a) \in R/\sigma^*$, where $a \in R$. Since R is clean, there exist $u \in U(R)$ and $e \in Id(R)$ such that $a \leq u + e$. Hence, there exists $x \in u + e$ such that $a \leq x$. So, $(a, x) \in \leq \subseteq \sigma$. Thus, $a\sigma x$. Since $a \in \sigma^*(a)$ and $x \in \sigma^*(x)$, we have $\sigma^*(a) \preceq_{\sigma^*} \sigma^*(x)$. Since $\sigma^*(x) \in \sigma^*(u) \oplus \sigma^*(e)$, it follows that $\sigma^*(a) \preceq_{\sigma^*} \sigma^*(u) \oplus \sigma^*(e)$. Now, by pervious step, $\sigma^*(a)$ is clean. Hence R/σ^* is clean. □

The following example illustrates this result.

Example 4.6. Let $(R, +, \cdot, \leq)$ be the clean ordered Krasner hyperring defined as in Example 3.5. Consider the pseudoorder

$$\sigma = \{(0,0), (a,a), (b,b), (c,c), (0,b), (b,0), (a,c), (c,a)\}.$$

Note that $\sigma^* = \sigma$, and that

$$R/\sigma^* = \{u_1, u_2\}, \text{ where } u_1 = \{0, b\} \text{ and } u_2 = \{a, c\}.$$

Now, $(R/\sigma^*, \oplus, \odot, \preceq_{\sigma^*})$ is a clean ordered ring, where $\oplus$ and $\odot$ are defined in the following tables:

$\oplus$	u_1	u_2
u_1	u_1	u_2
u_2	u_2	u_1

$\odot$	u_1	u_2
u_1	u_1	u_1
u_2	u_1	u_2

and $\preceq_{\sigma^*} = \{(u_1, u_1), (u_2, u_2)\}$.

An element x of an ordered Krasner hyperring $(R, +, \cdot, \leq)$ is said to be *regular* if there exists an element $a \in R$ such that $x \leq x \cdot a \cdot x$. An ordered Krasner hyperring R is said to be regular if every element of R is regular.

Corollary 4.7. *Let us follow the notations and definitions used in the Theorem 4.5. If R is regular, then R/σ^* is regular.*

Proof. Let R be regular and $\sigma^*(x) \in R/\sigma^*$, where $x \in R$. Then there exists $a \in R$ such that $x \leq x \cdot a \cdot x$. Clearly, $\leq \subseteq \sigma$, so $x\sigma x \cdot a \cdot x$. Since $x \in \sigma^*(x)$ and $x \cdot a \cdot x \in \sigma^*(x \cdot a \cdot x)$, clearly, we obtain $\sigma^*(x) \preceq_{\sigma^*} \sigma^*(x \cdot a \cdot x)$. This shows that $\sigma^*(x) \preceq_{\sigma^*} \sigma^*(x) \odot \sigma^*(a) \odot \sigma^*(x)$, so R/σ^* is regular. □

References

1. R. Ameri & A. Kordi: Clean multiplicative hyperrings. Ital. J. Pure Appl. Math. **35** (2015), 625-636.
2. T. Amouzegar & Y. Talebi: On clean hyperrings. Journal of Hyperstructures **4** (2015), no. 1, 1-10.
3. D.D. Anderson & V.P. Camillo: Commutative rings whose elements are a sum of a unit and an idempotent. Comm. Algebra **30** (2002), no. 7, 3327-3336.
4. A. Asokkumar: Derivations in hyperrings and prime hyperrings. Iran. J. Math. Sci. Inform. **8** (2013), no. 1, 1-13.
5. P. Corsini & V. Leoreanu: Applications of hyperstructure theory. Advances in Mathematics, Kluwer Academic Publishers, Dordrecht, 2003.
6. B. Davvaz: Semihypergroup theory. Elsevier, 2016.
7. B. Davvaz & V. Leoreanu-Fotea: Hyperring theory and applications. International Academic Press, USA, 2007.
8. B. Davvaz & S. Omidi: Basic notions and properties of ordered semihyperrings. Categ. General Alg. Structures Appl. **4** (2016), no. 1, 43-62.
9. B. Davvaz & A. Salasi: A realization of hyperrings. Comm. Algebra **34** (2006), no. 12, 4389-4400.
10. B. Davvaz, P. Corsini & T. Changphas: Relationship between ordered semihypergroups and ordered semigroups by using pseudoorder. European J. Combinatorics **44** (2015), 208-217.
11. Z. Gu & X. Tang: Ordered regular equivalence relations on ordered semihypergroups. J. Algebra **450** (2016), 384-397.
12. D. Heidari & B. Davvaz: On ordered hyperstructures. Politehn. Univ. Bucharest Sci. Bull. Ser. A Appl. Math. Phys. **73** (2011), no. 2, 85-96.
13. M. Krasner: A class of hyperrings and hyperfields. Intern. J. Math. and Math Sci. **6** (1983), no. 2, 307-312.
14. F. Marty: Sur une generalization de la notion de groupe. 8^{iem} Congres Math. Scandinaves, Stockholm, 1934, 45-49.
15. W.K. Nicholson: Lifting idempotent and exchange rings. Trans. Amer. Math. Soc. **229** (1977), 269-278.
16. S. Omidi & B. Davvaz: Foundations of ordered (semi)hyperrings. J. Indones. Math. Soc. **22** (2016), no. 2, 129-147.
17. S. Omidi & B. Davvaz: A short note on the relation $\mathcal{N}$ in ordered semihypergroups. GU J. Sci. **29** (2016), no. 3, 659-662.
18. S. Omidi, B. Davvaz & P. Corsini: Operations on hyperideals in ordered Krasner hyperrings. An. St. Univ. Ovidius Constanta, Ser. Mat. **24** (2016), no. 3, 275-293.

19. S. Omidi & B. Davvaz: Contribution to study special kinds of hyperideals in ordered semihyperrings. J. Taibah Univ. Sci. **11** (2017), 1083-1094.
20. S. Omidi & B. Davvaz: On ordered Krasner hyperrings. Iranian Journal of Mathematical Sciences and Informatics **12** (2017), no. 2, 35-49.
21. T. Vougiouklis: Hyperstructures and their representations. Hadronic Press, Florida, 1994.

[a]Department of Mathematics, Yazd University, Yazd, Iran
Email address: omidi.saber@yahoo.com

[b]Department of Mathematics, Yazd University, Yazd, Iran
Email address: davvaz@yazd.ac.ir