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β-G-BICONTINUOUS, β-G-COMPACTNESS AND β-G-STABLE IN DITOPOLOGICAL TEXTURE SPACES

Hariwan Z. Ibrahim

Abstract. The purpose of this paper is to introduce and study new notions of continuity, compactness and stability in ditopological texture spaces based on the notions of β-g-open and β-g-closed sets and some of their characterizations are obtained. Finally, the relationships between these concepts and the other related concepts are investigated.

1. Introduction

Textures and ditopological texture spaces were first introduced by L. M. Brown as a point-based setting for the study of fuzzy topology. α-open sets, pre-open sets, semi-open sets, b-open sets, β-open sets and g-closed sets in ditopological texture spaces were studied by [14], [12], [13], [7], [15] and [6], respectively. The study of compactness and stability in ditopological texture spaces was started to begin in [11]. The notions of α-g-closed, α-g-open, pre-g-closed, pre-g-open, semi-g-closed, semi-g-open, b-g-closed, b-g-open, α-g-bicontinuous, pre-g-bicontinuous, semi-g-bicontinuous, b-g-bicontinuous, α-g-compact, pre-g-compact, semi-g-compact, b-g-compact, α-g-stable, pre-g-stable, semi-g-stable and b-g-stable in ditopological texture spaces were introduced in [2], [3], [4] and [5].

2. Preliminaries

The following are some basic definitions of textures.

Texture space ([11])**:** Let S be a set. Then $\varphi \subseteq P(S)$ is called a *texturing* of S, and S is said to be *textured* by φ if

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(1) $(\varphi, \subseteq)$ is a complete lattice containing S and ϕ and for any index set I and $A_i \in \varphi$, $i \in I$, the meet $\bigwedge_{i\in I} A_i$ and the join $\bigvee_{i\in I} A_i$ in φ are related with the intersection and union in $P(S)$ by the equalities
$\bigwedge_{i\in I} A_i = \bigcap_{i\in I} A_i$
for all I, while
$\bigvee_{i\in I} A_i = \bigcup_{i\in I} A_i$
for all finite I.

(2) φ is completely distributive;

(3) φ separates the points of S. That is, given $s_1 \neq s_2$ in S we have $L \in \varphi$ with $s_1 \in L$, $s_2 \notin L$, or $L \in \varphi$ with $s_2 \in L$, $s_1 \notin L$.

If S is textured by φ then (S, φ) is called a *texture space*, or simply a texture.

Complementation ([11])**:** A mapping $\sigma : \varphi \to \varphi$ satisfying $\sigma(\sigma(A)) = A$, $\forall A \in \varphi$ and $A \subseteq B \Rightarrow \sigma(B) \subseteq \sigma(A)$, $\forall A, B \in \varphi$ is called a *complementation* on (S, φ) and (S, φ, σ) is then said to be a *complemented texture*.

For a texture (S, φ), most properties are conveniently defined in terms of the p-sets

$$P_s = \bigcap\{A \in \varphi : s \in A\}$$

and the q-sets,

$$Q_s = \bigvee\{A \in \varphi : s \notin A\}.$$

Ditopology ([11])**:** A dichotomous topology on a texture (S, φ), or ditopology for short, is a pair (τ, k) of subsets of φ, where the set of open sets τ satisfies

(1) $S, \phi \in \tau$,

(2) $G_1, G_2 \in \tau \Rightarrow G_1 \cap G_2 \in \tau$, and

(3) $G_i \in \tau, i \in I \Rightarrow \bigvee_i G_i \in \tau$,

and the set of closed sets k satisfies

(1) $S, \phi \in k$,

(2) $K_1, K_2 \in k \Rightarrow K_1 \cup K_2 \in k$, and

(3) $K_i \in k, i \in I \Rightarrow \bigcap K_i \in k$.

Hence a ditopology is essentially a "topology" for which there is no a priori relation between the open and closed sets.

For $A \in \varphi$ we define the closure $[A]$ and the interior $]A[$ of A under (τ, k) by the equalities

$[A] = \bigcap\{K \in k : A \subseteq K\}$ and $]A[= \bigvee\{G \in \tau : G \subseteq A\}$.
We refer to τ as the topology and k as the cotopology of (τ, k).

If (τ, k) is a ditopology on a complemented texture (S, φ, σ), then we say that (τ, k) is complemented if the equality $k = \sigma(\tau)$ is satisfied. In this study, a complemented ditopological texture space is denoted by $(S, \varphi, \tau, k, \sigma)$.
In this case we have $\sigma([A]) =]\sigma(A)[$ and $\sigma(]A[) = [\sigma(A)]$.

We denote by $O(S, \varphi, \tau, k)$, or when there can be no confusion by $O(S)$, the set of open sets in φ. Likewise, $C(S, \varphi, \tau, k)$, $C(S)$ will denote the set of closed sets.

Let (S_1, φ_1) and (S_2, φ_2) be textures. In the following definition we consider the product texture [8] $P(S_1) \otimes \varphi_2$, and denote by $\overline{P}_{(s,t)}$, $\overline{Q}_{(s,t)}$, respectively the p-sets and q-sets for the product texture $(S_1 \times S_2, P(S_1) \otimes \varphi_2)$.

Direlation ([10])**:** Let (S_1, φ_1) and (S_2, φ_2) be textures. Then

(1) $r \in P(S_1) \otimes \varphi_2$ is called a relation from (S_1, φ_1) to (S_2, φ_2) if it satisfies
 R1 $r \not\subseteq \overline{Q}_{(s,t)}$, $P_{s'} \not\subseteq Q_s \Rightarrow r \not\subseteq \overline{Q}_{(s',t)}$.
 R2 $r \not\subseteq \overline{Q}_{(s,t)} \Rightarrow \exists s' \in S_1$ such that $P_s \not\subseteq Q_{s'}$ and $r \not\subseteq \overline{Q}_{(s',t)}$.
(2) $R \in P(S_1) \otimes \varphi_2$ is called a *corelation* from (S_1, φ_1) to (S_2, φ_2) if it satisfies
 CR1 $\overline{P}_{(s,t)} \not\subseteq R$, $P_s \not\subseteq Q_{s'} \Rightarrow \overline{P}_{(s',t)} \not\subseteq R$.
 CR2 $\overline{P}_{(s,t)} \not\subseteq R \Rightarrow \exists s' \in S_1$ such that $P_{s'} \not\subseteq Q_s$ and $\overline{P}_{(s',t)} \not\subseteq R$.
(3) A pair (r, R), where r is a relation and R a corelation from (S_1, φ_1) to (S_2, φ_2) is called a *direlation* from (S_1, φ_1) to (S_2, φ_2).

One of the most useful notions of (ditopological) texture spaces is that of difunction. A difunction is a special type of direlation.

Difunctions ([10])**:** Let (f, F) be a direlation from (S_1, φ_1) to (S_2, φ_2). Then (f, F) is called a *difunction* from (S_1, φ_1) to (S_2, φ_2) if it satisfies the following two conditions.
DF1 For $s, s' \in S_1$, $P_s \not\subseteq Q_{s'} \Rightarrow \exists t \in S_2$ such that $f \not\subseteq \overline{Q}_{(s,t)}$ and $\overline{P}_{(s',t)} \not\subseteq F$.
DF2 For $t, t' \in S_2$ and $s \in S_1$, $f \not\subseteq \overline{Q}_{(s,t)}$ and $\overline{P}_{(s,t')} \not\subseteq F \Rightarrow P_{t'} \not\subseteq Q_t$.

Image and Inverse Image ([10])**:** Let $(f, F) : (S_1, \varphi_1) \to (S_2, \varphi_2)$ be a difunction.

(1) For $A \in \varphi_1$, the image $f^{\to}A$ and the co-image $F^{\to}A$ are defined by

$$f^{\to}A = \bigcap\{Q_t : \forall s, f \not\subseteq \overline{Q}_{(s,t)} \Rightarrow A \subseteq Q_s\},$$
$$F^{\to}A = \bigvee\{P_t : \forall s, \overline{P}_{(s,t)} \not\subseteq F \Rightarrow P_s \subseteq A\}.$$

(2) For $B \in \varphi_2$, the inverse image $f^{\leftarrow}B$ and the inverse co-image $F^{\leftarrow}B$ are defined by

$$f^{\leftarrow}B = \bigvee\{P_s : \forall t, f \not\subseteq \overline{Q}_{(s,t)} \Rightarrow P_t \subseteq B\},$$
$$F^{\leftarrow}B = \bigcap\{Q_s : \forall t, \overline{P}_{(s,t)} \not\subseteq F \Rightarrow B \subseteq Q_t\}.$$

For a difunction, the inverse image and the inverse co-image are equal, but the image and co-image are usually not.

Bicontinuity ([9])**:** The difunction $(f, F) : (S_1, \varphi_1, \tau_1, k_1) \to (S_2, \varphi_2, \tau_2, k_2)$ is called *continuous* if $B \in \tau_2 \Rightarrow F^{\leftarrow}B \in \tau_1$, *cocontinuous* if $B \in k_2 \Rightarrow f^{\leftarrow}B \in k_1$, and *bicontinuous* if it is both continuous and cocontinuous.

Surjective difunction ([10])**:** Let $(f, F) : (S_1, \varphi_1) \to (S_2, \varphi_2)$ be a difunction. Then (f, F) is called *surjective* if it satisfies the condition
SUR. For $t, t' \in S_2$, $P_t \not\subseteq Q_{t'} \Rightarrow \exists s \in S_1$ with $f \not\subseteq \overline{Q}_{(s,t')}$ and $\overline{P}_{(s,t)} \not\subseteq F$.

If (f, F) is surjective then $F^{\to}(f^{\leftarrow}B) = B = f^{\to}(F^{\leftarrow}B)$ for all $B \in \varphi_2$ [[10], Corollary 2.33]

Definition 2.1 ([10])**.** Let (f, F) be a difunction between the complemented textures $(S_1, \varphi_1, \sigma_1)$ and $(S_2, \varphi_2, \sigma_2)$. The complement $(f, F)' = (F', f')$ of the difunction (f, F) is a difunction, where $f' = \bigcap\{\overline{Q}_{(s,t)} | \exists u, v$ with $f \not\subseteq \overline{Q}_{u,v}, \sigma_1(Q_s) \not\subseteq Q_u$ and $P_v \not\subseteq \sigma_2(P_t)\}$ and $F' = \bigvee\{\overline{P}_{(s,t)} | \exists u, v$ with $\overline{P}_{u,v} \not\subseteq F, P_u \not\subseteq \sigma_1(P_s)$ and $\sigma_2(Q_t) \not\subseteq Q_v\}$.

If $(f, F) = (f, F)'$ then the difunction (f, F) is called complemented.

Definition 2.2. Let $(S, \varphi, \tau, k, \sigma)$ be a ditopological texture space. A set $A \in \varphi$ is called:

(1) *α-open (α-closed)* ([14]) if $A \subseteq][A][[$ $([][A][] \subseteq A)$.
(2) *pre-open (pre-closed)* ([12]) if $A \subseteq][A][$ $([A[] \subseteq A)$.
(3) *sem-iopen (semi-closed)* ([13]) if $A \subseteq []A[]$ $(][A][\subseteq A)$.
(4) *b-open (b-closed)* ([7]) if $A \subseteq][A][\cup[]A[]$ $(][A][\cap[]A[] \subseteq A)$.
(5) *β-open (β-closed)* ([15]) if $A \subseteq [][A][]$ $(][]A[][\subseteq A)$.

We denote by $O_\alpha(S, \varphi, \tau, k)$ (resp. $PO(S, \varphi, \tau, k)$, $SO(S, \varphi, \tau, k)$, $bO(S, \varphi, \tau, k)$ and $\beta O(S, \varphi, \tau, k)$), or when there can be no confusion by $O_\alpha(S)$ (resp. $PO(S)$, $SO(S)$, $bO(S)$ and $\beta O(S)$), the set of α-open (resp. pre-open, semi-open, b-open and β-open) sets in φ. Likewise, $C_\alpha(S, \varphi, \tau, k)$ (resp. $PC(S, \varphi, \tau, k)$, $SC(S, \varphi, \tau, k)$, $bC(S, \varphi, \tau, k)$ and $\beta C(S, \varphi, T, k)$), or $C_\alpha(S)$ (resp. $PC(S)$, $SC(S)$, $bC(S)$ and $\beta C(S)$)

will denote the set of α-closed (resp. pre-closed, semi-closed, b-closed and β-closed) sets.

Definition 2.3 ([6]). Let (S, φ, τ, k) be a ditopological texture space. A subset A of a texture φ is said to be *generalized closed* (g-closed for short) if $A \subseteq G \in \tau$ then $[A] \subseteq G$.

Definition 2.4 ([6]). Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space. A subset A of a texture φ is said to be *generalized open* (g-open for short) if $\sigma(A)$ is g-closed.

We denote by $gc(S, \varphi, \tau, k)$, or when there can be no confusion by $gc(S)$, the set of g-closed sets in φ. Likewise, $go(S, \varphi, \tau, k, \sigma)$, or $go(S)$ will denote the set of g-open sets.

Definition 2.5 ([1]). Let (S, φ, τ, k) be a ditopological texture space. A subset A of a texture φ is said to be *α-g-closed* (resp. *pre-g-closed, semi-g-closed, b-g-closed* and *β-g-closed*) if $A \subseteq G \in O_\alpha(S)$ (resp. $A \subseteq G \in PO(S)$, $A \subseteq G \in SO(S)$, $A \subseteq G \in bO(S)$ and $A \subseteq G \in \beta O(S)$) then $[A] \subseteq G$.

We denote by $\alpha gc(S, \varphi, \tau, k)$ (resp. pre$gc(S, \varphi, \tau, k)$, semi$gc(S, \varphi, \tau, k)$, b$gc(S, \varphi, \tau, k)$ and $\beta gc(S, \varphi, \tau, k)$), or when there can be no confusion by $\alpha gc(S)$ (resp. pre$gc(S)$, semi$gc(S)$, b$gc(S)$ and $\beta gc(S)$), the set of α-g-closed (resp. pre-g-closed, semi-g-closed, b-g-closed and β-g-closed) sets in φ.

Definition 2.6 ([1]). Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space. A subset A of a texture φ is called *α-g-open* (resp. *pre-g-open, semi-g-open, b-g-open* and *β-g-open*) if $\sigma(A)$ is α-g-closed (resp. pre-g-closed, semi-g-closed, b-g-closed and β-g-closed).

We denote by $\alpha go(S, \varphi, \tau, k, \sigma)$ (resp. pre$go(S, \varphi, \tau, k, \sigma)$, semi$go(S, \varphi, \tau, k, \sigma)$, b$go(S, \varphi, \tau, k, \sigma)$ and $\beta go(S, \varphi, \tau, k, \sigma)$), or when there can be no confusion by $\alpha go(S)$ (resp. pre$go(S)$, semi$go(S)$, b$go(S)$ and $\beta go(S)$), the set of α-g-open (resp. pre-g-open, semi-g-open, b-g-open and β-g-open) sets in φ.

Definition 2.7 ([1]). Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space. For $A \in \varphi$, we define the β-g-closure $[A]_{\beta-g}$ and the β-g-interior $]A[_{\beta-g}$ of A under (τ, k) by the equalities

$$[A]_{\beta-g} = \bigcap\{K \in \beta gc(S) : A \subseteq K\} \text{ and }]A[_{\beta-g} = \bigcup\{G \in \beta go(S) : G \subseteq A\}.$$

Definition 2.8. The difunction $(f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ is called:

(1) *α-g-continuous* ([2]) (resp. pre-g-continuous ([3]), semi-g-continuous ([4]) and b-g-continuous ([5]), if $F^{\leftarrow}(G) \in \alpha go(S_1)$ (resp. pre$go(S_1)$, semi$go(S_1)$ and b$go(S_1)$), for every $G \in O(S_2)$.

(2) *α-g-cocontinuous* ([2]) (resp. *pre-g-cocontinuous (*[3]*), semi-g-cocontinuous* ([4]) and *b-g-cocontinuous* ([5]), if $f^{\leftarrow}(G) \in \alpha gc(S_1)$ (resp. pre$gc(S_1)$, semi$gc(S_1)$ and b$gc(S_1)$), for every $G \in k_2$.

(3) *α-g-bicontinuous* ([2]), if it is α-g-continuous and α-g-cocontinuous.

(4) *pre-g-bicontinuous* ([3]), if it is pre-g-continuous and pre-g-cocontinuous.

(5) *semi-g-bicontinuous* ([4]), if it is semi-g-continuous and semi-g-cocontinuous.

(6) *b-g-bicontinuous* ([5]), if it is b-g-continuous and b-g-cocontinuous.

Definition 2.9. A complemented ditopological texture space $(S, \varphi, \tau, k, \sigma)$ is called *α-g-compact* ([2]) (resp. *pre-g-compact* ([3]), *semi-g-compact* ([4]), *b-g-compact* ([5]) and *g-compact* ([6])) if every cover of S by α-g-open (resp. pre-g-open, semi-g-open, b-g-open and g-open) has a finite subcover.

Definition 2.10. A complemented ditopological texture space $(S, \varphi, \tau, k, \sigma)$ is called *α-g-stable* ([2]) (resp. *pre-g-stable* ([3]), *semi-g-stable* ([4]), *b-g-stable* ([5]) and *g-stable* ([6])) if every α-g-closed (resp. pre-g-closed, semi-g-closed, b-g-closed and g-closed) set $F \in \varphi \setminus \{S\}$ is α-g-compact (resp. pre-g-compact, semi-g-compact, b-g-compact and g-compact) in S.

3. β-g-bicontinuous, β-g-bi-irresolute, β-g-compact and β-g-stable

Definition 3.1. The difunction $(f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ is called:

(1) *β-g-continuous* (*β-g-irresolute*), if $F^{\leftarrow}(G) \in \beta go(S_1)$, for every $G \in O(S_2)$ ($G \in \beta go(S_2)$).

(2) *β-g-cocontinuous* (*β-g-co-irresolute*), if $f^{\leftarrow}(G) \in \beta gc(S_1)$, for every $G \in k_2$ ($G \in \beta gc(S_2)$).

(3) *β-g-bicontinuous*, if it is β-g-continuous and β-g-cocontinuous.

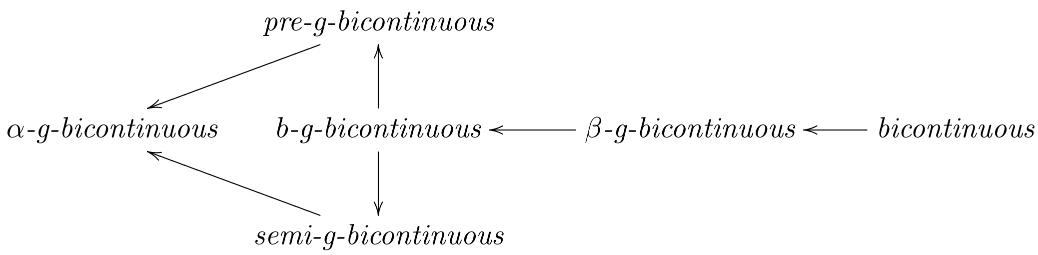
(4) *β-g-bi-irresolute*, if it is β-g-irresolute and β-g-co-irresolute.

Theorem 3.2. *Let* $(f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ *be a difunction. Then:*

(1) Every continuous is β-g-continuous.
(2) Every cocontinuous is β-g-cocontinuous.
(3) Every bicontinuous is β-g-bicontinuous.
(4) Every β-g-continuous is b-g-continuous.
(5) Every β-g-cocontinuous is b-g-cocontinuous.
(6) Every β-g-bicontinuous is b-g-bicontinuous.
(7) Every b-g-continuous is pre-g-continuous.
(8) Every b-g-cocontinuous is pre-g-cocontinuous.
(9) Every b-g-bicontinuous is pre-g-bicontinuous.
(10) Every b-g-continuous is semi-g-continuous.
(11) Every b-g-cocontinuous is semi-g-cocontinuous.
(12) Every b-g-bicontinuous is semi-g-bicontinuous.
(13) Every pre-g-continuous is α-g-continuous.
(14) Every pre-g-cocontinuous is α-g-cocontinuous.
(15) Every pre-g-bicontinuous is α-g-bicontinuous.
(16) Every semi-g-continuous is α-g-continuous.
(17) Every semi-g-cocontinuous is α-g-cocontinuous.
(18) Every semi-g-bicontinuous is α-g-bicontinuous.
(19) Every β-g-irresolute is β-g-continuous.
(20) Every β-g-co-irresolute is β-g-cocontinuous.

Proof. The proof is clear. □

Remark 3.3. The following diagram holds for a difunction $(f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$:



Theorem 3.4. *Let* $(f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ *be a difunction. Then:*

(1) *The following are equivalent:*

(a) (f,F) *is* β*-g-continuous.*

(b) $]F^{\rightarrow}A[^{S_2}\subseteq F^{\rightarrow}]A[^{S_1}_{\beta-g}$, $\forall A\in\varphi_1$.

(c) $f^{\leftarrow}]B[^{S_2}\subseteq]f^{\leftarrow}B[^{S_1}_{\beta-g}$, $\forall B\in\varphi_2$.

(2) *The following are equivalent:*

(a) (f,F) *is* β*-g-cocontinuous.*

(b) $f^{\rightarrow}[A]^{S_1}_{\beta-g}\subseteq[f^{\rightarrow}A]^{S_2}$, $\forall A\in\varphi_1$.

(c) $[F^{\leftarrow}B]^{S_1}_{\beta-g}\subseteq F^{\leftarrow}[B]^{S_2}$, $\forall B\in\varphi_2$.

Proof. We prove (1), leaving the dual proof of (2) to the interested reader.
$(a)\Rightarrow(b)$. Let $A\in\varphi_1$. From [10, Theorem 2.24 (2a)] and the definition of interior,

$$f^{\leftarrow}]F^{\rightarrow}(A)[^{S_2}\subseteq f^{\leftarrow}(F^{\rightarrow}(A))\subseteq A.$$

Since inverse image and co-image under a difunction is equal, $f^{\leftarrow}]F^{\rightarrow}(A)[^{S_2}= F^{\leftarrow}]F^{\rightarrow}(A)[^{S_2}$. Thus, $f^{\leftarrow}]F^{\rightarrow}(A)[^{S_2}\in\beta go(S_1)$, by β-g-continuity. Hence

$$f^{\leftarrow}]F^{\rightarrow}(A)[^{S_2}\subseteq]A[^{S_1}_{\beta-g}$$

and applying [10, Theorem 2.24 (2b)], we get

$$]F^{\rightarrow}(A)[^{S_2}\subseteq F^{\rightarrow}(f^{\leftarrow}(]F^{\rightarrow}(A)[^{S_2})\subseteq F^{\rightarrow}]A[^{S_1}_{\beta-g},$$

which is the required inclusion.
$(b)\Rightarrow(c)$. Take $B\in\varphi_2$. Applying inclusion (b) to $A=f^{\leftarrow}(B)$ and using [10, Theorem 2.24 (2b)], we obtain

$$]B[^{S_2}\subseteq]F^{\rightarrow}f^{\leftarrow}(B)[^{S_2}\subseteq F^{\rightarrow}]f^{\leftarrow}(B)[^{S_1}_{\beta-g}.$$

Hence, we have $f^{\leftarrow}]B[^{S_2}\subseteq f^{\leftarrow}F^{\rightarrow}]f^{\leftarrow}(B)[^{S_1}_{\beta-g}\subseteq]f^{\leftarrow}(B)[^{S_1}_{\beta-g}$ by [10, Theorem 2.24 (2a)].
$(c)\Rightarrow(a)$. Applying (c) for $B\in O(S_2)$, we get

$$f^{\leftarrow}(B)=f^{\leftarrow}]B[^{S_2}\subseteq]f^{\leftarrow}(B)[^{S_1}_{\beta-g},$$

so $F^{\leftarrow}(B)=f^{\leftarrow}(B)=]f^{\leftarrow}(B)[^{S_1}_{\beta-g}\in\beta go(S_1)$. Hence, (f,F) is β-g-continuous. □

Theorem 3.5. *Let* $(f,F):(S_1,\varphi_1,\tau_1,k_1,\sigma_1)\rightarrow(S_2,\varphi_2,\tau_2,k_2,\sigma_2)$ *be a difunction. Then:*

(1) *The following are equivalent:*

(a) (f,F) *is* β*-g-irresolute.*

(b) $]F^{\to}A[^{S_2}_{\beta-g}\subseteq F^{\to}]A[^{S_1}_{\beta-g}$, $\forall A\in\varphi_1$.
(c) $f^{\leftarrow}]B[^{S_2}_{\beta-g}\subseteq]f^{\leftarrow}B[^{S_1}_{\beta-g}$, $\forall B\in\varphi_2$.

(2) The following are equivalent:
(a) (f,F) *is* β*-g-co-irresolute.*
(b) $f^{\to}[A]^{S_1}_{\beta-g}\subseteq[f^{\to}A]^{S_2}_{\beta-g}$, $\forall A\in\varphi_1$.
(c) $[F^{\leftarrow}B]^{S_1}_{\beta-g}\subseteq F^{\leftarrow}[B]^{S_2}_{\beta-g}$, $\forall B\in\varphi_2$.

Proof. We prove (1), leaving the dual proof of (2) to the interested reader.
$(a)\Rightarrow(b)$. Take $A\in\varphi_1$. Then

$$f^{\leftarrow}]F^{\to}A[^{S_2}_{\beta-g}\subseteq f^{\leftarrow}(F^{\to}A)\subseteq A$$

by [10, Theorem 2.24 (2a)]. Now $f^{\leftarrow}]F^{\to}A[^{S_2}_{\beta-g}=F^{\leftarrow}]F^{\to}A[^{S_2}_{\beta-g}\in\beta go(S_1)$ by β-g-irresolute, so $f^{\leftarrow}]F^{\to}A[^{S_2}_{\beta-g}\subseteq]A[^{S_1}_{\beta-g}$ and applying [10, Theorem 2.24 (2b)], we obtain

$$]F^{\to}A[^{S_2}_{\beta-g}\subseteq F^{\to}(f^{\leftarrow}]F^{\to}A[^{S_2}_{\beta-g}\subseteq F^{\to}]A[^{S_1}_{\beta-g},$$

which is the required inclusion.
$(b)\Rightarrow(c)$. Take $B\in\varphi_2$. Applying inclusion (b) to $A=f^{\leftarrow}B$ and using [10, Theorem 2.24 (2b)], we obtain

$$]B[^{S_2}_{\beta-g}\subseteq]F^{\to}(f^{\leftarrow}B)[^{S_2}_{\beta-g}\subseteq F^{\to}]f^{\leftarrow}B[^{S_1}_{\beta-g}.$$

Hence, $f^{\leftarrow}]B[^{S_2}_{\beta-g}\subseteq f^{\leftarrow}F^{\to}]f^{\leftarrow}B[^{S_1}_{\beta-g}\subseteq]f^{\leftarrow}B[^{S_2}_{\beta-g}$ by [10, Theorem 2.24 (2a)].
$(c)\Rightarrow(a)$. Applying (c) for $B\in\beta go(S_2)$, we get

$$f^{\leftarrow}B=f^{\leftarrow}]B[^{S_2}_{\beta-g}\subseteq]f^{\leftarrow}B[^{S_1}_{\beta-g},$$

so $F^{\leftarrow}B=f^{\leftarrow}B=]f^{\leftarrow}B[^{S_1}_{\beta-g}\in\beta go(S_1)$. Hence, (f,F) is β-g-irresolute. □

Theorem 3.6. *Let* $(S_j,\varphi_j,\tau_j,k_j,\sigma_j)$*, for* $j\in\{1,2\}$*, be complemented ditopology and* $(f,F):(S_1,\varphi_1)\to(S_2,\varphi_2)$ *be complemented difunction. If* (f,F) *is* β*-g-continuous then* (f,F) *is* β*-g-cocontinuous.*

Proof. Since (f,F) is complemented, $(F',f')=(f,F)$. From [10, Lemma 2.20], $\sigma_1((f')^{\leftarrow}(B))=f^{\leftarrow}(\sigma_2(B))$ and $\sigma_1((F')^{\leftarrow}(B))=F^{\leftarrow}(\sigma_2(B))$ for all $B\in\varphi_2$. The proof is clear from these equalities. □

Corollary 3.7. *Let* $(S_j,\varphi_j,\tau_j,k_j,\sigma_j)$*,* $j=1,2$*, be complemented ditopology and* $(f,F):(S_1,\varphi_1)\to(S_2,\varphi_2)$ *be complemented difunction. If* (f,F) *is* β*-g-irresolute then* (f,F) *is* β*-g-co-irresolute.*

Proof. The proof is clear. □

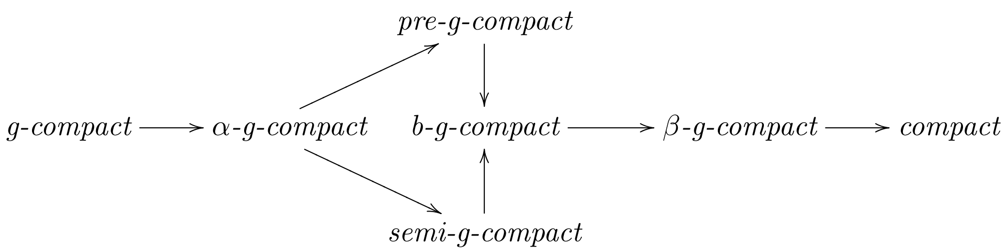
Definition 3.8. A complemented ditopological texture space $(S, \varphi, \tau, k, \sigma)$ is called *β-g-compact* if every cover of S by β-g-open has a finite subcover. Here we recall that $C = \{A_j : j \in J\}$, $A_j \in \varphi$ is a cover of S if $\bigvee C = S$.

Corollary 3.9. *Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space. Then:*

(1) Every α-g-compact is pre-g-compact.
(2) Every α-g-compact is semi-g-compact.
(3) Every pre-g-compact is b-g-compact.
(4) Every semi-g-compact is b-g-compact.
(5) Every b-g-compact is β-g-compact.
(6) Every β-g-compact is compact.
(7) Every g-compact is β-g-compact.

Proof. The proof is clear. □

Remark 3.10. *The following diagram holds for a complemented ditopological texture space $(S, \varphi, \tau, k, \sigma)$:*



Theorem 3.11. *If $(S, \varphi, \tau, k, \sigma)$ is β-g-compact and $L = \{F_j : j \in J\}$ is a family of β-g-closed sets with $\cap L = \phi$, then $\cap\{F_j : j \in J'\} = \phi$ for $J' \subseteq J$ finite.*

Proof. Suppose that $(S, \varphi, \tau, k, \sigma)$ is β-g-compact and let $L = \{F_j : j \in J\}$ be a family of β-g-closed sets with $\cap L = \phi$. Clearly $C = \{\sigma(F_j) : j \in J\}$ is a family of β-g-open sets. Moreover,

$$\bigvee C = \bigvee\{\sigma(F_j) : j \in J\} = \sigma(\cap\{F_j : j \in J\}) = \sigma(\phi) = S,$$

and so we have $J' \subseteq J$ finite with $\bigvee\{\sigma(F_j) : j \in J'\} = S$. Hence $\cap\{F_j : j \in J'\} = \phi$. □

Theorem 3.12. *Let $(f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ be a β-g-irresolute difunction. If $A \in \varphi_1$ is β-g-compact then $f^{\to}A \in \varphi_2$ is β-g-compact.*

Proof. Take $f^{\to}A \subseteq \bigvee_{j\in J} G_j$, where $G_j \in \beta go(S_2)$, $j \in J$. Now by [10, Theorem 2.24 (2a) and Corollary 2.12 (2)], we have

$$A \subseteq F^{\leftarrow}(f^{\to}A) \subseteq F^{\leftarrow}(\textstyle\bigvee_{j\in J} G_j) = \bigvee_{j\in J} F^{\leftarrow}G_j.$$

Also, $F^{\leftarrow}G_j \in \beta go(S_1)$ because (f, F) is β-g-irresolute. So by the β-g-compactness of A there exists $J' \subseteq J$ finite such that $A \subseteq \cup_{j\in J'} F^{\leftarrow}G_j$. Hence

$$f^{\to}A \subseteq f^{\to}(\cup_{j\in J'} F^{\leftarrow}G_j) = \cup_{j\in J'} f^{\to}(F^{\leftarrow}G_j) \subseteq \cup_{j\in J'} G_j$$

by [10, Corollary 2.12 (2) and Theorem 2.24 (2b)]. This establishes that $f^{\to}A$ is β-g-compact. □

Corollary 3.13. *Let $(f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ be a surjective β-g-irresolute difunction. Then, if $(S_1, \varphi_1, \tau_1, k_1, \sigma_1)$ is β-g-compact so is $(S_2, \varphi_2, \tau_2, k_2, \sigma_2)$.*

Proof. This follows by taking $A = S_1$ in Theorem 3.12 and noting that $f^{\to}S_1 = f^{\to}(F^{\leftarrow}S_2) = S_2$ by [10, Proposition 2.28 (1c) and Corollary 2.33 (1)]. □

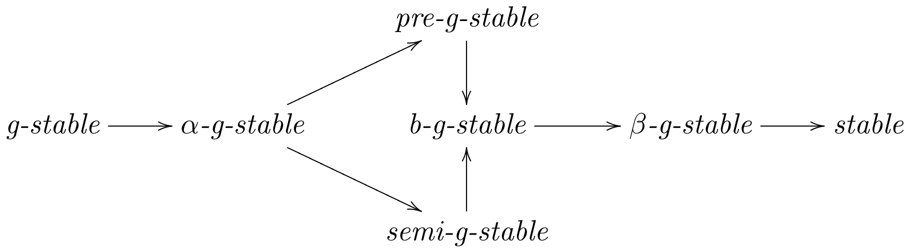
Definition 3.14. A complemented ditopological texture space $(S, \varphi, \tau, k, \sigma)$ is called *β-g-stable* if every β-g-closed set $F \in \varphi \setminus \{S\}$ is β-g-compact in S.

Corollary 3.15. *Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space. Then:*

1. *Every α-g-stable is pre-g-stable.*
2. *Every α-g-stable is semi-g-stable.*
3. *Every pre-g-stable is b-g-stable.*
4. *Every semi-g-stable is b-g-stable.*
5. *Every b-g-stable is β-g-stable.*
6. *Every β-g-stable is stable.*
7. *Every g-stable is β-g-stable.*

Proof. The proof is clear. □

Remark 3.16. *The following diagram holds for a complemented ditopological texture space* $(S, \varphi, \tau, k, \sigma)$*:*



Theorem 3.17. *Let* $(S, \varphi, \tau, k, \sigma)$ *be* β*-g-stable. If* G *is a* β*-g-open set with* $G \neq \phi$ *and* $D = \{F_j : j \in J\}$ *is a family of* β*-g-closed sets with* $\cap_{j\in J} F_j \subseteq G$ *then* $\cap_{j\in J'} F_j \subseteq G$ *for a finite subsets* J' *of* J.

Proof. Let $(S, \varphi, \tau, k, \sigma)$ be a β-g-stable, let G be an β-g-open set with $G \neq \phi$ and $D = \{F_j : j \in J\}$ be a family of β-g-closed sets with $\cap_{j\in J} F_j \subseteq G$. Set $K = \sigma(G)$. Then K is β-g-closed and satisfies $K \neq S$. Hence K is β-g-compact. Let $C = \{\sigma(F) | F \in D\}$. Since $\cap D \subseteq G$ we have $K \subseteq \bigvee C$, that is, C is a β-g-open cover of K. Hence, there exists $F_1, F_2, ..., F_n \in D$ so that

$$K \subseteq \sigma(F_1) \cup \sigma(F_2) \cup ... \cup \sigma(F_n) = \sigma(F_1 \cap F_2 \cap ... \cap F_n).$$

This gives $F_1 \cap F_2 \cap ... \cap F_n \subseteq \sigma(K) = G$, so $\cap_{j\in J'} F_j \subseteq G$ for finite subsets $J' = \{1, 2, ..., n\}$ of J. □

Theorem 3.18. *Let* $(S_1, \varphi_1, \tau_1, k_1, \sigma_1)$*,* $(S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ *be two complemented ditopological texture spaces with* $(S_1, \varphi_1, \tau_1, k_1, \sigma_1)$ *is* β*-g-stable, and* $(f, F) : (S_1, \varphi_1, \tau_1, k_1, \sigma_1) \to (S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ *be a* β*-g-bi-irresolute surjective difunction. Then* $(S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ *is* β*-g-stable.*

Proof. Take $K \in \beta gc(S_2)$ with $K \neq S_2$. Since (f, F) is β-g-co-irresolute, $f^{\leftarrow}K \in \beta gc(S_1)$. Let us prove that $f^{\leftarrow}K \neq S_1$. Assume the contrary. Since $f^{\leftarrow}S_2 = S_1$, by [10, Lemma 2.28 (1c)], we have $f^{\leftarrow}S_2 \subseteq f^{\leftarrow}K$, whence $S_2 \subseteq K$ by [10, Corollary 2.33 (1ii)] as (f, F) is surjective. This is a contradiction, so $f^{\leftarrow}K \neq S_1$. Hence, $f^{\leftarrow}(K)$ is β-g-compact in $(S_1, \varphi_1, \tau_1, k_1, \sigma_1)$ by β-g-stability. As (f, F) is β-g-irresolute, $f^{\rightarrow}(f^{\leftarrow}K)$ is β-g-compact for the ditopology (τ_2, k_2) by Theorem 3.12, and by [10, Corollary 2.33 (1)] this set is equal to K. This establishes that $(S_2, \varphi_2, \tau_2, k_2, \sigma_2)$ is β-g-stable. □

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Department of Mathematics, Faculty of Education, University of Zakho, Kurdistan-Region, Iraq
Email address: hariwan_math@yahoo.com