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PERFORMANCE ANALYSIS OF DEADLINE-CONSTRAINED SLOTTED ALOHA UNDER MULTI-PACKET RECEPTION

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Abstract. We consider the slotted Aloha-based access network where multiple transmitters try to send deadline-constrained packets to the access point. The delivery deadline of packets may be generally a random variable depending on the type of traffic or their urgency. The successful delivery probability (SDP), which is defined as the long-run fraction of packets successfully delivered to the receiver before their maximum allowable deadline, is a suitable measure for evaluating the performance of time-critical traffic. We derive the closed-form expression for the SDP for the slotted Aloha-based access network under the assumptions of MPR and general delivery deadline distribution.

1. Introduction

Multi-packet reception (MPR) is a promising technique for enhancing the network performance in such a way that it enables receivers to correctly decode multiple packets transmitted simultaneously. In the context of random access based networks, there have been much efforts to gain an insight into the impact of MPR on the behavior of random access protocols. Many studies focused on the stability, saturation throughput or delay issue for the slotted Aloha under MPR. Except for the recent work [15], little attention has been devoted to investigating the reliability issue for the slotted Aloha under MPR.

Delay-sensitive applications are widely used in wireless networks. Such applications include data collection between mobile nodes, real-time monitoring or surveillance, and real-time data broadcasting over wireless networks. For example, safety-related messages are broadcasted among vehicles for safety and comport in wireless

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vehicular networks and these messages are required to be delivered to the neighboring vehicles within a given delivery deadline. By allowing the receiver to be equipped with MPR-capability, it is expected that delay-sensitive applications can be serviced better. It is therefore required to investigate the impact of MPR-capability on the performance of deadline-constrained traffic.

In this paper, we consider a specific MPR channel, namely the M-MPR channel as adopted in [7]-[16], in which the receiver is able to correctly decode up M packets transmitted simultaneously by different nodes and otherwise all the packets are lost due to collisions. This paper investigates the successful delivery probability (SDP), which was introduced in [15, 17, 18] and is defined as the long-run proportion of packets delivered to the receiver in time, of the slotted Aloha under M-MPR channel model in the scenario that wireless devices transmit packets with a delivery deadline to the receiver. Such a scenario can be safety message dissemination in vehicular networks [19] or data collection in wireless sensor networks or machine-to-machine communications [20]. In general, the delivery deadline of a packet may not be constant and varies per packet due to the following reasons; the type of packet (emergent or not) and the processing time. The SDP is a suitable measure for evaluating the performance of deadline-constrained traffic.

Even if the performance of slotted Aloha access scheme has been extensively investigated during several decades, there is a lack of research on how well the slotted Aloha scheme works under deadline-constrained traffic and MPR scenario. The focus of this paper is to investigate the performance (in terms of SDP) of slotted Aloha under deadline-constrained traffic and MPR. The contributions of this paper are summarized as follows:

- We derive the closed-form expression for SDP of the slotted Aloha under M-MPR channel model when the delivery deadline of a packet follows a general probability distribution.
- The result shows that when the transmission probability is not chosen optimally, the SDP decreases as the variance of the delivery deadline increases while the mean delivery deadline is fixed.
- We show that choosing the transmission probability optimally can mitigate the negative effect of the variance of the delivery deadline on the SDP. As a result, almost the same maximum SDP can be achieved regardless of the variance of the delivery deadline.

The slotted Aloha is a simple access scheme which has been widely used for distributed channel access since Abramson's seminal work [1]. A lot of studies have been carried out from various perspective for evaluating the performance of the slotted Aloha under the assumptions of single packet reception or MPR. Ghez et al.[3, 4] first considered a slotted Aloha access protocol under a general MPR channel model. They dealt with the stability issue for slotted Aloha based network under the assumptions of MPR and infinite nodes. Naware et al.[2] extended the studies [3, 4] to Aloha based network under MPR and finite nodes. Chan et al. [10] extended the studies [3, 4] to CSMA (Carrier Sensing Multiple Access) based network under MPR. Gau et al. [5, 6] investigated the throughput performance of slotted Aloha under M-MPR channel model. Zhang et al. [7] showed that for the slotted Aloha with M-MPR capability the saturation throughput increases superlinearly with M. In [8], Zhang et al. showed that the superlinear scaling law also holds under bounded-delay moment requirements. In [16], Bae et al. derived the optimal transmission probability which maximizes the saturation throughput of the slotted Aloha under M-MPR channel model. However, all the studies mentioned so far focused on either the stability issue or throughput for the slotted Aloha under MPR.

Time-critical broadcasting services with strict delivery deadline constraint are widely used for data collection between mobile nodes and real-time data broadcasting over wireless networks. For instance, the dissemination of safety messages in vehicular network can benefit from deadline-constrained broadcasting [9, 11]. Campolo et al. [12, 13] considered IEEE 802.11p [14] based vehicular network in the scenario that each node tries to send periodically-generated safety messages to its neighbors and each safety message has a fixed delivery deadline constraint. They derived the SDP of safety messages.

Perhaps the closest to our work are [15] and [18]. Zhang et al. [15] derived the optimal transmission probability which maximizes the SDP of the slotted Aloha under M-MPR channel. Their results are derived based on the restrictive assumptions of saturated traffic condition and constant delivery deadline. The delivery deadline defined in [15] is just the head-of-line delay, not the whole sojourn time in the system. Bae [18] derived the SDP of the slotted Aloha under the assumptions of single packet reception and constant delivery deadline. Accordingly, this work generalizes the studies [15] and [18] in that we consider the general delivery deadline model and MPR channel model.

The rest of this work is organized as follows. In Section 2, system model is presented. In Section 3. we develop a Markov chain model for a single node and derive the SDP by using the stationary distribution of the Markov chain. Numerical results are presented in Section 4 and Section 5 concludes this paper.

2. System Model

2.1. Network Model We consider a fully-connected one-hop wireless network consisting of N transmitters (or nodes) and a receiver. All the nodes are within the transmission range of each other and try to send their data packets to the common receiver. A single wireless channel is shared among the nodes. The channel time is divided into time slots of equal length and every transmitted packet occupies the duration of one time slot.

A node l, $1 \leq l \leq N$, has a data queue for storing data packets. We assume that data packets arrive to the node l queue according to a Bernoulli process with mean λ_l. That is, one packet arrives with probability λ_l or not with probability $1 - \lambda_l$ in a time slot. The packet arrival processes are assumed to be independent across the nodes.

Each node generates deadline-constrained data traffic. Each packet has a delivery deadline within which it should be transmitted successfully to the receiver. The delivery deadline X of a data packet may vary in general. We assume that the delivery deadline X of a data packet is a random variable independent and identically distributed across the data packets and the nodes. Let $q_n = \mathrm{P}(X = n)$ $1 \leq n \leq D$ be the probability mass function of X where D is the maximum delivery deadline. Data packets exceeding the delivery deadline X are useless and thus are dropped from the data queue even when they wait in the queue or are in service.

As adopted in [7, 8, 15, 16], the receiver is assumed to have an M MPR-capability, all-or-nothing model. That is, the receiver is able to correctly decode all the packets if not more than M nodes transmit simultaneously in a slot. Otherwise, if more than M nodes transmit in a slot at the same time, collisions occur and any data packet can not be decoded by the receiver. We assume that there is no channel error and the only source of packet loss is collisions. A transmitted data packet is neither acknowledged by the receiver nor retransmitted even if a transmission failure occurs.

The node l communicates with the receiver based on the slotted-Aloha with a fixed transmission probability μ_l $0 < \mu_l \leq 1$. In other words, the node l which has data packets to send attempts to transmit a data packet with probability μ_l at the beginning of a time slot or not with probability $1 - \mu_l$.

2.2. Performance Metric Successful delivery probability (SDP) is considered as a performance metric for evaluating a time-critical traffic as in [15, 17, 18]. The SDP of the node l is defined as the probability that a packet generated by the node l will be successfully delivered to the receiver within the delivery deadline X. Consider the tagged node l. Let $A_l(t)$ be the total number of packets arriving in the tagged node's queue during the time window $[0,\ t]$. Let $S_l(t)$ be the number of packets which are successfully delivered to the receiver by the tagged node during the time window $[0,\ t]$. The SDP, denoted by P_{sdp}, of the node l is then defined as follows:

$$P_{sdp} := \lim_{t\to\infty} \frac{\mathrm{E}[S_l(t)]}{\mathrm{E}[A_l(t)]} = \lim_{t\to\infty} \frac{\mathrm{E}[S_l(t)]/t}{\lambda_l}, \tag{1}$$

The second equality comes from the fact $\mathrm{E}[A_l(t)] = \lambda_l t$. Hence, the SDP means the long-run fraction of packets which are successfully delivered to the receiver by the node l before the expiry of delivery deadline of a packet.

3. Mathematical Analysis

For discrete-time modeling, the following assumptions are made in a slot t:

- A packet arrival, if any, occurs at the beginning of slot t.
- A packet departure (either successful transmission or loss due to the expiry of delivery deadline), if any, occurs at the end of slot t
- We observe the stochastic behavior of the system immediately before the end of slot t.

Consider the node l, $1 \le l \le N$. For the node l, we define S_t^l as the elapsed sojourn time (including the waiting time in the queue and the elapsed service time) of the leading packet (the packet in the head of line) in service, if any, immediately before the end of slot t. Then, $S_t^l \in \{0, 1, 2, \cdots, D\}$ because the maximum delivery deadline is equal to D. '$S_t^l = 0$' denotes the state of empty queue and '$S_t^l = i$' denotes the state in which the queue of the node l is not empty and the sojourn time of the leading packet is equal to i. Then, it is easy to see that $\{(S_t^1, \cdots, S_t^N) :\ t = 0, 1, 2, \cdots\}$ forms a Markov chain. It is worth noting that $S_t^1, \cdots, S_t^N$ are independent across the nodes because any collided packet is never retransmitted so that for each node packet arrival and transmission processes evolve independently over time. As a result, we only need to analyze the single process S_t^l for the node l.

3.1. Markov Chain Model for a Single Node We assume homogeneous nodes in that $\lambda_l = \lambda$ and $\mu_l = \mu$ for all nodes.[1)] We choose an arbitrary node among N homogeneous nodes, called it the *tagged node*. Consider the Markov chain $\{S_t : t = 0, 1, 2, \cdots\}$ for the tagged node. Suppose that $S_t = i$ in slot t. Let $\boldsymbol{P}$ be the transition probability matrix of the Markov chain, whose (i, j) component is denoted by $P_{i,j} = \mathrm{P}(S_{t+1} = j | S_t = i)$. For convenience, we introduce the following probabilities:

- $r_n = \sum_{k=n}^{D} q_k$; the probability that an arriving packet has a deadline not less than n.
- $h_n = \frac{q_n}{r_n}$; the probability that the packet delivery deadline equal to n, given that the packet deadline is not less than n
- $\lambda_n = \lambda r_n$; the probability that a packet arrives in a slot and its delivery deadline is larger or equal to n

In what follows, we denote $\bar{a} = 1 - a$ for $0 \leq a \leq 1$. Then, the one-step transition probabilities $P_{i,j},\ 0 \leq i, j \leq D$ are given as follows:

- For $i = 0$, $P_{0,0} = \bar{\lambda}$, the probability of the event that there is no packet arrival at the beginning of slot $t + 1$; $P_{0,1} = \lambda$, the probability of the event that a packet arrives at the beginning of slot $t + 1$.
- For $1 \leq i \leq D-1$, $P_{i,i+1} = \bar{h}_i \bar{\mu}$, the probability of the event that i) the deadline of the leading packet is larger than i, given its elapsed deadline i, i.e., its deadline is not less than i, and ii) the leading packet is not transmitted at the slot.

  $P_{i,i-k} = (h_i + \bar{h}_i \mu)\lambda_{i-k} \Pi_{n=i-k+1}^{i} (1 - \lambda_n)$, $0 \leq k \leq i$ with the conventions that $\prod_{n}^{i}(1 - \lambda_n) = 1$ when $n > i$, $\lambda_0 = 1$, and $h_0 = 1$. This represents the probability of the event that i) the leading packet is removed from the queue due to either transmission or the expiry of deadline; ii) a packet arrived in the $k + 1$-th slot since the arrival of the leading packet and its deadline is larger or equal to $i - k$; iii) all the other packets, if any, which arrived before the packet arrived in the $k + 1$-th slot, have been dropped from the queue due to the expiry of deadline. Note that some packets arrived earlier than the packet may be dropped from the queue because the delivery deadline is a random variable.
- For $i = D$, $P_{D,D-k} = \lambda_{D-k} \Pi_{n=D-k+1}^{D} (1 - \lambda_n)$ for $0 \leq k \leq D$ with the conventions that $\prod_{n}^{i}(1 - \lambda_n) = 1$ when $n > i$ and $\lambda_0 = 1$.

[1)] Our developed model can be straightforwardly extended to the heterogeneous nodes case.

Let $\pi_i = \lim_{t\to\infty} \mathrm{P}(S_t = i)$. We denote by $\boldsymbol{\pi} = (\pi_0, \pi_1, \cdots, \pi_D)$ the steady-state probability vector of the Markov chain. Then, the balance equation $\boldsymbol{\pi} P = \boldsymbol{\pi}$ yields

$$\pi_0 = \bar{\lambda}\pi_0 + \sum_{k=1}^{D}\left[(h_k + \bar{h}_k\mu)\prod_{n=1}^{k}(1-\lambda_n)\right]\pi_k \tag{2}$$

$$\pi_1 = \lambda\pi_0 + \lambda_1\sum_{k=1}^{D}\left[(h_k + \bar{h}_k\mu)\prod_{n=2}^{k}(1-\lambda_n)\right]\pi_k \tag{3}$$

$$\pi_i = \bar{h}_{i-1}\bar{\mu}\pi_{i-1} + \lambda_i\sum_{k=i}^{D}\left[(h_k + \bar{h}_k\mu)\prod_{n=i+1}^{k}(1-\lambda_n)\right]\pi_k, \quad 2 \le i \le D, \tag{4}$$

where $h_D + \bar{h}_D\mu = 1$ because $h_D = 1$ and we use the convention that $\prod_n^k(1-\lambda_n) = 1$ if $n > k$. The equations (2), (3) and (4) are simplified in the next lemma.

Lemma 1. *For $2 \le i \le D$, we have*

$$\pi_i = \sum_{k=i}^{D}\left[(h_k + \bar{h}_k\mu)\prod_{n=i+1}^{k}(1-\lambda_n)\right]\pi_k \tag{5}$$

$$\pi_i = \frac{\bar{h}_{i-1}\bar{\mu}}{1-\lambda_i}\pi_{i-1}. \tag{6}$$

Proof. We show it inductively backward starting from $i = D$ using (2), (3) and (4). If $i = D$, The left-hand side of (5) becomes $(h_D + \bar{h}_D\mu)\pi_D = \pi_D$ because $h_D + \bar{h}_D\mu = 1$ and thus (5) holds for $i = D$. Also, (6) holds from (4) when $i = D$. Suppose that (5) and (6) hold for $2 < i+1 < D$. For i, we have

$$\begin{aligned}
&\sum_{k=i}^{D}\left[(h_k + \bar{h}_k\mu)\prod_{n=i+1}^{k}(1-\lambda_n)\right]\pi_k \\
=\;& (h_i + \bar{h}_i\mu)\pi_i + \sum_{k=i+1}^{D}\left[(h_k + \bar{h}_k\mu)\prod_{n=i+1}^{k}(1-\lambda_n)\right]\pi_k \\
=\;& (h_i + \bar{h}_i\mu)\pi_i + (1-\lambda_{i+1})\pi_{i+1} \\
=\;& (h_i + \bar{h}_i\mu)\pi_i + \bar{h}_i\bar{\mu}\pi_i \\
=\;& \pi_i,
\end{aligned}$$

where the second equality comes from the induction hypothesis (5) for $i+1$ and the third equality from the induction hypothesis (6) for $i+1$. This proves the result (5) for i. Combining this result and (4) leads to (6) for i. □

Using (5) and (6), from (3) we easily get $\pi_1 = \frac{\lambda}{\bar{\lambda}}\pi_0$ since $\lambda_1 = \lambda$. Hence, the steady-state probabilities of the Markov chain are derived as

$$\pi_i = \lambda_1(\bar{\mu})^{i-1}\prod_{n=1}^{i}\frac{\bar{h}_{n-1}}{\bar{\lambda}_n}\pi_0,\ 1 \leq i \leq D \tag{7}$$

$$\pi_0 = \left[1 + \lambda_1\sum_{i=1}^{D}\left((\bar{\mu})^{i-1}\prod_{n=1}^{i}\frac{\bar{h}_{n-1}}{\bar{\lambda}_n}\right)\right]^{-1}, \tag{8}$$

where we use the convention $\bar{h}_0 = 1$ and (8) comes from the normalization condition

$$\sum_{i=0}^{D}\pi_i = 1.$$

3.2. Successful Delivery Probability To derive the SDP given by (1), of the tagged node, we first drive the conditional successful transmission probability, denoted by P_s, which is defined as the probability that given the tagged node makes a transmission attempt in a slot, the result is successful without collisions. Under M-MPR channel model, a transmitted packet will be successfully delivered to the receiver if there is no collision, i.e., not more than M nodes transmit simultaneously in a slot. Note that an arbitrary node independently makes a transmission attempt in a slot with probability $\bar{\pi}_0\mu$ because of the assumption of homogeneous nodes. Therefore, given the tagged node makes a transmission attempt, the conditional successful probability P_s of the packet is given by

$$P_s = \sum_{k=0}^{M-1}\binom{N-1}{k}(\bar{\pi}_0\mu)^k(1-\bar{\pi}_0\mu)^{N-k-1}. \tag{9}$$

The SDP given by (1) is then the ratio between i) the average number of packets successfully delivered to the receiver by the tagged node in a slot before the deadline and ii) the average number of packets arriving to the tagged node's queue in a slot. Hence, the SDP P_{sdp} is obtained as

$$P_{sdp} = \frac{\bar{\pi}_0\mu P_s}{\lambda}, \tag{10}$$

where P_s is given by (9). The optimal transmission probability μ^* which maximizes the SDP P_{sdp} is then obtained as

$$\arg\max_{\mu\in(0,1]} P_{sdp}. \tag{11}$$

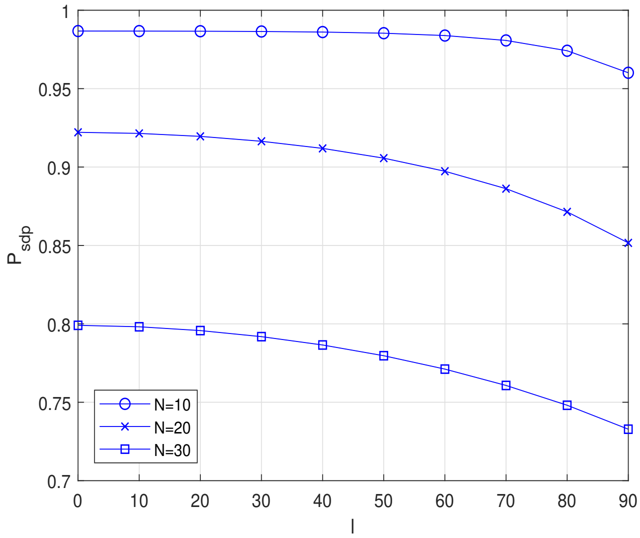

Figure 1. Successful delivery probability for the varying l when $\lambda = 0.02$, $M = 2$ and $\mu = 1/N$

4. Numerical Results

For numerical examples, system parameters are set as follows:

- The mean delivery deadline is set to $\mathrm{E}[X] = 100$ slots.
- The packet arrival probability is set to $\lambda = 0.02$, i.e., the mean of packet inter-arrival time is equal to 50 slots.
- The delivery deadline X is uniformly distributed on $\{n : 100 - l \leq n \leq 100 + l\}$ for some integer $0 \leq l \leq 99$, i.e.,

$$q_n = 1/(2l + 1),\ 100 - l \leq n \leq 100 + l \tag{12}$$

In this model, the maximum delivery deadline D is equal to $D = 100 + l$ and $\mathrm{E}[X] = 100$. When $l = 0$, this model reduces the deterministic model, i.e., $\mathrm{P}(X = 100) = 1$. As l increases from 0 to 99, the variance of X increases while the mean delivery deadline is always fixed.

Fig.1 shows the SDP as a function of l, which determines the probability distribution of the delivery deadline X and is given by (12), for different values of M when $\lambda = 0.02$, $M = 2$ and $\mu = 1/N$. Note that a large l leads to large variance of the delivery deadline while the mean deadline is fixed. For each N, the SDP has its

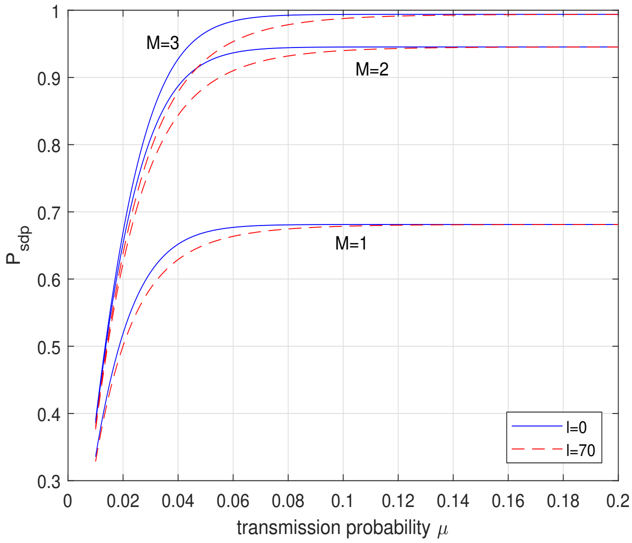

Figure 2. Successful delivery probability versus μ for different values of M and l when $\lambda = 0.02$

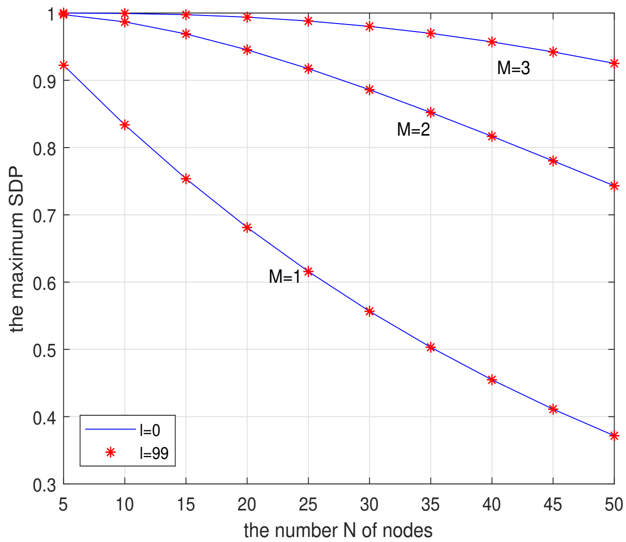

Figure 3. The maximum successful delivery probability as a function of N for different values of M and l when $\lambda = 0.02$

maximum when $l = 0$, i.e., the delivery deadline is deterministic, and then decreases as the variance of the delivery deadline increases. Therefore, we see that increasing the variance of the delivery deadline causes an additional loss in terms of SDP. On the one hand, we see that the SDP decreases sharply with the increase of N. This is because the collided packet is never acknowledged in our model and μ is simply set to $1/N$, which is not the optimal one.

Fig.2 plots the SDP versus μ for $M = 1, 2, 3$ and $l = 0, 70$ when $\lambda = 0.02$. For each M, we notice that a smaller l leads to a higher SDP regardless of μ. We also observe that the SDP increases for both cases $l = 0$ and $l = 70$ as μ increases. It is worth noting that the gap of SDP between both cases $l = 0$ and $l = 70$ can be minimized by optimally setting the respective μ. This result provides a motivation to design the optimal transmission probability which maximizes the SDP. In addition, as expected, it is seen that a larger M guarantees a higher SDP.

Fig.3 shows the maximum SDP, which is evaluated at the corresponding optimal transmission probability μ^* given by (11), as a function of N for the different values of M and l when $\lambda = 0.02$. Note that in our delivery deadline model $l = 0$ and $l = 99$ correspond to the lowest and the largest variances, respectively. Different from the result in Fig.1, it is interesting to see that there is little difference between the two SDPs for both $l = 0$ and $l = 99$. This result implies that choosing the transmission probability optimally can mitigate the negative effect of the uncertainty of the delivery deadline on the SDP. We also notice that with only $M = 3$ the maximum SDP is larger than 0.9 even in a highly competing scenario. This gives a strong motivation for the receiver to be equipped with MPR-capability.

5. Conclusions

We provided an analytical model for evaluating the SDP of the slotted Aloha under general delivery deadline constraint and MPR channel. The SDP was derived in a closed-form. The result showed that when the transmission probability is not selected optimally, the variance of the delivery deadline had a negative effect on the SDP. This negative effect of the variance of the deadline can be mitigated by choosing the transmission probability optimally.

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