J. Korean Soc. Math. Educ. Ser. B: Pure Appl. Math.
https://doi.org/10.7468/jksmeb.2024.31.2.179
Volume 31, Number 2 (May 2024), Pages 179–187

ISSN(Print) 1226-0657
ISSN(Online) 2287-6081

CURVELET TRANSFORM AS AN EXTENSION OF WAVELET TRANSFORM AND ITS OPERATIONAL CALCULUS

Sachin Mane [a,*], Bharat Bhosale [b] and Shubham D. Shedge [c]

Abstract. In image and signal processing, the wavelet transform is frequently employed. However, it has the drawback of having weak directionality, which has limited its use in many applications. A recent addition to the wavelet transform, the curvelet transform attempts to address crossing phenomena that occur along curved edges in 2-D images.

As an extension of the wavelet transform, we discuss various curvelet transform features in this paper. There are numerous uses for the curvelet and wavelet transforms in image and signal processing.

1. Introduction

A multi-scale geometric analysis tool for images is the wavelet transform. Its uses in the field of image denoising are numerous, and the advantage of the wavelet transform is that it can reflect one-dimensional continuous signal singularity while preserving the singularity of the edge of two-dimensional images, such as a variety of straight lines, curves, etc. In a higher-dimensional plane, it is difficult to use the wavelet transform to express its features [10]. Wavelet analysis is useful for modelling acoustic scattering and sonar [6]. The curvelet transform, a new multi-scale representation suitable for objects that smooth away discontinuities across curves, was introduced by Candes and Donoho (1999) [7]. Wavelet-based multi-resolution approaches have close ties to optical data analysis, biological and computer vision, image and signal processing, and scientific computing. Wavelet functions are used as the object in multi-resolution analysis to specify the signal as a collection of its successive approximations[1]. Wavelets use the multi-resolution technique, which is deeply related to signal processing [2]. Ridgelets are specially adapted only to

Received by the editors September 30, 2023. Revised December 1, 2023. Accepted Dec. 20, 2023.
2020 *Mathematics Subject Classification.* 44A05.
Key words and phrases. wavelet transform, curvelet transform, extended curvelet transform.
*Corresponding author.

straight singularities [5]. The wavelet transform decomposes a signal into a representation that shows signal details and tends as a function of time [9].

Many fields have confirmed the effectiveness of the multi-resolution geometric analysis technique using curvelets as basis functions [8]. In order to address intriguing phenomena that appear along curved edges in 2D images, the wavelet transform has recently undergone an extension known as the curvelet transform [4]. Therefore, compared to all other multi-scale transforms employed in image and signal processing, the curvelet transform is more appropriate. As an extension of the wavelet transform, we will now analyze the characteristics of the curvelet transform.

2. MATHEMATICAL PRELIMINARIES

2.1. Wavelet Transform :

The classical wavelet transform, also known as the Continuous Wavelet Transform (CWT), is a decomposition of a function $f(x)$ with regard to a fundamental wavelet (x), produced by convolution of a function with a scaled and translated version of $\psi(x)$.

$$W_{\psi}(a,b)[f] = |a|^{-\frac{1}{2}} \int f(x)\psi^{*}(\frac{x-b}{a})dx \tag{2.1}$$

$$W_{\psi}(a,b)[f] =< f, |a|^{-\frac{1}{2}}\psi(\frac{x-b}{a}) >$$
$$=< f, \psi_{a,b} >$$

Where $<,>$ is the inner product. The functions f and ψ satisfy the admissibility condition and are square-integrable functions.

$$C_{\psi} = \int \frac{|\hat{\psi}(w)|^2}{|w|} dw < \infty \tag{2.2}$$

C_{ψ} is called the admissibility constant. Complex conjugation is indicated by the superscript $*$, with $'a'$ standing for the scale parameter $(a > 0)$ and $'b'$ for the translation parameter.

2.2. Curvelet Transform :

A discrete version of the continuous curvelet transform called the discrete curvelet transform is defined as [3],

$$C_{j,k,l}(f) = \int_{R^2} f(x)\overline{\gamma_{j,k,l}}(x)dx = \int_{R^2} \hat{f}(\xi)\overline{\hat{\gamma}_{j,k,l}}(\xi)d\xi \tag{2.3}$$

Where,

$$\hat{f}(\xi) = \frac{1}{2\pi}\int_{R^2} f(x)e^{-i<x,\xi>}dx, \tag{2.4}$$

$$\hat{\gamma}_{j,k,l}(\xi) = e^{-i<b_k^{j,l},\xi>}U_j(R_{\theta_{j,l}},\xi) = e^{-ib_k^{j,l}\xi}2^{\frac{-3}{4}j}W(2^{-j}\gamma)V\frac{(w+\theta_{j,l})}{\theta_{j,l}} \tag{2.5}$$

2.3. Curvelet as an Extension of Wavelet Transform :
From equation (2.4) and (2.5), equation (2.3) becomes,

$$C_{j,k,l}[f(x)] = \frac{1}{2\pi}\int_{R^2} e^{-ix\xi}e^{ib_k^{j,l}x}U_j(R_{\theta_{j,l}},x)f(x)dx$$
$$= \frac{1}{2\pi}\int_{R^2} e^{-ix\xi}e^{isx}U_j(R_{\theta_{j,l}},x)f(x)dx, \text{ where } s = b_k^{j,l}$$
$$= \frac{1}{\sqrt{|a|}}\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{iwx}\psi(x)f(x)dx, \text{ where } w = s-\xi.$$

Using $\psi(x) = \exp(i\pi x^2) = \exp[i\pi(\frac{x-b}{a})^2]$ as a mother wave.

$$C_{j,k,l}[f(x)] = \frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{iwx}exp\big[i\pi(\frac{x-b}{a})^2\big]f(x)dx\right]$$
$$= \frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{i[wx+\pi(\frac{x-b}{a})^2]}f(x)dx\right] \tag{2.6}$$
$$C_{j,k,l}[f(x)] = \frac{1}{\sqrt{|a|}}W_\psi[f(x)](a,b).$$

Curvelet Transform $= \frac{1}{\sqrt{|a|}}$(Wavelet Transform), where f and ψ are square integrable functions, and ψ satisfies the admissibility condition.

2.4. The test function space S :
An infinitely differentiable complex valued function ϕ on R^n is said to belong to the test function space $S(R^n)$ if, $\gamma_{\nu,\beta}(\phi) = \sup_{x\in R^n}|D^\beta\phi(x)| < \infty$, for all $\beta \in N_0^n$
The dual space S' is the space of tempered distributions.

2.5. Generalized Curvelet Transform :
The distributional curvelet transform of $f(x) \in S^*(R^n)$ is defined by,

$$C_{j,k,l}\{f(x)\} = C_{j,k,l}(w,a,b) = < f(x), K(x,w,a,b) > \tag{2.7}$$

where, $K(x,w,a,b) = \frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}} e^{i[wx+\pi(\frac{x-b}{a})^2]}\right]$, here $K(x,w,a,b) \in S$ and $f \in S^*$.

2.6. Graph of Mother Wavelet & Its Fourier Transform :

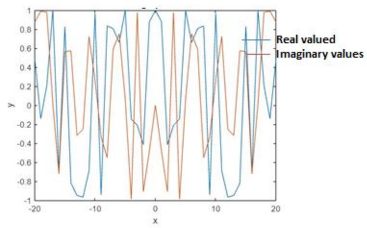

Figure 1. Graph of mother wavelet $\exp(i\pi x^2)$

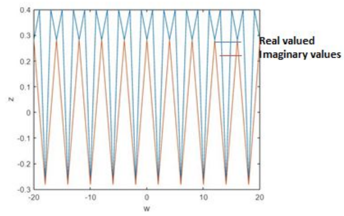

Figure 2. Fourier Transform of mother wavelet $\exp(i\pi x^2)$

3. RESULTS

3.1. Linearity Property of Extended Curvelet Transform :

$$C_{j,k,l}\{af(x) + bg(x)\} = aC_{j,k,l}\{f(x)\} + bC_{j,k,l}\{g(x)\}.$$

Proof. : Consider,

$$\begin{aligned} C_{j,k,l}\{af(x)+bg(x)\} &= \frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{i[wx+\pi(\frac{x-b}{a})^2]}\{af(x)+bg(x)\}dx\right] \\ &= a\frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{i[wx+\pi(\frac{x-b}{a})^2]}f(x)dx\right] \end{aligned}$$

$$+ b\frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{i[wx+\pi(\frac{x-b}{a})^2]}g(x)dx\right]$$
$$= aC_{j,k,l}\{f(x)\} + bC_{j,k,l}\{g(x)\}.$$

□

3.2. Extended Curvelet Transform of Translation :

$$C_{j,k,l}\{f(x-x_o)\} = e^{iwx_0+\frac{i\pi}{a^2}\{x_0^2-2bx_0\}}C_{j,k,l}\left[f(x)e^{\frac{2xi\pi x_0}{a^2}}\right].$$

Proof.

$$C_{j,k,l}\{f(x-x_o)\} = \frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{i\left[wx+\pi(\frac{x-b}{a})^2\right]}f(x-x_o)dx\right].$$

Put $x - x_0 = t$ then $x = t + x_0,\ \ dx = dt$.

$$\begin{aligned}
C_{j,k,l}f(x-x_o) &= \frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{i[w(t+x_0)+\pi(\frac{t+x_0-b}{a})^2]}f(t)dt\right]\\
&= \frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{iwt}e^{iwx_0}e^{i\frac{\pi}{a^2}[(t-b)^2+2(t-b)x_0+x_0^2]}f(t)dt\right]\\
&= e^{iwx_0}\frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{i\left[wt+\pi(\frac{t-b}{a})^2\right]}e^{i\frac{\pi x_0^2}{a^2}}e^{\frac{i\pi}{a^2}[2(t-b)x_0]}f(t)dt\right]\\
&= e^{iwx_0}e^{i\frac{\pi x_0^2}{a^2}}\frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{i\left[wt+\pi(\frac{t-b}{a})^2\right]}e^{\frac{i\pi 2tx_0}{a^2}}e^{\frac{-i\pi 2bx_0}{a^2}}f(t)dt\right]\\
&= e^{iwx_0}e^{i\frac{\pi x_0^2}{a^2}}e^{\frac{-i\pi 2bx_0}{a^2}}\frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{i\left[wt+\pi(\frac{t-b}{a})^2\right]}e^{\frac{i\pi 2tx_0}{a^2}}f(t)dt\right]\\
&= e^{iwx_0+\frac{i\pi}{a^2}\{x_0^2-2bx_0\}}C_{j,k,l}\left[f(x)e^{\frac{2xi\pi x_0}{a^2}}\right].
\end{aligned}$$

□

3.3. Scaling Property of Extended Curvelet Transform :

$$C_{j,k,l}\{f(\alpha x)\} = \frac{1}{\alpha}C_{j,k,l}\left[f(x)e^{-(\frac{\alpha-1}{\alpha})iwt}e^{i\frac{\pi}{a^2}[(-\frac{\alpha^2-1}{\alpha^2})t^2-2(\frac{\alpha-1}{\alpha})t]}\right].$$

Proof. : Consider,

$$C_{j,k,l}\{f(\alpha x)\} = \frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{i[wx+\pi(\frac{x-b}{a})^2]}f(\alpha x)dx\right].$$

Put $\alpha x = t$ then $x = \frac{t}{\alpha}$, $dx = \frac{dt}{\alpha}$.

$$\begin{aligned}
C_{j,k,l}\{f(\alpha x)\} &= \frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{i[w\frac{t}{\alpha}+\pi(\frac{\frac{t}{\alpha}-b}{a})^2]}f(t)\frac{dt}{\alpha}\right] \\
&= \frac{1}{\alpha}\frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{i\left[\frac{w}{\alpha}t+\pi\left[\frac{(\frac{t}{\alpha})^2-2\frac{t}{\alpha}b+b^2}{a^2}\right]\right]}f(t)dt\right] \\
&= \frac{1}{\alpha}\frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{iwt(1-\frac{\alpha-1}{\alpha})+i\pi\left[\frac{(1-\frac{\alpha^2-1}{\alpha^2})t^2-2(1-\frac{\alpha-1}{\alpha})tb+b^2}{a^2}\right]}f(t)dt\right] \\
&= \frac{1}{\alpha}\frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{i[wt+\pi(\frac{t-b}{a})^2]}f(t)dt\ e^{-(\frac{\alpha-1}{\alpha})iwt}.\right. \\
&\qquad\qquad \left.\cdot\, e^{i\pi\left[(-\frac{\alpha^2-1}{\alpha^2})\frac{t^2}{a^2}-2(\frac{\alpha-1}{\alpha})\frac{t}{a^2}\right]}\right] \\
&= \frac{1}{\alpha}\ C_{j,k,l}\{f(x)e^{-(\frac{\alpha-1}{\alpha})iwt}\ e^{i\frac{\pi}{\alpha^2}\left[(-\frac{\alpha^2-1}{\alpha^2})t^2-2(\frac{\alpha-1}{\alpha})t\right]}\}.
\end{aligned}$$

□

3.4. Differentiation of Extended Curvelet Transform :

$$\frac{d}{dx}[C_{j,k,l}\{f(x)\}] = [iw+2\pi i(\frac{-b}{a^2})]\ C_{j,k,l}\{f(x)\} + \frac{2\pi ix}{a^2}\ C_{j,k,l}[xf(x)] + C_{j,k,l}\{f^{'}(x)\}.$$

Proof. : Consider,

$$\begin{aligned}
\frac{d}{dx}[C_{j,k,l}\{f(x)\}] &= \frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty}\frac{\partial}{\partial x}e^{i[wx+\pi(\frac{x-b}{a})^2]}f(x)dx\right] \\
&= \frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty}\{e^{i[wx+\pi(\frac{x-b}{a})^2]}i[w+2\pi(\frac{x-b}{a^2})]f(x)\right. \\
&\qquad \left. + e^{i[wx+\pi(\frac{x-b}{a})^2]}f^{'}(x)\}dx\right] \\
&= \frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty}\{e^{i[wx+\pi(\frac{x-b}{a})^2]}[iw+2\pi i(\frac{-b}{a^2})+\frac{2\pi ix}{a^2}]f(x)\}dx\right] \\
&\qquad + C_{j,k,l}f^{'}(x) \\
&= [iw+2\pi i(\frac{-b}{a^2})]\ C_{j,k,l}\{f(x)\} + \frac{2\pi ix}{a^2}\ C_{j,k,l}[xf(x)] + C_{j,k,l}\{f^{'}(x)\}.
\end{aligned}$$

□

Similarly,

3.5. Extended Curvelet Transform of Differentiation :

$$C_{j,k,l}\{f^{'}(x)\} = \frac{d}{dx}[C_{j,k,l}\{f(x)\}] - \{[iw+2\pi i(\frac{-b}{a^2})]\ C_{j,k,l}f(x) + \frac{2\pi i}{a^2}\ C_{j,k,l}[xf(x)]\}.$$

4. Modulation of Curvelet Transform as an Extension of Wavelet Transform

4.1. Property 1 :

$$C_{j,k,l}[f(x)\cos tx] = \frac{1}{2}\big[C_{j,k,l}\{f(x)((w+t),a,b)\} + C_{j,k,l}\{f(x)((w-t),a,b)\}\big].$$

Proof.

$$\begin{aligned}
C_{j,k,l}[f(x)\cos tx] &= \frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{i[wx+\pi(\frac{x-b}{a})^2]} f(x)\cos tx dx\right] \\
&= \frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{i[wx+\pi(\frac{x-b}{a})^2]} f(x)[\frac{e^{itx}+e^{-itx}}{2}]dx\right] \\
&= \frac{1}{2}\frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{i[wx+\pi(\frac{x-b}{a})^2]} f(x)e^{itx}dx\right] \\
&\quad + \frac{1}{2}\frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{i[wx+\pi(\frac{x-b}{a})^2]} f(x)e^{-itx}dx\right] \\
&= \frac{1}{2}\frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{i[(w+t)x+\pi(\frac{x-b}{a})^2]} f(x)dx\right] \\
&\quad + \frac{1}{2}\frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{i[(w-t)x+\pi(\frac{x-b}{a})^2]} f(x)dx\right] \\
&= \frac{1}{2}C_{j,k,l}\{f(x)((w+t),a,b)\} + \frac{1}{2}C_{j,k,l}\{f(x)((w-t),a,b)\} \\
&= \frac{1}{2}\big[C_{j,k,l}\{f(x)((w+t),a,b)\} + C_{j,k,l}\{f(x)((w-t),a,b)\}\big].
\end{aligned}$$

□

4.2. Property 2 :

$$C_{j,k,l}[f(x)\sin tx] = \frac{1}{2}\big[C_{j,k,l}\{f(x)((w+t),a,b)\} - C_{j,k,l}\{f(x)((w-t),a,b)\}\big].$$

Proof.

$$\begin{aligned}
C_{j,k,l}[f(x)\sin tx] &= \frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{i[wx+\pi(\frac{x-b}{a})^2]} f(x)\sin tx dx\right] \\
&= \frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{i[wx+\pi(\frac{x-b}{a})^2]} f(x)[\frac{e^{itx}-e^{-itx}}{2}]dx\right] \\
&= \frac{1}{2}\frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{i[wx+\pi(\frac{x-b}{a})^2]} f(x)e^{itx}dx\right]
\end{aligned}$$

$$
\begin{aligned}
&- \frac{1}{2}\frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{i[wx+\pi(\frac{x-b}{a})^2]} f(x)e^{-itx}dx\right] \\
&= \frac{1}{2}\frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{i[(w+t)x+\pi(\frac{x-b}{a})^2]} f(x)dx\right] \\
&\quad - \frac{1}{2}\frac{1}{\sqrt{|a|}}\left[\frac{1}{2\pi}|a|^{\frac{1}{2}}\int_{-\infty}^{\infty} e^{i[(w-t)x+\pi(\frac{x-b}{a})^2]} f(x)dx\right] \\
&= \frac{1}{2}C_{j,k,l}\{f(x)((w+t),a,b)\} - \frac{1}{2}C_{j,k,l}\{f(x)((w-t),a,b)\} \\
&= \frac{1}{2}\big[C_{j,k,l}\{f(x)((w+t),a,b)\} - C_{j,k,l}\{f(x)((w-t),a,b)\}\big].
\end{aligned}
$$

□

5. Applications of Extended Curvelet Transform

Extended Curvelet transform and its properties are useful to solve partial and ordinary differential equations like time independent Schrodinger equation for the quantum harmonic oscillator, Korteweg-De Vries equation. The curvelet transform is also widely used in many other fields, including fluid mechanics, seismic data exploration, signal and image processing, and the solution of partial differential equations encountered in non-linear physical phenomena.

6. Conclusion

A multi-scale directional transform known as the curvelet transform enables the best non-adaptive sparse representation of objects with edge data, particularly in higher-dimensional signals. This paper presents some properties of the curvelet transform as an extension of the wavelet transform with applications.

References

1. G. Beylkin: On the representation of operators in bases of compactly supported wavelets. SIAM Journal on Numerical Analysis **29** (1992), no. 6, 1716-1740. `https://doi.org/10.1137/0729097`
2. B. Bhosale: Wavelet analysis of randomized solitary wave solutions. J. Math. Anal. Appl. **1** (2014), no. 1, 20-26.
3. B. Bhosale: Curvelet Interaction with Artificial Neural Networks Artificial Neural Network Modelling (2016), 109-125. `https://doi.org/10.1007/978-3-319-28495-8_6`

4. B. Bhosale, A. Jain & et.al.: Curvelet based multiresolution analysis of graph neural networks. International Journal of Applied Physics and Mathematics **4** (2014), no. 5, 313-316. https://doi.org/10.7763/IJAPM.2014.V4.304
5. E.J. Candès & D.L. Donoho: Ridgelets: A key to higher-dimensional intermittency. Philosophical Transactions of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences **357** (1999), no. 1760, 2495-2509. https://doi.org/10.1098/rsta.1999.0444
6. L. Debnath & Shah, F.A. : Wavelet transforms and their applications. Boston: Birkhä user (2002), 12-14. https://doi.org/10.1007/978-0-8176-8418-1
7. D.L. Donoho & M.R. Duncan: Digital curvelet transform: strategy, implementation, and experiments. In Wavelet applications VII SPIE **4056** (2000), 12-30. https://doi.org/10.1117/12.381679
8. J. Ma & G. Plonka: A review of curvelets and recent applications. IEEE Signal Processing Magazine **27** (2000), no. 2, 118-133.
9. S.D. Shedge & B.N. Bhosale: Operational Calculus On Wavelet Transform As An Extension Of Fractional Fourier Transform. Journal of the Oriental Institute M.S. University of Baroda **71** (2022), 56-59.
10. J.S. Walker & Y. Chen: Image denoising using tree-based wavelet subband correlations and shrinkage. Optical Engineering **39** (2000), no. 11, 2900-2908. https://doi.org/10.1117/1.1315571

[a]Research Scholar: Department of Mathematics,Shivaji University Kolhapur, Maharashtra, India.
Email address: sachinm3116@gmail.com

[b]Principal, Prof.: Sambhajirao Kadam College, Deur, Satara, Maharashtra, India
Email address: bnbhosale15@gmail.com

[c]Professor: Department of Mathematics, Rajarshi Chhatrapati Shahu College, Kolhapur, Maharashtra, India
Email address: shedge.shubham@rcsc.ac.in