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GENERALIZED RICCI SOLITONS ON 3-DIMENSIONAL CONTACT METRIC MANIFOLDS

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Abstract. In the present paper we study 3-dimensional contact metric manifolds with $\varphi Q = Q\varphi$ admitting generalized Ricci solitons and generalized gradient Ricci solitons. It is proven that if a 3-dimensional contact metric manifold satisfying $\varphi Q = Q\varphi$ admits a generalized Ricci soliton with non zero soliton vector field V being pointwise collinear with the characteristic vector field ξ, then the manifold is Sasakian. Also it is shown that if a 3-dimensional compact contact metric manifold with $\varphi Q = Q\varphi$ admits a generalized gradient Ricci soliton then either the soliton is trivial or the manifold is flat or the scalar curvature is constant.

1. Introduction

In 1982, Hamilton introduced the notion of Ricci soliton. Ricci solitons are self-similar solutions to the Ricci flow which are given by the partial differential equation

$$\frac{\partial}{\partial t} g_t = -2Ric, \tag{1}$$

where g_t is a Riemannian metric for each t in some interval of the real line and Ric is the Ricci tensor of g_t. Ricci solitons which are also generalization of Einstein manifolds are defined as

$$\mathcal{L}_V g + 2Ric + 2\lambda g = 0, \tag{2}$$

where $\mathcal{L}_V g$ denotes the Lie derivative of the metric tensor g along the vector field V and λ is a constant.

If V is gradient of some smooth function f on M, then the Ricci soliton is called gradient Ricci soliton and the foregoing equation takes the form

$$Hess f + Ric = \lambda g, \tag{3}$$

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where $Hessf$ is the Hessian of f.

In 2014, Nurowski et al. ([11]) introduced the notion of generalized Ricci solitons. The generalized Ricci solitons are the generalisation of Ricci solitons.
A $(2n+1)$-dimensional smooth Riemannian manifold (M^{2n+1}, g) is said to be generalized Ricci soliton if there exists a smooth vector field V such that

$$\pounds_V g(X,Y) + 2c_1 V^{\#}(X) \otimes V^{\#}(Y) = 2c_2 Ric(X,Y) + 2\lambda g(X,Y), \tag{4}$$

for all smooth vector fields X, Y on M and for arbitrary real constants c_1, c_2 and λ. $V^{\#}$ is the non zero 1-form given by $V^{\#}(X) = g(X,V)$ for any smooth vector field X on M and Ric is the Ricci tensor of g.

- For $c_1 = c_2 = \lambda = 0$ equation (1.1) is a generalization of Killing equation.
- For $c_1 = c_2 = 0$ equation (1.1) is homothetic equation.
- For $c_1 = 0,\ c_2 = -1$ the equation (1.1) presents Ricci soliton.
- When $c_1 = 1, c_2 = \frac{-1}{n-1}, \lambda = 0$ equation (1.1) is generalization of metric projective structures with skew symmetric Ricci tensor in projective class.

When the soliton vector field V is gradient of some smooth function $f : M \to \mathbb{R}$, the equation (1.1) becomes

$$Hessf + c_1 df \otimes df = c_2 Ric + \lambda g, \tag{5}$$

where the Hessian of f is given by $Hessf(X,Y) = g(\nabla_X Df, Y)$, where Df is the gradient of f. In this case f is called *potential function* of the generalized Ricci soliton and g is called generalized gradient Ricci soliton.

In the recent years many works has been done on generalized Ricci soliton (([8], [11], [18]). M. D. Siddiqi ([8]) studied generalized Ricci solitons on trans-sasakian manifolds.
On the other hand, in 1872 Sophus Lie introduced the notion of contact transformation as a geometric tool to study systems of differential equations. In 1990, Blair ([5]), Koufogiorgos and Sharma characterized 3-dimensional contact metric manifolds with $\varphi Q = Q\varphi$ where φ is the operator given by the contact structure and Q is the Ricci operator. Based on this result, we study those manifolds which are generalized Ricci solitons. More precisely, we study generalized Ricci solitons on 3-dimensional contact metric manifolds with $\varphi Q = Q\varphi$ in the following cases:
(1) The soliton vector field is parallel to the characteristic vector field.
(2) The soliton vector field is the gradient of a smooth function.

The paper is organized as follows: After the introduction, in section 2 we discuss about preliminaries of contact metric manifolds. In section 3 we consider generalized Ricci solitons where the soliton vector field is collinear with the Reeb vector field ξ. Section 4 deals with generalized gradient Ricci solitons. In this paper, we study generalized Ricci solitons in 3-dimensional contact metric manifold satisfying $\varphi Q = Q\varphi$ and first we prove the following:

Theorem 1.1. *If a 3-dimensional contact metric manifold M such that $\varphi Q = Q\varphi$ admits a generalized Ricci soliton with non zero soliton vector field V being pointwise collinear with the characteristic vector field ξ, then V is a constant multiple of ξ, the scalar curvature is constant and the manifold is Sasakian.*

Again, considering the soliton vector field is gradient of some smooth function we prove

Theorem 1.2. *If a 3-dimensional compact contact metric manifold M with $\varphi Q = Q\varphi$ admits a generalized gradient Ricci soliton then either the soliton is trivial or the manifold is flat or the scalar curvature is constant.*

2. Preliminaries

This section consists of some basic definitions and properties of contact metric manifolds (see [1], [9], [15]). A $(2n+1)$ dimensional smooth Riemannian manifold (M^{2n+1}, g) is said to be almost contact metric manifold if there exist on M a $(1,1)$ tensor field φ, a 1-form η and a vector field ξ such that

$$\varphi^2 X = -X + \eta(X)\xi, \;\; \varphi(\xi) = 0, \;\; \eta \circ \varphi = 0, \;\; \eta(\xi) = 1, \tag{6}$$

for any vector field X on M. In an almost contact metric manifold we also have

$$g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{7}$$

for all smooth vector fields X, Y on M. The fundamental 2-form Φ is given by $\Phi(X, Y) = g(X, \varphi Y)$. An almost contact metric manifold is said to be contact metric manifold if $d\eta(X, Y) = g(X, \varphi Y)$. The almost contact structure (φ, ξ, η) is said to be normal [9] if and only if

$$N^{(1)}(X, Y) = N_\varphi(X, Y) + 2d\eta(X, Y)\xi = 0, \tag{8}$$

for any smooth vector fields X, Y on M, where N_φ denotes the Nijenhuis torson of φ, given by

$$N_\varphi(X, Y) = \varphi^2[X, Y] + [\varphi X, \varphi Y] - \varphi[\varphi X, Y] - \varphi[X, \varphi Y]. \tag{9}$$

On a contact metric manifold we consider two self adjoint operators

$$h = \frac{1}{2}\pounds_\xi \varphi, \; l = R(.,\xi)\xi,$$

where R is the Riemann curvature tensor of g. These two operators satisfy the following identities:

$$trh = 0, \;\; trh\varphi = 0, \;\; h\xi = 0, \;\; h\varphi = -\varphi h.$$

Furthermore, we have

$$\nabla_X \xi = -\varphi X - \varphi h X, \tag{10}$$

$$trl = Ric(\xi, \xi) = 2n - trh^2, \tag{11}$$

where ∇ is the Levi-Civita connection of g. The Ricci operator Q is given by $Ric(X,Y) = g(QX,Y)$.

On 3-dimensional contact metric manifold satisfying $\varphi Q = Q\varphi$, the following relations hold ([4]):

$$\begin{aligned} R(X,Y)Z =& (\frac{r}{2} - trl)\{g(Y,Z)X - g(X,Z)Y\} + \frac{1}{2}(3trl - r)\{g(Y,Z)\eta(X)\xi \\ & - g(X,Z)\eta(Y)\xi + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y\}, \end{aligned} \tag{12}$$

$$Ric(X,Y) = \frac{1}{2}(r - trl)g(X,Y) + \frac{1}{2}(3trl - r)\eta(X)\eta(Y), \tag{13}$$

and from (13) we have

$$QX = \frac{1}{2}(r - trl)X + \frac{1}{2}(3trl - r)\eta(X)\xi. \tag{14}$$

Now we state the following Lemma which will be used later.

Lemma 2.1 ([4])**.** *Let M be a contact metric manifold with a contact metric structure (φ, ξ, η, g) such that $\varphi Q = Q\varphi$. Then the function trl is constant everywhere on M and $\xi(r) = 0$. Further, if $trl = 0$ then, M is flat.*

3. Generalized Ricci Solitons

In this section we discuss about generalized Ricci soliton where the soliton vector V is pointwise collinear with the Reeb vector field ξ.

Proof of The Theorem 1.1. Generalized Ricci soliton equation is given by

$$\pounds_V g(X,Y) + 2c_1 V^\#(X) \otimes V^\#(Y) = 2c_2 Ric(X,Y) + 2\lambda g(X,Y). \tag{15}$$

Taking $V = \delta\xi \neq 0$, for some $\delta \in C^{\infty}(M)$ from (15) we get

$$g(\nabla_X \delta\xi, Y) + g(\nabla_Y \delta\xi, X) + 2c_1\delta^2\eta(X)\eta(Y) = 2c_2 Ric(X,Y) + 2\lambda g(X,Y), \tag{16}$$

where X, Y are smooth vector fields on M. Using (10) in (16) we have

$$\begin{aligned} X(\delta)\eta(Y) + Y(\delta)\eta(X) - 2\delta g(\varphi hX, Y) \\ +2c_1\delta^2\eta(X)\eta(Y) =& 2c_2 Ric(X,Y) + 2\lambda g(X,Y). \end{aligned} \tag{17}$$

Substituting $X = Y = \xi$ in (17) we obtain

$$2\xi(\delta) + 2c_1\delta^2 = 2c_2 trl + 2\lambda. \tag{18}$$

After replacing $Y = \xi$, equation (17) transforms into

$$X(\delta) + \xi(\delta)\eta(X) + 2c_1\delta^2\eta(X) = 2c_2 Ric(X,\xi) + 2\lambda\eta(X). \tag{19}$$

Using of (18) and (13) in (19) yields

$$X(\delta) = \xi(\delta)\eta(X), \tag{20}$$

which further infer that

$$D\delta = \xi(\delta)\xi. \tag{21}$$

Now differentiating (21) along an arbitrary vector field $X$and using (10) we obtain

$$\nabla_X D\delta = X(\xi(\delta))\xi + \xi(\delta)(-\varphi X - \varphi hX). \tag{22}$$

Taking inner product of (22) with an arbitrary vector field Y and noting that $g(\nabla_X D\delta, Y) = g(\nabla_Y D\delta, X)$, we get

$$X(\xi(\delta))\eta(Y) - Y(\xi(\delta))\eta(X) + 2\xi(\delta)d\eta(X,Y) = 0. \tag{23}$$

Taking X and Y perpendicular to ξ we obtain $\xi(\delta) = 0$, since $d\eta$ is non zero.
Using this result in (21) we get δ is constant.
Using $\delta =$ contant in (17) and tracing over X and Y we get

$$r = \frac{c_1\delta^2 - 3\lambda}{c_2},$$

provided $c_2 \neq 0$. The above expression of r shows that r is constant.
Therefore from (17) we have

$$\delta g(\varphi hX, Y) + c_1\delta^2\eta(X)\eta(Y) = c_2 Ric(X,Y) + \lambda g(X,Y) \tag{24}$$

Using of (13) in (24) yields

$$-\delta g(\varphi hX, Y) + \{c_1\delta^2 - \frac{c_2}{2}(3trl - r)\}\eta(X)\eta(Y) = \{\frac{c_2}{2}(r - trl) + \lambda\}g(X,Y). \tag{25}$$

Now differentiating the above equation along an arbitraty vector field Z and using (10) we have

$$-\delta g((\nabla_Z\varphi h)X,Y) = \{c_1\delta^2 - \frac{c_2}{2}(3trl - r)\}\{\eta(Y)g(X,\varphi Z + \varphi hZ) + \eta(X)g(Y,\varphi Z + \varphi hZ)\}. \tag{26}$$

Contracting over Y and Z in (26) and in view of $tr\varphi = tr\varphi h = 0$ we obtain

$$\delta(div(\varphi h)X) = 0. \tag{27}$$

It is known that in a contact metric manifold [5] $div(\varphi h)X = 2n\eta(X) - g(Q\xi, X)$. Therefore from (27) we have

$$\delta(2 - trl) = 0. \tag{28}$$

The equation (28) implies $trl = 2$, as $\delta = 0$ implies $V = 0$ which is a contradiction. Putting $trl = 2$ in the expression $trl = 2n - trh^2$ we get $trh^2 = 0$, from which we have $h = 0$ on M as h is symmetric. Therefore M is Sasakian and this proves Theorem 1.1. □

4. Generalized Gradient Ricci Soliton

In this section we consider 3-dimensional contact metric manifold admitting generalized gradient Ricci solitons. First we prove the following:

Lemma 4.1. If a 3-dimensional contact metric manifold with $\varphi Q = Q\varphi$ admits generalized gradient Ricci soliton, then

(i)
$$R(X,Y)Df = c_1(c_2QY + \lambda Y)X(f) - c_1(c_2QX + \lambda X)Y(f) + c_2\{(\nabla_X Q)Y - (\nabla_Y Q)X\}.$$

(ii) $Ric(Y,Df) = \frac{1}{1-c_1c_2}(-2c_1\lambda - c_1c_2r)Y(f) + c_2(divQ)Y - c_2Y(r).$

Proof. In a generalized gradient Ricci soliton

$$Hessf(X,Y) + c_1df(X)df(Y) = c_2Ric(X,Y) + \lambda g(X,Y). \tag{29}$$

From the equation (29) we infer that

$$\nabla_Y Df + c_1Dfg(Df,Y) = c_2QY + \lambda Y. \tag{30}$$

Differentiation of (30) along an arbitrary vector field X entails that

$$\begin{aligned}\nabla_X\nabla_Y Df =& - c_1\{g(\nabla_X Df, Y)Df + g(Df, \nabla_X Y)Df) + \nabla_X Df g(Df, Y)\} \\ &+ c_2\{(\nabla_X Q)Y + Q(\nabla_X Y)\} + \lambda\nabla_X Y.\end{aligned} \tag{31}$$

Making use of (30) and (31) in the expression of curvature tensor

$$R(X, Y)Z = \nabla_X\nabla_Y Z - \nabla_Y\nabla_X Z - \nabla_{[X,Y]}Z,$$

we obtain

$$\begin{aligned}R(X, Y)Df =& c_1(c_2QY + \lambda Y)X(f) - c_1(c_2QX + \lambda X)Y(f) \\ &+ c_2\{(\nabla_X Q)Y - (\nabla_Y Q)X\},\end{aligned} \tag{32}$$

and this proves the first part of Lemma 4.1.
Now contracting the vector field X in the equation (32) yields

$$Ric(Y, Df) = c_1c_2Ric(Y, Df) - (2c_1\lambda + c_1c_2r)Y(f) + c_2(divQ)Y - c_2Y(r). \tag{33}$$

Hence,

$$Ric(Y, Df) = \frac{1}{1 - c_1c_2}(-2c_1\lambda - c_1c_2r)Y(f) + c_2(divQ)Y - c_2Y(r), \tag{34}$$

this proves the second part of the Lemma 4.1. □

Proof of The Theorem 1.2. Taking inner product of (32) with ξ we get

$$\begin{aligned}g(R(X, Y)Df, \xi) =& \{c_1c_2\eta(QY) + c_1\lambda\eta(Y)\}X(f) \\ &- \{c_1c_2\eta(QX) + c_1\lambda\eta(X)\}Y(f) \\ &+ c_2\{g((\nabla_X Q)Y, \xi) - g((\nabla_Y Q)X, \xi)\}.\end{aligned} \tag{35}$$

Using of $Q\xi = trl\xi$ and Lemma 2.1. give us

$$(\nabla_X Q)\xi = Q\varphi X + Q\varphi hX + trl(-\varphi X - \varphi hX). \tag{36}$$

Equations (35) and (36) together imply

$$\begin{aligned}g(R(X, Y)Df, \xi) =& \{c_1c_2\eta(QY) + c_1\lambda\eta(Y)\}X(f) \\ &- \{c_1c_2\eta(QX) + c_1\lambda\eta(X)\}Y(f) \\ &+ c_2\{2g(Q\varphi X, Y) - g(hX, Q\varphi Y) \\ &+ g(hY, Q\varphi X) - 2trlg(\varphi X, Y)\}.\end{aligned} \tag{37}$$

Now replacing Z by ξ in (12) and taking inner product with Df yields

$$g(R(X, Y)\xi, Df) = \frac{trl}{2}(X(f)\eta(Y) - Y(f)\eta(X)). \tag{38}$$

Comparing (37) with (38) we obtain

$$\{c_1c_2\eta(QY)+c_1\lambda\eta(Y)\}X(f)-\{c_1c_2\eta(QX)+c_1\lambda\eta(X)\}Y(f) + c_2\{g((\nabla_X Q)Y,\xi)-g((\nabla_Y Q)X,\xi)\} = -\frac{trl}{2}\{(X(f)\eta(Y)-Y(f)\eta(X))\}. \tag{39}$$

Now Substituting $Y=\xi$ in (39) and using $h\xi=0$, $\varphi\xi=0$ and equation (14), we get

$$\{X(f)-\eta(X)\xi(f)\}(c_1c_2trl+c_1\lambda+\frac{trl}{2})=0. \tag{40}$$

Therefore we have the following two cases:

CASE (I): $X(f)-\eta(X)\xi(f)=0$.

$X(f)-\eta(X)\xi(f)=0$ and this implies $Df=\xi(f)\xi$.

Now differentiating this along X and using (10) we get

$$\nabla_X Df = X(\xi(f))\xi-\xi(f)\{\varphi X+\varphi hX\}.$$

Using of the foregoing expression in the relation $g(\nabla_X Df,Y)=(\nabla_Y Df,X)$ entails that

$$X(\xi(f))\eta(Y)-Y\xi(f)\eta(X)+2\xi(f)d\eta(X,Y)=0. \tag{41}$$

Choosing X and Y perpendicular to ξ we have $\xi(f)=0$ and this implies $Df=0$. Which infer that f is constant. So $V=0$.

CASE (II): $(c_1c_2trl+c_1\lambda+\frac{trl}{2})=0$.

In this case we have $trl=\frac{-2c_1\lambda}{2c_1c_2+1}$.

From (30) we obtain

$$g(\nabla_\xi Df,\xi)=-c_1(\xi(f))^2+c_2trl+\lambda,$$

which implies

$$\xi(\xi(f))=-c_1(\xi(f))^2+c_2trl+\lambda. \tag{42}$$

Putting $Y=Df$ in (13) and comparing with (34) gives us

$$\frac{1}{1-c_1c_2}(-2c_1\lambda-c_1c_2r)Y(f)+c_2(divQ)Y-c_2Y(r) = \frac{1}{2}(r-trl)Y(f)+\frac{1}{2}(3trl-r)\eta(Y)\xi(f). \tag{43}$$

Now replacing Y by ξ and using the result of Lemma 2.1. the equation (43) transforms into

$\xi(f)\{\frac{1}{1-c_1c_2}(-2c_1\lambda-c_1c_2r)-trl\}=0.$

If $\xi(f) = 0$ then from equation (42) $\lambda = -c_2 trl$. Utilization of this in $trl = \frac{-2c_1\lambda}{2c_1c_2+1}$ gives $trl = 0$. From Lemma 2.1. we conclude M is flat.
If $\frac{1}{1-c_1c_2}(-2c_1\lambda - c_1c_2 r) - trl = 0$ then $trl = \frac{-2c_1\lambda}{2c_1c_2+1}$ implies $r = \frac{trl}{3}$. This completes the proof of Theorem 1.2. □

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